Habit Formation and Evidence from Micro Data

Zongwu Cai
Department of Mathematics and Statistics
Department of Economics, University of North Carolina at Charlotte
Haiyong Liu
Department of Economics, East Carolina University
XuanLiu
Department of Economics, East Carolina University

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Abstract

Given a model with habit formation, the response of a household's share of risky assets to changes of wealth is ambiguous. The direction of this response depends on the endowment flow of households and the level of habit persistence.

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JEL classification: E21; E24; D91

1 Introduction

Habit formation is widely used in the areas of finance and macroeconomics. For example, Jermann (1998) and Boldrin et al. (2001) adopt habit formation to jointly match both business cycle moments and the equity returns. Campbell and Cochrane (1999) use habit formation model to replicate the observed mean and countercyclicality of equity premia. As we turn to micro data to find empirical evidence that supports the notion of habit formation, the findings are mixed. Brunnermeier and Nagel (2008) conclude that the habit formation preferences are not supported by the micro data. Specifically their theoretical model with habit formation predicts that the share of risky assets will negatively respond to a decrease in today's wealth. Their econometric results, however, fail to show such a negative response in the micro data. Nevertheless, opposite findings are presented by Wachter and Yogo (2009), who are able to match the observed patterns in consumption and portfolio choice to the model predictions. The objective of this note is to underline the relation between the level of endowments and the proportion of risky assets in household investment portfolio in response to a change in the level of liquid wealth. With the existence of persistent habits and time-varying endowments, we find that the share of risky assets in household investment portfolio responds to a wealth change in an ambiguous way. The direction of the response depends on the endowment flow of households, as well as the intensity of habit. This finding reconciles aforementioned inconsistencies in the literature and also provides a promising framework to examine habit formation models with micro level data.

2 Model 1: An endowment economy with risky asset

We first develop a simple portfolio choice model, where a household chooses a risky asset, a, to smooth their consumption in a three-period economy. Its return, r, can be only known in period 2. The household receives time-varying endowment flows: y_1 in period 1 and y_2 in period 2. The households consume c_1 in the first period, and consume c_2 in the second period. Formally, the problem is to choose a, c_1, and c_2 to maximize the expected utility

$$\max \mathbb{E}_1 U \quad = \quad \log(c_1 - xc_0) + \mathbb{E}_1 \beta \log(c_2 - xc_1), \tag{1}$$

where β is a discount factor and x is the habit coefficient. We therefore assign infinitely negative utility to consumption choices with $c_t \leq xc_{t-1}, t = 1, 2$. Note that our specification of the habit, xc_{t-1}, may be time-varying while Brunnermeier and Nagel (2008) assume it to be constant. Unlike our model and that of Wachter and Yogo (2009), moreover, Brunnermeier and Nagel (2008) do not allow endowment flows, even though both of their studies control for current and lagged endowment flows (labor incomes) in empirical estimations. The period budget constraints in periods 1 and 2 are given respectively by $c_1 = y_1 - a$ and $c_2 = y_2 + ra$.

To solve the optimization problem, we first set up the Lagrangian as

$$\mathbb{L} = \log(c_1 - xc_0) + \mathbb{E}_1 \beta \log(c_2 - xc_1) + \lambda (y_1 - c_1 - a) + \mathbb{E}_1 \mu (y_2 - c_2 + ra)$$

The optimality conditions are then given by

$$\begin{aligned} \frac{1}{c_1 - xc_0} - \mathbb{E}_1 \frac{\beta x}{c_2 - xc_1} &= \lambda, \\ \frac{\beta}{c_2 - xc_1} &= \mu, \\ \mathbb{E}_1 \mu r &= \lambda, \end{aligned}$$

which can be rearranged as follows:

$$\begin{aligned} \mathbb{E}_1 \frac{\beta r}{c_2 - xc_1} &= \frac{1}{c_1 - xc_0} - \mathbb{E}_1 \frac{\beta x}{c_2 - xc_1}, \\ \Rightarrow \mathbb{E}_1 \frac{\beta(r+x)}{c_2 - xc_1} &= \frac{1}{c_1 - xc_0} \end{aligned}$$

