

Time Consistency of Optimal Monetary and Fiscal Policy in a Small Open Economy

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Abstract

This paper characterizes conditions under which optimal monetary and fiscal policy is time consistent in a stylized small open economy with a flexible foreign exchange rate regime. It shows that these conditions depend on the way in which leisure is assumed to enter preferences and/or on the process which productivity is assumed to follow. This paper further argues that these conditions may fail to be sufficient if the small open economy implements a fixed foreign exchange rate regime. Thus, a credible fixed exchange rate regime does not necessarily help render optimal policy time consistent.

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The paper is available at: <http://sites.google.com/site/macroiinterest/Home/research/tcsoe>.

1 Introduction

Time inconsistency has been a central issue in the analysis of monetary and fiscal policy and has been widely studied. The literature on time consistency (hereafter TC) could at least be traced back to Kydland and Prescott (1977) and Barro and Gordon (1983), both of which have shown that when it has the opportunity to reoptimize, a government will have an incentive to renege on the policy announced by its predecessor. Time inconsistency has been regarded as one important reason for inflation bias [Kydland and Prescott (1977), Barro and Gordon (1983)], financial crises [Chari et al. (1998), Albanesi and Christiano (2001), Albanesi et al. (2003a)], and the failure of some monetary unions [Chari and Kehoe (2008)].² Besides, the 2010 sovereign debt crisis in Greece provides one more example indicating the importance and the empirical relevance of the TC issue, particularly in a small open economy with a fixed exchange rate regime (hereafter FERR).

To restore TC, different types of commitment technology have been proposed, such as rules [Kydland and Prescott (1977)], reputation [Backus and Driffill (1985)], “conservative” central bankers [Rogoff (1985)], incentive contracts with inflation targets [Svensson (1997)], etc.. Recent developments in the literature have emphasized the use of maturity structures of public debt as a commitment technology that is capable of rendering optimal monetary and fiscal policy (hereafter OMFP) time consistent in a closed economy.³ For example, both Alvarez et al. (2004) and Persson et al. (2006), building on the work of Lucas and Stokey (1983), Persson et al. (1987) and Calvo and Obstfeld (1990), show OMFP is time consistent in a closed economy by imposing appropriate restrictions on the maturity structure of public debt. These restrictions are designed to neutralize both the incentive of using “surprise inflation” to finance public spending, an incentive that was formally analyzed in Calvo (1978), and the incentive of devaluing inflation-indexed debt, an incentive discussed in Lucas and Stokey (1983).⁴

In this paper, we argue that the aforementioned commitment technology breaks down in a

stylized small open economy with conventional debt instruments. In particular, whether the commitment technology is capable of rendering time consistent OMFP in this economy depends on the way in which leisure is assumed to enter preferences and on the process which productivity is assumed to follow. One important point of departure of our analysis is that we formally explore the role played by the labor market in determining TC of OMFP. Specifically, we study the restriction imposed by the requirement of time consistent policy continuation (hereafter TCPC) of hours, or labor input, on TC. Such a restriction arises because the period resource constraint does not necessarily hold with equality in a small open economy so that a government has fewer implicit policy instruments to use.⁵

The main findings are as follows. First, the commitment technology discussed in Alvarez et al. (2004) and Persson et al. (2006) is not sufficient any more for TC in a small open economy with a flexible FERR.⁶ The reason for time inconsistency is that the period resource constraint does not necessarily hold in each period so that the government has fewer implicit policy instruments to use to guarantee TC of OMFP than in a closed economy. As a result, for any chosen maturity structure of public and external debt, there exist two forces imposing orthogonal restrictions on marginal financing costs across governments⁷. On the one hand, in order to have TCPC of consumption and real money balances, it requires that marginal financing costs vary across governments. On the other hand, the requirement of TCPC of hours requires constant marginal financing costs across governments. Since these requirements are orthogonal to each other, there does not exist a maturity structure of public and external debt that is capable of rendering OMFP time consistent.

Second, prompted by the failure of the aforementioned commitment technology in this economy, we further explore additional conditions *within* the model under which, if combined with the aforementioned technology, will be sufficient to render OMFP time consistent. For this purpose, we generalize the formal analysis of TC by directly exploring the impact of the requirement of

TCPC of hours on the TC property of OMFP. With the generalized method, we find that when preferences and/or productivity processes satisfy certain restrictive conditions, the requirement of TCPC of hours will not impose the constant marginal financing cost requirement any more, and OMFP will become time consistent. In this case, there exist many maturity structures of external and public debt capable of rendering OMFP time consistent.

Third, these conditions, which are sufficient in the case of a flexible FERR, may fail to be sufficient in the case of a fixed FERR. In the latter case, the monetary economy effectively reduces to a real economy and the aforementioned orthogonal restrictions still exist. The difference is that now both the money market and the labor market jointly impose an orthogonal restriction on marginal financing costs from that of the good market. The reason behind the new role of the money market is that the government cannot manipulate nominal debt to affect optimal real money balances when nominal interest rates are exogenously given. This finding is related to those in Persson and Svensson (1986), Huber (1992), and Liu (2011), and we discuss the differences among them in Section 3. As a result, OMFP is time inconsistent even if the fixed FERR itself is credible. This result extends the existing understanding about the relationship between a credible fixed FERR and TC of policies: according to Kydland and Prescott (1977), rules such as a credible fixed FERR will assure TC of monetary policy.

The rest of this paper is organized as follows: Section 2 explores TC of OMFP in a stylized small open economy with a flexible FERR. Section 3 studies the same problem in the same stylized small open economy but with a fixed FERR. Section 4 discusses policy implications. And Section 5 concludes.

2 Time Consistency under A Flexible FERR

In this section, we study whether the time-inconsistent incentive of the government in a small open economy could be neutralized by choosing an appropriate maturity structure of public and external debt. The model is a highly stylized small open economy model: a representative household with separable concave preferences over consumption, real money balances and leisure, perfect competitive firms with constant return to scale technology, a Ramsey (benevolent) government that maximizes households' utility, one tradable good, perfect capital mobility, perfect foresight, and a credible flexible FERR.⁸ This economy can be regarded as a small open economy counterpart of that in Persson et al. (2006).

2.1 The Model

2.1.1 The Representative Household

In this economy, a representative household is given the price of consumption good, p_t , the price of the net claim on consumption good, q_t , the labor income tax rate, τ_t , and the nominal interest rate, i_{t+1} .⁹ It chooses the time profile of consumption, c_t , real money balances, m_{t+1} , and hours, h_t , to maximize its lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, m_t, h_t),$$

where β denotes the subjective discount factor, which weights consumption bundles over time. Here we restrict the discussion to the case in which the period utility function is separable, $u(c, m, h) = u(c) + v(m) + g(h)$.¹⁰ In addition to the usual concavity assumption of the utility function, we further assume that $u_{mt} \geq 0$ to guarantee that nominal interest rates are always non-negative in the equilibrium.

We impose Svensson timing of markets, which means that the good market meets before the financial market in each period [Svensson (1985)]. Real money balances are defined by $m_t = \frac{M_{t-1}}{p_t}$, where M denotes the nominal money balance. There are several reasons for the use of Svensson timing. First, it introduces a cost of inflation. When the price level suddenly increases (higher than expected) due to the re-optimization of the new government, the real money balance will decrease, and the household will receive less utility from the given nominal money balance. One way to understand such a cost is that inflation forces households to economize money thus bringing costs to holding money [Bailey (1956) and Tower (1971)]. Another rationale is that it gives the government more flexibility, compared to the use of Lucas timing, in choosing the maturity structure of public debt to guarantee TC.¹¹ For example, TC requires that the nominal public debt that will mature at each and every period be zero in Alvarez et al. (2004) which assume Lucas timing; while it does not impose such a restrictive requirement in Persson et al. (2006) which assume Svensson timing.