To simplify the discussion, we assume that r has two possible values of equal probability, r^{H}, and r^{L}. Thus, we can rewrite the above equations as

$$\frac{\beta(r^{\mathrm{H}} + x)}{(y_2 - xy_1) + (r^{\mathrm{H}} + x)\, a} + \frac{\beta(r^{\mathrm{L}} + x)}{(y_2 - xy_1) + (r^{\mathrm{L}} + x)\, a} = \frac{2}{y_1 - xc_0 - a}$$

The left hand side can be written as

$$\begin{aligned} \mathrm{LHS} &= \frac{\beta(r^{\mathrm{H}} + x)\left[(y_2 - xy_1) + \left(r^{\mathrm{L}} + x\right) a\right] + \beta(r^{\mathrm{L}} + x)\left[(y_2 - xy_1) + \left(r^{\mathrm{H}} + x\right) a\right]}{\left[(y_2 - xy_1) + (r^{\mathrm{H}} + x)\, a\right]\left[(y_2 - xy_1) + (r^{\mathrm{L}} + x)\, a\right]} \\ &= \frac{\beta(r^{\mathrm{H}} + r^{\mathrm{L}} + 2x)\,(y_2 - xy_1) + \beta(r^{\mathrm{H}} + x)(r^{\mathrm{L}} + x)a}{\left[(y_2 - xy_1) + (r^{\mathrm{H}} + x)\, a\right]\left[(y_2 - xy_1) + (r^{\mathrm{L}} + x)\, a\right]}. \end{aligned}$$

Thus

$$\frac{\beta(r^{\mathrm{H}} + r^{\mathrm{L}} + 2x)\,(y_2 - xy_1) + \beta(r^{\mathrm{H}} + x)(r^{\mathrm{L}} + x)a}{\left[(y_2 - xy_1) + (r^{\mathrm{H}} + x)\, a\right]\left[(y_2 - xy_1) + (r^{\mathrm{L}} + x)\, a\right]} = \frac{2}{y_1 - xc_0 - a}$$

This equation about a can be solved numerically.

2.1 Numerical results

In our numerical exercise, we set 9 possible values for y_1 and y_2, which are evenly distributed between [20000 : 180000], and 9 possible values for x between [0.1 : 0.9]. Fig 1 shows the results on the risky