The representative household is subject to the period budget constraint, which is given by:

$$\begin{aligned} & q_t \left[(1 - \tau_t) w_t h_t + \frac{M_{t-1}}{p_t} \right] + \sum_{s=t}^{\infty} q_s \left({}_{t-1}b_s^P + \frac{{}_{t-1}B_s^P}{p_s} \right) \\ & \geq q_t \left(c_t + \frac{M_t}{p_t} \right) + \sum_{s=t+1}^{\infty} q_s \left({}_t b_s^P + \frac{{}_t B_s^P}{p_s} \right). \end{aligned} \quad (1)$$

w_t denote the real wage rate. $({}_{t-1}b_s^P)$ denotes net claims by the domestic household when entering period t on the amount of goods to be delivered in period s . The superscript P means that the bond is held by the households. These bonds are real because their purchasing power will not change when the domestic price level changes. $({}_{t-1}B_s^P)$ denotes the net claims on money to be delivered in period s . The sum $\sum_{s=t}^{\infty} q_s \left({}_{t-1}b_s^P + \frac{{}_{t-1}B_s^P}{p_s} \right)$ denotes the representative household's initial bond holding position. We apply the same discount factor on both internal and external bonds because

of perfect capital mobility.

The representative household is also subject to the no-Ponzi game condition:

$$\lim_{j \rightarrow \infty} \left[q_{t+j} \frac{M_{t+j}}{p_{t+j}} + \sum_{s=t+j+1}^{\infty} q_s \left({}_{t+j}b_s^P + \frac{{}_{t+j}B_s^P}{p_s} \right) \right] \geq 0, \quad \forall t \geq 0. \quad (2)$$

This condition means that the representative household has to keep non-negative financial assets in the limit and it must hold in each period.

In this economy, nominal interest rates are denoted by i_t . With a flexible FERR, i_t , $\forall t \geq 0$, is a policy choice and its value is chosen by the government as long as it is non-negative. Note that when the government commits to a fixed FERR, i_t will be exogenously determined, a case which will be discussed in Section 3. In this economy, we always have the following no-arbitrage condition:

$$\frac{q_{t+1}/p_{t+1}}{q_t/p_t} = \frac{1}{1 + i_{t+1}} \leq 1, t \geq 0. \quad (3)$$

Combining the period budget constraint and the no-Ponzi game condition, the intertemporal budget constraint of the representative household is given by:

$$\sum_{t=0}^{\infty} q_t [(1 - \tau_t)w_t h_t + \Pi_t] + \frac{M_{-1}}{p_0} + \sum_{t=0}^{\infty} q_t \left(-{}_1b_t^P + \frac{{}_1B_t^P}{p_t} \right) = \sum_{t=0}^{\infty} q_t c_t + \sum_{t=1}^{\infty} q_t m_t i_t. \quad (4)$$

It can be shown that the time sequences for $\{c_t, m_{t+1}, h_t\}_{t=0}^{\infty}$ satisfying the constraints (1) and (2) are the same as those satisfying the single constraint (4) [Woodford (1994)]. Thus, the representative household maximizes its lifetime utility function subject to the single constraint (4). Let λ denote the Lagrangian multiplier associated with Eq. (4). The optimality conditions for the representative

household are the single intertemporal budget constraint (4) and the following:

$$\beta^t u_{ct} = \lambda q_t, t \geq 0 \quad (5)$$

$$\tau_t = 1 + \frac{1}{w_t} \frac{u_{ht}}{u_{ct}}, t \geq 0 \quad (6)$$

$$i_{t+1} = \frac{u_{mt+1}}{u_{ct+1}}, t \geq 0. \quad (7)$$

Eq. (5) says that the marginal utility of consumption should equal the marginal cost of consumption; Eq. (6) shows that the introduction of labor income tax distorts the marginal rate of substitution between consumption and hours; and Eq. (7) states that there is a cost to holding money when nominal interest rates are strictly positive.

2.1.2 Competitive firms

In each period, perfect competitive firms use constant returns-to-scale technology in production, $y_t = z_t h_t, t \geq 0$. y_t denotes output. One point worth mentioning is the possibility of zero output in this small open economy. There are several ways to rule out that possibility. One way is to assume that the net foreign asset accumulated by the representative household is not large enough so that the household will work at any given real wage rate in each period. Another way is to assume decreasing returns-to-scale technology as in Schmitt-Grohé and Uribe (2003). As a result, the marginal product of labor at a low level of labor input will be extremely high and the possibility of a corner solution will be ruled out. However, in this case, firms will have non-zero profits which should be taxed. Such profit taxes will inevitably complicate the discussion of TC of OMFP. To simplify the discussion, we assume the former way. In a closed economy model, zero-output is not an issue because consumption, a part of output, is always positive with standard preferences under usual regularity conditions. z_t denotes the total factor productivity. It can be either time-varying or constant over time, but not stochastic in this paper.

Output is sold in both the domestic and the international good markets so that the “law of one price” for the one tradable good holds in each period:

$$p_t = S_t p^{**}, t \geq 0, \quad (8)$$

where S_t denotes the nominal exchange rate at time t and the variable p^{**} denotes the world price.¹²

Firms maximize profit:

$$\Pi_t = z_t h_t - w_t h_t, t \geq 0. \quad (9)$$

The optimality condition for labor demand is standard:

$$w_t = z_t, t \geq 0. \quad (10)$$

2.1.3 The government

The government finances an exogenous flow of time-varying government expenditures, g_t , by levying labor income taxes, by printing money and by trading multi-period nominal and real bonds with both domestic households and international investors. The monetary/fiscal regime consists of plans for the explicit policy instruments: money and bonds of different maturity dates; and for the policy choices: nominal interest rates and labor income tax rates.¹³ Here we assume that lump-sum taxes are not available to the government, a standard assumption in the literature [Alvarez et al. (2004) and Persson et al. (2006)]. The period budget constraint of the government is given by:

$$g_t + \frac{M_{t-1}}{p_t} + \sum_{s=t}^{\infty} \frac{q_s}{q_t} \left({}_{t-1}b_s^G + \frac{{}_{t-1}B_s^G}{p_s} \right) \leq \tau_t w_t h_t + \frac{M_t}{p_t} + \sum_{s=t+1}^{\infty} \frac{q_s}{q_t} \left({}_t b_s^G + \frac{{}_t B_s^G}{p_s} \right). \quad (11)$$

The superscript G means that the bond is issued by the government. $({}_{t-1}b_s^G)$ denotes total net claims on the amount of goods to be delivered by the government in period s . $({}_{t-1}B_s^G)$ denotes the net claims on money to be delivered by the government in period s .

The no-Ponzi game condition for the government is given by:

$$\lim_{j \rightarrow \infty} \left[q_{t+j} \frac{M_{t+j}}{p_{t+j}} + \sum_{s=t+j+1}^{\infty} q_s \left({}_{t+j}b_s^G + \frac{{}_{t+j}B_s^G}{p_s} \right) \right] \leq 0, \forall t \geq 0. \quad (12)$$

This condition rules out the possibility that the government borrows infinitely to finance its expenditures. The government's intertemporal budget constraint is given by:

$$\sum_{t=0}^{\infty} q_t \left(-{}_1b_t^G + \frac{{}_{-1}B_t^G}{p_t} \right) + \frac{M_{-1}}{p_0} = \sum_{t=0}^{\infty} q_t (\tau_t w_t h_t - g_t) + \sum_{t=1}^{\infty} q_t i_t m_t. \quad (13)$$

2.1.4 International investors

International investors can always borrow and lend at a nominal interest rate of i^{**} in the international capital market. Due to perfect capital mobility, the uncovered interest rate parity condition holds:

$$(1 + i_{t+1}) = \frac{S_{t+1}}{S_t} (1 + i^{**}) = \frac{1 + i^{**}}{1 + \pi^{**}} \frac{p_{t+1}}{p_t}, t \geq 0. \quad (14)$$

where π^{**} denote the inflation rate in the world economy. The second equality in Eq. (14) comes from the assumed purchasing power parity condition. In addition, we assume that

$$\beta \frac{1 + i^{**}}{1 + \pi^{**}} = 1, \quad (15)$$

Since $\frac{1+i^{**}}{1+\pi^{**}} = 1 + r^{**}$ where r^{**} denotes the real interest rate, Eq. (15) is the non-stochastic steady state version of the standard Euler equation with respect to asset accumulation.

2.1.5 Competitive equilibrium

Definition 1 *A competitive equilibrium is defined as a sequence $\{c_t, m_{t+1}, h_t, w_t, \Pi_t, q_{t+1}, p_{t+1}\}_{t=0}^{\infty}$, a positive constant λ , and an initial price level $p_0 > 0$, satisfying Eqs. (3), (4), (5), (6), (7), (9), (10), (13), (14), given the initial asset conditions of $\{M_{-1}, (-_1b_t^P), (-_1b_t^G), \text{ and } (-_1B_t^P), (-_1B_t^G), \forall t \geq 0\}$, the exogenous processes $\{z_t, g_t\}_{t=0}^{\infty}$, and a sequence of government policies $\{\tau_t, i_{t+1}\}_{t=0}^{\infty}$*

Equation (3) defines q_{t+1} for any given prices and nominal interest rates. Eqs. (4)–(7) solve the representative household’s utility maximization problem. Eqs. (9) and (10) solve the firms’ profit maximization problem. Eq. (13) pins down the initial price level. And Eq. (14) pins down the prices. Given Eq. (15) and the separable utility function assumption, we obtain the following:

$$q_t = \left(\frac{1 + \pi^{**}}{1 + i^{**}} \right)^t = \beta^t, t \geq 0, \quad (16)$$

$$u_{ct} = \lambda q_t / \beta^t = \lambda \left(\frac{1 + \pi^{**}}{1 + i^{**}} \right)^t / \beta^t = \lambda, t \geq 0. \quad (17)$$

Eqs. (16)–(17) essentially show that the discount factors will shrink at the rate of β and consumption is constant over time. We focus on this simple example because it is sufficient to discuss our main results.

Given the nature of the question discussed in the paper, the notation is generally complicated because of the use of the maturity structure of debt and because of the discussion on Ramsey equilibrium. It helps to clarify the main differences between two similar models: the small open economy model in this paper and the closed economy model in Persson et al. (2006). First, the

government in a small open economy, by definition, does not have any influence on the world real interest rates, while the government in Persson et al. (2006) can affect the equilibrium real interest rates with its policy. This difference implies different policy choices and different policy instruments to governments, thus different TC properties of OMFP.

Second, households in this small open economy can borrow and lend in the international bond market to smooth consumption. Thus, the period resource constraint does not necessarily bind in each period. As a result, we cannot substitute hours out to facilitate the discussion. In addition, these no-binding-period-resource-constraints do impose a restriction with respect to the TC property of OMFP and whether productivity is constant or time-varying matters in terms of the TC property of OMFP in our model. On the contrary, the period resource constraint must bind in each period in the Persson et al. (2006) model. As a result, Persson et al. (2006) can use the binding period resource constraint to substitute hours out to simplify their discussion. In addition, the labor market does not impose any restriction on the TC property of OMFP in a closed economy; and Persson et al. (2006) assume constant productivity without affecting their conclusion. We elaborate this point further in Section 2.3.1.

2.1.6 Intertemporal constraints

Using the optimality conditions, we rewrite the government's intertemporal budget constraint containing only the initial price level, p_0 , a constant, λ , real money balances, $\{m_{t+1}\}_{t=0}^{\infty}$, hours $\{h_t\}_{t=0}^{\infty}$, and the maturity structure of public debt it inherits from the previous government, $(-{}_1B_t^G)$ and $(-{}_1b_t^G)$:

$$\frac{\lambda}{p_0} \left[\sum_{t=0}^{\infty} Q_t (-{}_1B_t^G) + M_{-1} \right] = \sum_{t=0}^{\infty} \beta^t [\lambda(z_t h_t - g_t - {}_{-1}b_t^G) + u_{ht} h_t] + \sum_{t=1}^{\infty} \beta^t u_{mt} m_t, \quad (18)$$

where $Q_t = \prod_{j=1}^t (1 + \frac{u_{mj}}{\lambda})^{-1}$ and $Q_0 = 1$. For the mathematical procedures of obtaining equations such as Eq. (18), a reference paper is Schmitt-Grohé and Uribe (2004).

Similarly, combining the intertemporal budget constraint of the representative household with that of the government, we obtain the intertemporal resource constraint for the small open economy as:

$$\sum_{t=0}^{\infty} \beta^t [c_t + g_t + (-_1b_t^F) - z_t h_t] = -\frac{1}{p_0} \sum_{t=0}^{\infty} Q_t (-_1B_t^F), \quad (19)$$

where $_{-1}B_t^F =_{-1} B_t^G -_{-1} B_t^P$, and $_{-1}b_t^F =_{-1} b_t^G -_{-1} b_t^P$. The superscript F means that the bond is traded with the international investors.

2.2 OMFP under Commitment

Proposition 1 *A Ramsey allocation problem under commitment is to choose a constant λ [i.e., constant consumption, a point that can be seen from Eq. (17)], an initial price level p_0 , and a sequence of $\{m_{t+1}, h_t\}_{t=0}^{\infty}$ to maximize the representative household's lifetime utility:*

$$u\left(c(\lambda), \frac{M_{-1}}{p_0}, h_0\right) + \sum_{t=1}^{\infty} \beta^t u(c(\lambda), m_t, h_t), \quad (20)$$

subject to Eqs. (18) and (19), given the initial asset position, $\{_{-1}B_t^G, _{-1}B_t^F, _{-1}b_t^G, _{-1}b_t^F\}_{t=0}^{\infty}$ and M_{-1} , and the exogenous processes $\{z_t, g_t\}_{t=0}^{\infty}$.

Proof The proof is standard. The key is to show that when the governments commits to the announced policy, a sequence $\{c_t, m_{t+1}, \text{ and } h_t\}_{t=0}^{\infty}$ satisfies Eqs. (3), (4), (5), (6), (7), (9), (10), (13), and (14), if and only if it satisfies Eqs. (18) and (19). The details are available upon request and a reference paper of a standard proof can be found in Schmitt-Grohé and Uribe (2004). ■

Let μ_0^G and μ_0^E be the Lagrange multipliers associated with Eq. (18), the $t = 0$ government's

intertemporal budget constraint, and Eq. (19), the economy's intertemporal resource budget constraint, respectively. μ^G represents the marginal public financing cost while μ^E represents the marginal external financing cost. The optimality condition with respect to λ is:

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t u_{ct} \frac{\partial c}{\partial \lambda} &= \mu_0^E \sum_{t=0}^{\infty} \beta^t \frac{\partial c}{\partial \lambda} - \mu_0^G \left\{ \sum_{t=0}^{\infty} \beta^t (z_t h_t - g_t - {}_{-1}b_t^G) \right\} + \frac{\mu_0^E}{p_0} \sum_{t=1}^{\infty} ({}_{-1}B_t^F) \frac{\partial Q_t}{\partial \lambda} \\ &+ \frac{\mu_0^G}{p_0} \left\{ \left[\sum_{t=0}^{\infty} Q_t ({}_{-1}B_t^G) + M_{-1} \right] + \lambda \sum_{t=1}^{\infty} ({}_{-1}B_t^G) \frac{\partial Q_t}{\partial \lambda} \right\}. \end{aligned} \quad (21)$$

Eq. (21) is a single equation. The lefthand side represents the marginal cost in terms of utility due to an increase of λ . The foregone discounted present value of utility due to the decrease in consumption is the marginal cost of the increase in λ . The righthand side of Eq. (21) represents the corresponding marginal benefit, which contains four components. The first represents the increase in the discounted present value of lifetime utility if the economy's intertemporal resource constraint is relaxed due to the decrease of consumption. The second represents the change of the discounted present value of disutility when the government's intertemporal budget constraint is relaxed for the same reason. The third is the marginal benefit caused by the change in the discounted present value of external debt. And the last component comes from the associated change in the discounted present value of public debt.