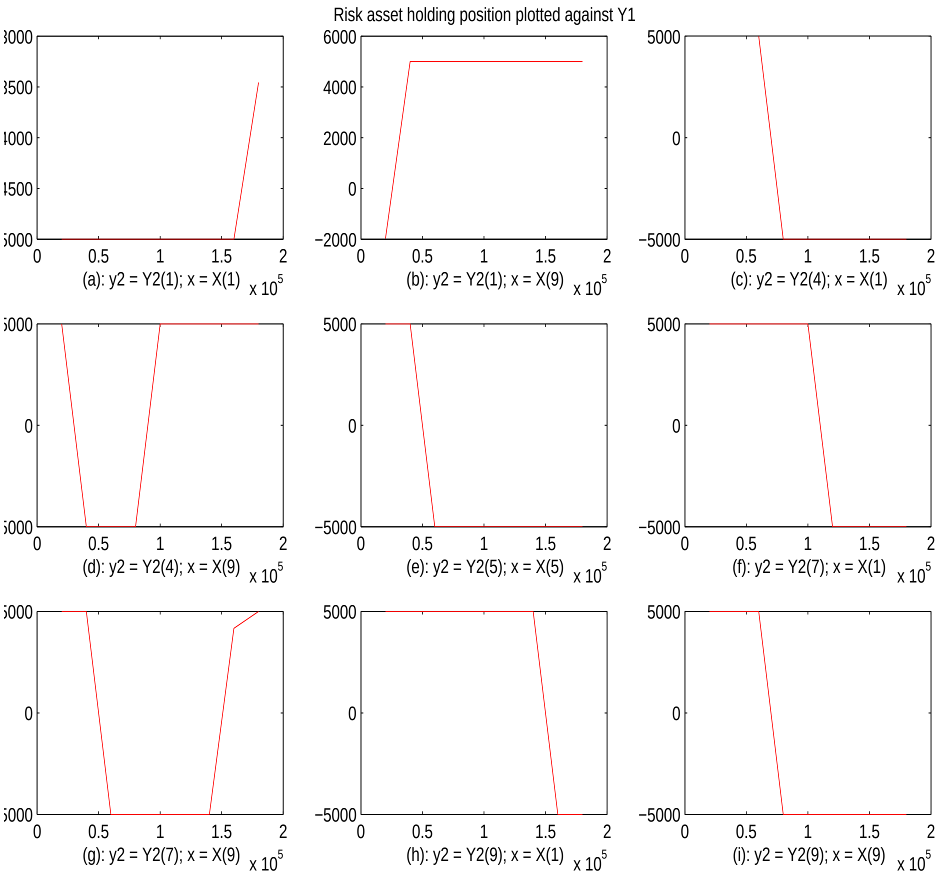

Figure 1: Nine possible scenarios

asset holding position in nine possible scenarios. The vertical axis represents the holding position of risky asset. The horizontal axis represents today's endowment. In each panel, we plot the risky asset holding position against $Y1$, a vector describing the possible values of today's endowment, for any given y_2 and x. In this picture, X denotes a vector of possible values for the habit formation parameter, which ranges from 0.1 to 0.9. If $x = X(1)$, the first element of X and its lowest value, it implies that the household puts less emphasis on its habit. $Y2$ denotes a vector with possible states of tomorrow's endowment, whose values range from 0.1 to 0.9. When $y_2 = Y2(1)$, the first element of $Y2$, it indicates that tomorrow's endowment will be low and the household is likely to be a lender. When $y_2 = Y2(9)$, the last element of $Y2$, it indicates that tomorrow's endowment is going to be high and the household is likely to be a borrower.

When tomorrow's endowment is low, the holding position will increase and it does not depend on the value of x, as shown in panel (a) and panel (b) from Figure 1. This is exactly the theoretical prediction of Brunnermeier and Nagel (2008). However, when tomorrow's endowment is high, the holding position will decrease and it does not depend on the value of x, as can be seen from panel (h) and panel (i) in Figure 1. This is contrary to the model prediction in Brunnermeier and Nagel (2008). In other scenarios, the predictions are mixed, depending on the value of y_2 and x.

In addition, we plot the risky asset holding position to y_1 ratio against y_1 in Fig 2. It shows that

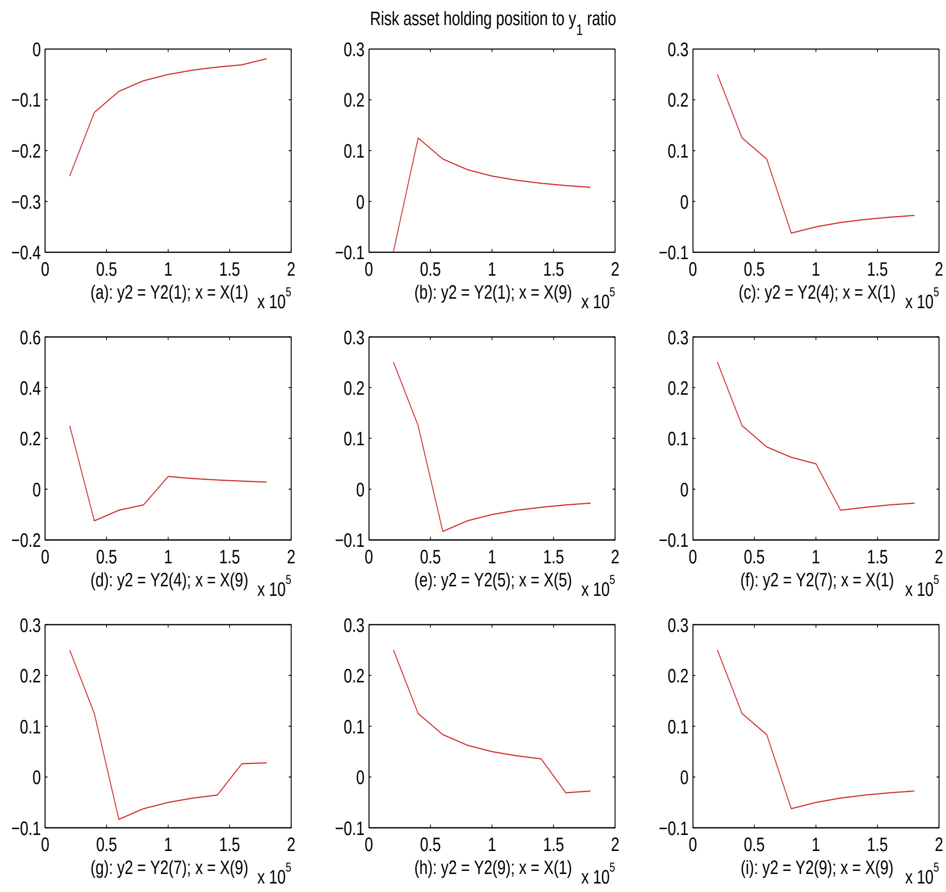

Figure 2: Nine possible scenarios

with the increase of y_1, the share spent on the risky asset will in general decline.