The optimality condition with respect to m_t is:

$$u_{mt} = -\mu_0^G (u_{mmt} m_t + u_{mt}) + \frac{\mu_0^G \lambda}{\beta^t p_0} \sum_{s=t}^{\infty} ({}_{-1}B_s^G) \frac{\partial Q_s}{\partial m_t} + \frac{\mu_0^E}{\beta^t p_0} \sum_{s=t}^{\infty} ({}_{-1}B_s^F) \frac{\partial Q_s}{\partial m_t}, t \geq 1. \quad (22)$$

Eq. (22) denotes a system of equations. Its left-hand side represents the marginal cost in units of utility if the real money balance decreases. The righthand side represents the corresponding marginal benefit in units of utility, which has three sources: the first source is the relaxing of the government's intertemporal budget constraint; the second is the change in the discounted present

value of public debt due to the resulted change in nominal interest rates; and the last source comes from the change in the discounted present value of external debt due to the resulted change in nominal interest rates.

The optimality condition with respect to h_t is:

$$-u_{ht} = \mu_0^G (\lambda z_t + u_{hht} h_t + u_{ht}) + \mu_0^E z_t, t \geq 0. \quad (23)$$

Eq. (23) also denotes a system of equations. It shows that optimal hours are determined by equating the marginal benefit to the marginal cost. This optimality condition is similar to that in the corresponding closed economy. The only difference is that here the Lagrange multiplier in the second component of the right-hand side is the Lagrange multiplier associated with the intertemporal resource budget constraint, while in the closed economy the corresponding multipliers are the ones that are associated with period resource constraints.

The optimality condition with respect to p_0 is:

$$u_{m0} M_{-1} = \mu_0^G \lambda \left[\sum_{t=0}^{\infty} Q_t (-{}_1B_t^G) + M_{-1} \right] + \mu_0^E \sum_{t=0}^{\infty} Q_t (-{}_1B_t^F). \quad (24)$$

Eq. (24) is a single equation. There is a marginal benefit in units of utility due to an increase in the price level since inflation reduces the real value of outstanding nominal public debt and nominal external debt. This marginal benefit is given by the righthand side of Eq. (24). On the other hand, there is marginal cost because the real money balance is reduced. In the equilibrium, the marginal cost exactly offsets the marginal benefit.

Definition 2 A Ramsey equilibrium is defined as a choice of $(\lambda, p_0, \{h_t\}_{t=0}^{\infty}, \{m_{t+1}\}_{t=0}^{\infty})$ satisfying Eqs. (18), (19), (21), (22), (23), and (24), given the initial asset position, $\{-{}_1B_t^G, -{}_1B_t^F, -{}_1b_t^G, -{}_1b_t^F\}_{t=0}^{\infty}$ and M_{-1} , and the exogenous processes $\{z_t, g_t\}_{t=0}^{\infty}$.

2.3 OMFP with Discretion

We follow the same methodology as in the literature to discuss the TC property of OMFP in this stylized small open economy. In particular, we look for sufficient conditions for TC, i.e., whether there exist Lagrange multipliers and a maturity structure of public and external debt such that Ramsey policy is invariant to an ex post reoptimization, i.e., policy continuation satisfies the optimality conditions of the successor government. If a solution exists, OMFP is time consistent; otherwise, it is time inconsistent.

Formally, the TC problem becomes whether the $t = 0$ government can find Lagrange multipliers, μ_1^G and μ_1^E and a maturity structure $\{ {}_0B_t^G, {}_0B_t^F, {}_0b_t^G, {}_0b_t^F \}_{t=1}^\infty$, such that the policy continuation of the $t=0$ government satisfies the optimality conditions of the $t = 1$ government [Lucas and Stokey (1983), Persson et al. (2006), and Alvarez et al. (2004)]. The policy continuation of the $t = 0$ government refers to the $t = 0$ government's optimal choices, $\lambda^*, h_t^*, m_{t+1}^*, p_1^*$.¹⁴ The optimality conditions of the $t = 1$ government are the one-period ahead updated version of the optimality conditions of the $t = 0$ government. Given this standard methodology, we can define Lagrange multipliers as implicit policy instruments and bonds of different maturity dates as explicit policy instruments.¹⁵

One point worth emphasizing is that our discussion does not depend on whether issued bonds are denominated in the domestic currency or in a foreign currency (such as the US dollar). The reason is as follows. (1) We can interpret real bond denominated in the US dollar as real bond denominated in the domestic currency. To see this, note that $b_t^{**} = \frac{B_t^{**}}{p^{**}} = \frac{B_t S}{p^{**}} = \frac{B_t}{p_t} = b_t$, the first equality comes from the non-arbitrage condition in the foreign bond market, the second equality comes from the non-arbitrage condition across borders, the third equality comes from the law of one-price, and the last equality comes from the non-arbitrage condition in the domestic bond market. (2) We can also interpret nominal bond denominated in the US dollar as real bonds

denominated in the domestic currency but inflated by the exogenous world price. To see this, note that $B_t^{**} = b_t^{**} p^{**} = b_t p^{**}$, where p^{**} is exogenous. Put together, our result does not depend on whether the issued bonds are denominated in the domestic currency or in a foreign currency.

2.3.1 Marginal Financing Costs and the Labor Market

Defining $\Lambda_{t,s} = \begin{pmatrix} \lambda^* z_t + u_{hht}^* h_t^* + u_{ht}^* & z_t \\ \lambda^* z_s + u_{hhs}^* h_s^* + u_{hs}^* & z_s \end{pmatrix}$, $\forall t, s \geq 1$ and $t \neq s$, we have the following:

Proposition 2 *In a small open economy whose Ramsey allocation problem under commitment is defined in Definition 2, both marginal public and marginal external financing costs should be constant across governments in order to have TCPC of hours when $\Lambda_{t,s}, \forall t, s \geq 1$ and $t \neq s$, is not singular. Both marginal public and marginal external financing costs could vary across governments when $\Lambda_{t,s}, \forall t, s \geq 1$ and $t \neq s$, is singular.*

Proof The key is to show that with a non-singular $\Lambda_{t,s}$, the following must be true:

$$\mu_1^G = \mu_0^G = \hat{\mu}^G; \mu_1^E = \mu_0^E = \hat{\mu}^E. \quad (25)$$

The proof is in Appendix A. ■

First, when the matrix $\Lambda_{t,s}$ is non-singular, the Lagrange multipliers are uniquely determined in order to have TCPC of hours. From Eq. (23), optimal policy of hours in each period is jointly determined by the period productivity and the Lagrange multipliers associated with the two intertemporal budget constraints. When $\Lambda_{t,s}, \forall t, s \geq 1$ and $t \neq s$, is not singular, the $t = 1$ government will deviate from the policy continuation of hours if the Lagrange multipliers are different and productivity is time-varying. In other words, marginal financing costs have to be kept constant across governments in order to have TCPC of hours when productivity is time-varying and $\Lambda_{t,s}$ is not singular. In the context of this small open economy, the constant marginal financing

costs do not necessarily imply constant optimal labor income tax rates because hours could change over time if productivity is time-varying. The constant marginal financing costs also do not imply constant nominal interest rates because real money balances can change over time.

Second, there are two sufficient conditions for $\Lambda_{t,s}$ to be singular:

Proposition 3 *With separable preferences and/or the total factor productivity satisfying one of the following:*

$$\frac{u_{hh}h}{u_h} = -\zeta, \quad (26)$$

$$z_t \equiv z, \quad (27)$$

where ζ is a constant, $\Lambda_{t,s}, \forall t, s \geq 1$ and $t \neq s$, is singular.

Proof The proof is in Appendix B. ■

When $\Lambda_{t,s}$ is singular, the Lagrange multipliers are subject to one linear equation even though they are not uniquely determined by the requirement of TCPC of hours. Appendix E shows two “one-linear” equations, one for each sufficient condition.

Both Huber (1992) and Liu (2011) discuss conditions similar to Eq. (26) as sufficient conditions. There are several differences among them. (1) We show that it is a sufficient condition for TC of OMFP in a monetary small open economy while they show that it is a sufficient condition for TC of optimal fiscal policy in a real small open economy. (2) In this paper, we also show that it fails to be sufficient even in a real small open economy, which is slightly different from those in Huber (1992) and Liu (2011). The major difference is that the representative household has preferences over real money balances in our model while it does not in theirs. Thus, TC of OMFP in a real small open economy depends on people’s preferences and whether there is a money market or not does make a difference with respect to TC of optimal fiscal policy. (3) We derive Eq. (26) directly

from the discussion of the singularity of $\Lambda_{t,s}$ while Huber (1992) simply guesses and confirms a condition similar to Eq. (26) without talking about $\Lambda_{t,s}$.

In the corresponding closed economy [the one discussed in Persson et al. (2006)], the requirement of TCPC of hours does not impose restrictions on the TC property of OMFP. To see this, note that the relevant optimality condition becomes

$$\mu_1^G (u_{ct}^* z_t + u_{hht}^* h_t^* + u_{ht}^*) + \mu_{1t}^E z_t = -u_{ht}^*, \forall t \geq 1. \quad (28)$$

where μ_{1t}^E denotes the Lagrange multiplier associated with the period resource constraint in period t . After the Ramsey government chooses the value for μ_1^G , μ_{1t}^E will adjust in such a way that policy continuation of hours will automatically satisfy Eq. (28). For this very reason, Persson et al. (2006) skip the discussion on the effect of the requirement of TCPC of hours on TC of OMFP.

2.3.2 Time Inconsistency When $\Lambda_{t,s}$ Is Not Singular

Proposition 4 *In a small open economy whose Ramsey allocation problem under commitment is defined in Definition 2, OMFP is time inconsistent if $\Lambda_{t,s}$ is not singular.*

Proof The proof is in Appendix C. ■

Two optimality conditions are important with respect to Proposition 4. One is the optimality condition with respect to consumption, and the other is the optimality condition with respect to money balances in period $t = 2$. They are rewritten as following:

$$\mu_1^E A_2 ({}_0B_2^F) + \mu_1^G \lambda^* A_2 ({}_0B_2^G) = \tilde{D}_{37}, \quad (29)$$

$$\mu_1^E Q_2^1 ({}_0B_2^F) + \mu_1^G \lambda^* Q_2^1 ({}_0B_2^G) = \tilde{D}_{38,2}, \quad (30)$$

The notations are defined in Appendix C. Here we replace $\hat{\mu}^E$ and $\hat{\mu}^G$ in \hat{D}_{37} and $\hat{D}_{38,2}$ with μ_1^E and μ_1^E , respectively, to obtain \tilde{D}_{37} and $\tilde{D}_{38,2}$.

When $\Lambda_{t,s}$ is not singular, Lagrange multipliers will be constant across governments as we have shown in Proposition 2; and \tilde{D}_{37} and $\tilde{D}_{38,2}$ will become \hat{D}_{37} and $\hat{D}_{38,2}$, i.e., their values will become pre-determined. This will in turn lead to a singularity problem and thus no solution to the two unknowns, $({}_0B_2^F)$ and $({}_0B_2^G)$. To see this singularity problem, note that the ratio of $\hat{D}_{37}/\hat{D}_{38,2}$ is pre-determined and is not necessarily equal to A_2/Q_2^1 , while an equality is a necessary condition for the existence of solutions in such a two linear equation system.

To turn-around the singularity problem, we must allow Lagrange multipliers to vary across governments. In other words, the good market and the money market jointly ask that marginal financing costs vary across governments. Put together, when $\Lambda_{t,s}$ is not singular, there are two forces in this small open economy that are important with respect to TC of OMFP. One force, which arises from the labor market when $\Lambda_{t,s}$ is not singular, asks for constant marginal financing costs across government. The other force, which arises from the good market and the money market, asks for varying marginal financing costs across governments. These restrictions are orthogonal to each other. As a result, OMFP is time inconsistent.

2.3.3 Time Consistency When $\Lambda_{t,s}$ Is Singular

Proposition 5 *In a small open economy whose Ramsey allocation problem under commitment is defined in Definition 2, OMFP is time consistent if $\Lambda_{t,s}$ is singular. Further, there are many maturity structures of debt that are capable of rendering OMFP time consistent.*

Proof The proof is in Appendix D. ■

Intuitively, when $\Lambda_{t,s}$ is singular, the Lagrange multipliers are not uniquely determined by the requirement of TCPC of hours. In other words, the force asking for constant marginal financing

costs disappears. Therefore, OMFP becomes time consistent.

3 Time Consistency under A Fixed FERR

Kydland and Prescott (1977) argue that a credible fixed FERR helps solve the time inconsistency problem. One relevant question this paper answers is whether such a claim holds in a small open economy version of those closed economy models in Lucas and Stokey (1983), Alvarez et al. (2004) and Persson et al. (2006).

When the government commits to the fixed FERR, i.e., $S_t \equiv S$, the monetary economy effectively becomes a real economy. There are several major changes. First, $i_t \equiv i^{**}$, a feature meaning that the government loses its control over monetary policy, as predicted by the impossible trinity theorem established in the Mundell-Fleming model. Second, $p_t \equiv Sp^{**}$, a feature implying that p_0 is not a choice variable any more. Third, $Q_t \equiv (1 + i^{**})^{-t}$, which implies that Q_t is fully determined by i^{**} and is independent of λ and m_t . Finally, nominal bonds (external and public) of different maturity dates are not effective policy instruments and only the total bond holding position, $\sum_{t=0}^{\infty} Q_t(-_1B_t^G) = \sum_{t=0}^{\infty} (i^{**})^{-t}(-_1B_t^G)$ and $\sum_{t=0}^{\infty} Q_t(-_1B_t^F) = \sum_{t=0}^{\infty} (i^{**})^{-t}(-_1B_t^F)$, matters in the discussion of TC.

The intertemporal budget constraints are given by:

$$\lambda \mathbb{B}_t^G = \sum_{t=0}^{\infty} \beta^t [\lambda(z_t h_t - g_t) + u_{ht} h_t] + \sum_{t=1}^{\infty} \beta^t u_{mt} m_t - \frac{\lambda M_{-1}}{p^{**}}, \quad (31)$$

$$\mathbb{B}_t^F = \sum_{t=0}^{\infty} \beta^t [c_t + g_t - z_t h_t], \quad (32)$$

where $\mathbb{B}_t^G = \frac{1}{p^{**}} \sum_{t=0}^{\infty} (i^{**})^{-t}(-_1B_t^G) + \sum_{t=0}^{\infty} \beta^t(-_1b_t^G)$ and $\mathbb{B}_t^F = -\frac{1}{p^{**}} \sum_{t=0}^{\infty} (i^{**})^{-t}(-_1B_t^F) - \sum_{t=0}^{\infty} \beta^t(-_1b_t^F)$. The Ramsey allocation problem under commitment becomes:

Definition 3 *A Ramsey allocation problem under commitment is to choose a constant λ and a*

sequence of $\{m_{t+1}, h_t\}_{t=0}^{\infty}$ to maximize the representative household's lifetime utility:

$$u\left(c(\lambda), \frac{M_{-1}}{p^{**}}, h_0\right) + \sum_{t=1}^{\infty} \beta^t u(c(\lambda), m_t, h_t),$$

subject to Eqs. (31) and (32), given the initial asset position, $\{-1B_t^G, -1B_t^F, -1b_t^G, -1b_t^F\}_{t=0}^{\infty}$ and M_{-1} , and the exogenous processes $\{z_t, g_t\}_{t=0}^{\infty}$.