3 Discussion and Conclusion

There is a critical difference between our model and the model used by Brunnermeier and Nagel (2008). The latter rules out the possibility that households may become borrowers. There is an inherent discrepancy between their modeling constraint and empirical data from the Panel Study of Income Dynamics (PSID), in which households may be lenders or borrowers. Subsequently, the empirical results in Brunnermeier and Nagel (2008) suggest that the share of risky assets is not affected by liquid wealth changes, while their theoretical model implies the risky proportion moves in the same direction as the change of wealth. The simple model presented in previous section relaxes their modeling constraint by allowing households to be either borrowers or lenders. The predictions based on above numerical exercises indicate that the correlation between the share of risky assets and wealth is indeed ambiguous, depending on the magnitude of households' endowment flows as well as the habit formation coefficient. To test the robustness of our findings, we enrich our basic model by allowing households to invest in both risk-free and risky assets, and adding initial wealth to the model. Corresponding numerical results do not change our findings . Furthermore, we can demonstrate the importance of the value of the habit coefficient in generating time-varying risk aversion in lieu of wealth fluctuations. Suppose the habit formation preferences gives the following period utility function:

$$u_t = \log(c_t - x c_{t+1})$$

Households face an endowment today and an endowment tomorrow. They invest in assets which provide a return of R. Figure 3 plot the critical value of x against the endowment flow.[1] The critical value of x is defined as the following: if the true value of x is above that critical value for the given endowment flow, risk averse households like uncertainty of R; while if the true value of x is below that critical value for the given endowment flow; risk averse households dislike uncertainty of R.

This picture illustrates why Brunnermeier and Nagel (2008) predict a negative response. A very likely explanation is that the parameter values implied in their paper corresponds to an area, in which risk averse households always dislike uncertainty. For instance their model could characterize a household situated at point A in Figure 3, where their endowments are 2 and 0 for today and tomorrow respectively. In other words, it is possible that they only focus on a very special case.

[1] In calculating the critical value, we set the maximum value of x at 1.

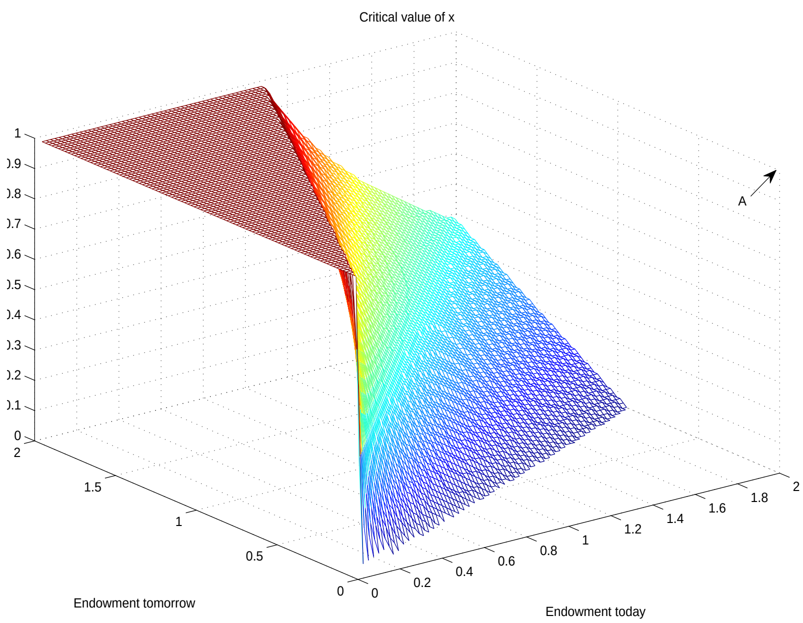

Figure 3: Critical value of x

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