Proposition 6 *In a small open economy whose Ramsey allocation problem under commitment is defined in Definition 3, OMFP is time inconsistent.*

Proof The proof takes three steps. In the first step, we show that if $\Lambda_{t,s}$ is not singular, Eq. (25) holds by considering the requirement of TCPC of hours only. In the second step, we show that if $\Lambda_{t,s}$ is singular, Eq. (25) also holds by considering both the requirement of TCPC of hours and that of real money balances. Mathematically, we not only consider Eq. (36), but also consider the optimality conditions associated with real money balances, which are given by

$$u_{mt} = -\mu_0^G(u_{mmt}m_t + u_{mt}), t \geq 1; \quad (33)$$

$$u_{mt} = -\mu_1^G(u_{mmt}m_t + u_{mt}), t \geq 1; \quad (34)$$

Eqs. (33) and (34) are the conditions of the $t = 0$ government and of the $t = 1$ government, respectively. In the last step, we show that when Eq. (25) holds, then OMFP is time inconsistent.

The proof is in Appendix E. ■

The orthogonal restrictions on financial costs across governments remain when the FERR changes from a flexible FERR to a fixed FERR. This leads to time inconsistency of OMFP. It is true regardless of the singularity of $\Lambda_{t,s}$. The failure of Eqs. (26) and (27) to be sufficient is due to the new role played by the requirement of TCPC of real money balances in this economy (with a

fixed FERR). The reason for the new role is mainly because the government could not manipulate nominal debt to influence the choices of real money balances. To see this, it is useful to compare Eq. (22) to Eq. (33): the nominal debt which shows up in Eq. (22) when the FERR is flexible and it drops out in Eq. (33) when the FERR is fixed.

Note that we obtain the time-inconsistent OMFP in a small open economy with a fixed FERR, no matter whether the sufficient conditions that have been identified by both Huber (1992) and Liu (2011) are true or not. In this sense, we revive the finding in Persson and Svensson (1986) in a more general environment in which households have preferences over real money balances in addition to consumption and leisure. In addition, Proposition 6 implies the following:

Corollary 1 *It is impossible to have fixed FERR, perfect capital mobility, and time consistent OMFP at the same time in a stylized small open economy.*

Even though the monetary policy is time consistent (because monetary policy is not independent anymore) when the fixed FERR is credible, OMFP is time inconsistent because the government does not have sufficient implicit policy instruments. Our new result complements the finding in Kydland and Prescott (1977) and Barro and Gordon (1983) in the following sense: credible rules may help render certain optimal policy time consistent; but at the same time, they may lead to the time inconsistency of other optimal policies. Note that Corollary 1 is different from the impossible trinity theorem, which states that it is impossible to have the fixed FERR, perfect capital mobility, and an independent monetary policy at the same time. Instead, Corollary 1 states that OMFP is time inconsistent if the government commits to a fixed FERR and capital is perfectly mobile. Note that our conclusion is weaker because it requires that certain assumptions be satisfied in our stylized economy.

This finding is also empirically relevant and has the potential in explaining the recent sovereign debt crisis in Greece. It is widely regarded as the outcome of a burst of the fiscal problem ac-

cumulated in Greece over time. This is in line with the theoretical prediction of Corollary 1. To see that, note that we show in this paper that optimal fiscal policy is time inconsistent and time inconsistency has been regarded as an important reason for financial crises [Albanesi et al. (2003a)].

4 Policy Implication

Our analysis has strong policy implications. In line with the literature, it confirms the importance of the appropriate management of debt instruments (bonds of different maturity dates). There are several innovative points in our analysis. First, the government could implement debt instruments in such a way that changes the role played by one particular market with respect to TC of optimal policy. To see this, we compare what we find in the flexible FERR case to what we find in the fixed FERR case. On the one hand, when the government could manipulate nominal debt (in the flexible FERR case), the requirement of TCPC of real money balances does not impose the constant marginal financing cost requirement. On the other hand, when the government could not manipulate nominal debt (in the fixed FERR case), the requirement of TCPC of real money balances does impose the constant marginal financing cost requirement.

Second, our analysis shows the importance of designing new types of debt instruments. This is true because one commitment technology itself, an appropriate maturity structure of conventional debt as argued in the existing literature, fails to be sufficient in our highly stylized small open economy. To see new debt instruments as a potential rescue, we discuss how they could possibly affect the restriction of the requirement of TCPC of hours on TC. With the conventional debt instruments, the optimality conditions with respect to hours are given by Eq. (23):

$$-u_{ht} = \mu_0^G (\lambda z_t + u_{hht} h_t + u_{ht}) + \mu_0^E z_t, t \geq 0.$$

Now suppose we have a new type of debt, for example, promises of leisure as recommended in Faig (1991), $(1 - \tau_t)w_t(-_1A_t), \forall t \geq 0$. With the new debt instruments, the optimality conditions could be

$$-u_{ht} = \mu_0^G [\lambda z_t + u_{hht}(h_t +_{-1}A_t) + u_{ht}] + \mu_0^E z_t, t \geq 0.$$

According to the new optimality condition, there is no direct link between the singularity of $\Lambda_{t,s}$ and Eq. (25). In this case, even if $\Lambda_{t,s}$ is not singular, Eq. (25) will not necessarily hold solely because of the requirement of TCPC of hours. Intuitively, the new debt instruments provide more degrees of freedom to governments such that they can render OMFP TC.

5 Conclusion

This paper makes a small step in exploring the TC property of OMFP in a small open economy with perfect capital mobility. We show that one commitment technology recently proposed in the literature fails to be sufficient in a small open economy. Even though we derive the sufficient conditions for TC of OMFP within the model with a flexible FERR, they are clearly restrictive. Furthermore, these conditions fail to be sufficient in a stylized real small open economy. We summarize that OMFP in a small open economy is in general time inconsistent with conventional debt instruments.

In addition, we show that the commitment to the fixed FERR does not help render OMFP time consistent, not to mention the credibility of the fixed exchange rate commitment itself. This result clearly extends our understanding between the credibility of a fixed FERR and TC of OMFP.

Although intuitive, our results are still preliminary in the sense that they are far from being useful for the quantitative analysis in this important field. Nevertheless, our results point out one

important extension for the future research, namely, to pursue Schmitt-Grohé and Uribe (2007)'s recommendation that “the most urgent step ... is to characterize credible policy ...”. In particular, our results imply that the future research about OMFP in a small open economy should focus on time consistent discretionary policy rather than Ramsey policy under commitment. In addition, our analysis strongly recommends the development of new debt instruments and an appropriate use of the existing debt instruments.

References

- Albanesi, S., Chari, V. V., Christiano, L. J., 2003a. Expectation traps and monetary policy. *Review of Economic Studies* 70 (4), 715–741.
- Albanesi, S., Chari, V. V., Christiano, L. J., 2003b. How severe is the time-inconsistency problem in monetary policy? *Quarterly Review*, 17–33.
- Albanesi, S., Christiano, L. J., 2001. Comment on the costs of losing monetary independence: The case of Mexico. *Journal of Money, Credit and Banking* 33 (2), 398–403.
- Alvarez, F., Kehoe, P. J., Neumeyer, P. A., 2004. The time consistency of fiscal and monetary policies. *Econometrica* 72 (2), 541–567.
- Backus, D. K., Driffill, J., 1985. Rational expectations and policy credibility following a change in regime. *Review of Economic Studies* 52, 211–221.
- Bailey, M. J., 1956. The welfare cost of inflationary finance. *Journal of Political Economy* 64, 93–110.
- Barro, R. J., Gordon, D. B., 1983. Rules, discretion and reputation in a model of monetary policy. *Journal of Monetary Economics* 12 (1), 101–121.
- Calvo, G. A., 1978. On the time consistency of optimal policy in a monetary economy. *Econometrica* 46 (6), 1411–1428.
- Calvo, G. A., Obstfeld, M., 1990. Time consistency of fiscal and monetary policy: A comment. *Econometrica* 58 (5), 1245–1247.
- Chari, V. V., Christiano, L. J., Eichenbaum, M., 1998. Expectation traps and discretion. *Journal of Economic Theory* 81 (2), 462–492.
- Chari, V. V., Christiano, L. J., Kehoe, P. J., 1991. Optimal fiscal and monetary policy: Some recent results. *Journal of Money, Credit and Banking* 23 (3), 519–39.
- Chari, V. V., Kehoe, P. J., 2008. Time inconsistency and free-riding in a monetary union. *Journal of Money, Credit and Banking* 40 (7), 1329–1356.
- Faig, M., 1991. Time inconsistency, capital mobility and debt restructuring in a small open economy. *Scandinavian Journal of Economics* 93 (3), 447–55.

- Huber, B., 1992. Time consistency of government financial policy in a small open economy. *European Economic Review* 36 (8), 1545–1555.
- Kydland, F. E., Prescott, E. C., 1977. Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy* 85, 473–490.
- Liu, X., 2011. Liquidity constraints, maturity structure and time-consistency of fiscal policy. *Economic Record* 87 (278), 480–493.
- Lucas, R. J., Stokey, N. L., 1983. Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12 (1), 55–93.
- Lucas, R. J., Stokey, N. L., 1987. Money and interest in a cash-in-advance economy. *Econometrica* 55 (3), 491–513.
- Persson, M., Persson, T., Svensson, L. E. O., 1987. Time consistency of fiscal and monetary policy. *Econometrica* 55, 1419–1431.
- Persson, M., Persson, T., Svensson, L. E. O., 2006. Time consistency of fiscal and monetary policy: A solution. *Econometrica* 74 (1), 193–212.
- Persson, T., Svensson, L. E. O., 1986. International borrowing and time-consistent fiscal policy. *Scandinavian Journal of Economics* 88 (1), 273–95.
- Rogoff, K., 1985. The optimal degree of precommitment to an intermediate monetary target. *Journal of International Economics* 18, 1169–1190.
- Schmitt-Grohé, S., Uribe, M., 2003. Anticipated ramsey reforms and the uniform taxation principle: The role of international financial markets. Tech. rep.
- Schmitt-Grohé, S., Uribe, M., 2004. Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory* 114 (2), 198–230.
- Schmitt-Grohé, S., Uribe, M., 2007. Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics* 54 (6), 1702–1725.
- Svensson, L., 1997. Optimal inflation targets, conservative central bankers and linear inflation contracts. *American Economic Review* 87, 98–114.
- Svensson, L. E. O., 1985. Money and asset prices in a cash-in-advance economy. *Journal of Political Economy* 93 (5), 919–44.
- Tower, E., 1971. More on the welfare cost of inflationary finance. *Journal of Money, Credit and Banking* 3 (4), 850–60.
- Uribe, M., 2006. A fiscal theory of sovereign risk. *Journal of Monetary Economics* 53, 1857–1875.
- Woodford, M., 1994. Monetary policy and price level determinacy in a cash-in-advance economy. *Economic Theory* 4 (3), 345–80.

Notes

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²The forward-looking component (not necessarily stochastic) has a non-negligible impact on OMFP here. We do not consider the cases in which OMFP is solely determined by the contemporaneous state, for example, the ones discussed in Albanesi and Christiano (2001) and Albanesi et al. (2003b).

³The meaning of OMFP follows the tradition of Lucas and Stokey (1983) and Chari et al. (1991) in the sense that the Ramsey government maximizes the utility of households by choosing the least distortional monetary and fiscal policy.

⁴In line with the literature, whether OMFP is time consistent depends on whether the government can use policy instruments to influence its successor's policy choices in such a way that the successor will follow the announced policy continuation [Alvarez et al. (2004) among others]. According to Alvarez et al. (2004), explicit policy instruments are defined as bonds of various maturity dates; and policy choices are defined as both labor income tax rates and nominal interest rates in the case of flexible exchange rate regimes, and as labor income tax rates in the case of fixed exchange rate regimes.

⁵The standard methodology to discuss the TC property of OMFP is to find whether there exists a maturity structure of debt and Lagrange multipliers such that Ramsey policy is invariant to an ex post reoptimization. Given this standard methodology, we can define Lagrange multipliers as implicit policy instruments and bonds of different maturity dates as explicit policy instruments.

⁶In this paper, we assume that the government's commitment to a FERR is credible.

⁷There is one government per period. For each government, it solves an infinite-period optimization problem.

⁸Throughout the paper, we assume perfect capital mobility.

⁹We index the initial period with 0 in order to simplify the notation. We can index the initial period with ι and in this case, we have the same main results but with more complicated notations.

¹⁰We introduce this assumption in order to discuss the TC problem in a simple model and our practice is in line with the literature [Persson et al. (2006)].

¹¹With Lucas timing, the financial market meets before the good market in every period [Lucas and Stokey (1987)].

¹²Even though here p^{**} and later i^{**} and π^{**} are assumed to be constant over time, our main results about TC will be the same when they are time-varying.

¹³In the case of a flexible FERR, nominal interest rates are not effective policy choices any more when capital is perfect mobile.

¹⁴To make the discussion as clear as possible, the policy continuation will be represented by a superscript of $*$.

¹⁵One point worth emphasizing is that the order in which these optimality conditions are discussed does not matter since: (1) to show time inconsistency, we do not have to use all of them; and (2) to establish TC, we have to use all of them. Given the model's setup, we discuss the optimality conditions in such an order that we analyze solutions to Lagrange multipliers first and then the maturity structure. Note that Persson et al. (2006) discuss the maturity structure first then the Lagrange multipliers. One common point is that both our paper and Persson et al. (2006) have used all these optimality conditions to establish TC of OMFP.

A Proof of Proposition 2

Proof Here is a sketch of the proof. The optimality conditions with respect to hours for the $t = 1$ government are given by:

$$\mu_1^G (\lambda^* z_t + u_{hht}^* h_t^* + u_{ht}^*) + \mu_1^E z_t = -u_{ht}^*, \forall t \geq 1. \quad (35)$$

Eq. (35) is the one-period ahead version of Eq. (23). It represents a system of linear equations with two unknowns, μ_1^G and μ_1^E . Note that the $t = 0$ government has the similar optimality conditions

$$\mu_0^G (\lambda^* z_t + u_{hht}^* h_t^* + u_{ht}^*) + \mu_0^E z_t = -u_{ht}^*, \forall t \geq 1.$$

Thus, for any two periods, t and s , we have the following if we want to guarantee TCPC of hours:

$$\Lambda_{t,s} \begin{pmatrix} \mu_1^G \\ \mu_1^E \end{pmatrix} = \begin{pmatrix} -u_{ht}^* \\ -u_{hs}^* \end{pmatrix} \equiv \Lambda_{t,s} \begin{pmatrix} \mu_0^G \\ \mu_0^E \end{pmatrix}, \forall t, s \geq 1 \text{ and } t \neq s. \quad (36)$$

This is Eq. (36). It is immediately clear that when the matrix $\Lambda_{t,s}$ is non-singular, the Lagrange multipliers are uniquely determined in order to have TCPC of hours:

$$\mu_1^G = \mu_0^G = \hat{\mu}^G; \mu_1^E = \mu_0^E = \hat{\mu}^E.$$

This is Eq. (25). When the matrix $\Lambda_{t,s}$ is singular, the Lagrange multipliers are not uniquely determined by the requirement of TCPC of hours. ■

B Proof of Proposition 3

Proof First, under Eq. (26), we have the following

$$\begin{aligned} -u_{ht}^* &= \mu_0^G [\lambda^* z_t - (\zeta - 1) u_{ht}^*] + \mu_0^E z_t, \Rightarrow u_{ht}^* = \frac{\mu_0^G \lambda^* z_t + \mu_0^E z_t}{(\zeta - 1) \mu_0^G - 1}, \forall t \geq 1, \\ \Rightarrow \Lambda_{t,s} &= \begin{pmatrix} \lambda^* z_t - \frac{(\zeta-1)(\mu_0^G \lambda^* z_t + \mu_0^E z_t)}{(\zeta-1)\mu_0^G - 1} & z_t \\ \lambda^* z_s - \frac{(\zeta-1)(\mu_0^G \lambda^* z_s + \mu_0^E z_s)}{(\zeta-1)\mu_0^G - 1} & z_s \end{pmatrix} \Rightarrow |\Lambda_{t,s}| = 0, \forall t, s \geq 1 \text{ and } t \neq s. \end{aligned}$$

Second, when Eq. (27) holds, the system of equations reduces to one single equation and $\Lambda_{t,s}$ is clearly singular. ■

C Proof of Proposition 4

Proof The key is to show that we cannot find a maturity structure $\mathbf{X}_1 = \{ {}_0B_t^G, {}_0B_t^F, {}_0b_t^G, {}_0b_t^F \}_{t=1}^\infty$ such that policy continuation of the $t = 0$ government will satisfy the optimality conditions of the $t = 1$ government's Ramsey problem. For this purpose, it is sufficient to show the non-existence of $\mathbf{X}_2 = \{ {}_0B_2^G, {}_0B_2^F \}$. Two equations are crucial. One is the optimality condition with respect to consumption, and the other is the optimality condition with respect to money balances in period $t = 2$. With some manipulation, we obtain the following two equations

$$\hat{\mu}^E A_2 ({}_0B_2^F) + \hat{\mu}^G \lambda^* A_2 ({}_0B_2^G) = \hat{D}_{37}, \quad (37)$$

$$\hat{\mu}^E Q_2^1 ({}_0B_2^F) + \hat{\mu}^G \lambda^* Q_2^1 ({}_0B_2^G) = \hat{D}_{38,2}, \quad (38)$$

Here we have

$$\begin{aligned} Q_t^1 &= \prod_{j=2}^t \left(1 + \frac{u_{mj}^*}{\lambda^*} \right)^{-1}, A_t = Q_t^1 \left[\sum_{i=2}^t \frac{u_{mi}^*}{\lambda^* (\lambda^* + u_{mi}^*)} \right], \\ D_{37} &= p_1^* \frac{\lambda^* - \hat{\mu}^E}{1 - \beta} \left(\frac{\partial c}{\partial \lambda} \right)^* - p_1^* \hat{\mu}^G \left[\sum_{t=1}^\infty \beta^{t-1} \frac{u_{ht}^*}{\lambda^*} h_t^* + \sum_{t=2}^\infty \beta^{t-1} \frac{u_{mt}^*}{\lambda^*} m_t^* \right], \end{aligned}$$

$$D_{38,t} = \hat{\mu}^G m_t^* \beta^{t-1} p_1^* (\lambda^* + u_{mt}^*) + \frac{u_{mt}^* (1 + \hat{\mu}^G) \beta^{t-1} p_1^* (\lambda^* + u_{mt}^*)}{u_{mmt}^*},$$

$$\hat{D}_{37} = D_{37} - \sum_{t=3}^{\infty} \frac{A_t}{Q_t^1} (D_{38,t-1} - D_{38,t}), \hat{D}_{38,2} = D_{38,2} - D_{38,3}.$$

Eq. (37) is the $t = 1$ government's optimality condition with respect to consumption and Eq. (38) is the $t = 1$ government's optimality condition with respect to real money balance in period $t = 2$. Under the condition that $\Lambda_{t,s}$ is not singular (which is the case here), Eq. (25) must hold. In other words, Lagrange multipliers are uniquely determined in order to have time TCPC of hours. For this reason, we put a hat on top of the two Lagrange multipliers. There are two unknowns, ${}_0B_2^G$ and ${}_0B_2^F$, in these two linear equations. The rest of the variables are functions of policy continuation and the pinned down Lagrange multipliers.

There is no solution to this system. To see this, simply check Eq. (37) and Eq. (38). It is clear that the solution to $({}_0B_2^F)$ and $({}_0B_2^G)$ exists only if the ratio of $\hat{D}_{37}/\hat{D}_{38,2}$ is the same as the ratio of A_2/Q_2^1 . Since both ratios are functions of pre-determined variables, no solution exists and OMFP is time inconsistent.

Note that we have applied a "pre-determined argument": if two ratios are pre-determined, they are generally not equal. Even though this pre-determined argument is not standard, it has been used in the literature, for example, Uribe (2006). ■

D Proof of Proposition 5

Proof We prove Proposition 5 by construction as in the literature [Alvarez et al. (2004) and Persson et al. (2006)]. To proceed, we leave the Lagrange multipliers undetermined and make the following arbitrary assumptions to facilitate the discussion:

$$({}_0b_t^F) = ({}_0\hat{b}_t^F), t \geq 2; ({}_0b_t^G) = ({}_0\hat{b}_t^G), t \geq 1; ({}_0B_t^G) = ({}_0\hat{B}_t^G), t \geq 3. \quad (39)$$

where the variables $({}_0\hat{b}_t^F)$, $({}_0\hat{b}_t^G)$, and $({}_0\hat{B}_t^G)$ denote the values arbitrarily chosen for $({}_0b_t^F)$, $({}_0b_t^G)$, and $({}_0B_t^G)$, respectively.

We start with the optimality condition with respect to real money balances

$$\sum_{s=t}^{\infty} \mu_1^E Q_s^1 ({}_0B_s^F) + \sum_{s=t}^{\infty} \mu_1^G \lambda^* Q_s^1 ({}_0B_s^G) = \tilde{D}_{38,t}, t \geq 2 \quad (40)$$

Here we replace the $\{\hat{\mu}^E, \hat{\mu}^G\}$ in $D_{38,t}$ with $\{\mu_1^E, \mu_1^G\}$ to obtain $\tilde{D}_{38,t}$. Given the undetermined Lagrange multipliers, subtracting Eq. (40) held at $t = S$ from the equation (40) held at $t = S + 1$ produces the following equation involving $({}_0B_S^F)$ and $({}_0B_S^G)$:

$$\mu_1^E Q_S^1 ({}_0B_S^F) + \mu_1^G \lambda^* Q_S^1 ({}_0B_S^G) = \tilde{D}_{38,S} - \tilde{D}_{38,S+1}, S \geq 2. \quad (41)$$

Eq. (41) and assumption (39) are sufficient to pin down $({}_0B_S^F)$ and $({}_0B_S^G)$ for any $S \geq 3$ as functions of the undetermined Lagrange multipliers.

We then rewrite Eqs. (21) and (22) at $t = 2$ as

$$\mu_1^E A_2 ({}_0B_2^F) + \mu_1^G \lambda^* A_2 ({}_0B_2^G) = \tilde{D}_{37}, \quad (42)$$

$$\mu_1^E Q_2^1 ({}_0B_2^F) + \mu_1^G \lambda^* Q_2^1 ({}_0B_2^G) = \tilde{D}_{38,2} - \tilde{D}_{38,3}, \quad (43)$$

Here we replace the $\{\hat{\mu}^E, \hat{\mu}^G\}$ in \hat{D}_{37} with $\{\mu_1^E, \mu_1^G\}$ to obtain \tilde{D}_{37} . We can get rid of both $({}_0B_2^G)$ and $({}_0B_2^F)$ to obtain one equation with the two undetermined Lagrange multipliers:

$$0 = Q_2^1 \tilde{D}_{37} - A_2 \left(\tilde{D}_{38,2} - \tilde{D}_{38,3} \right), \quad (44)$$

Note that \tilde{D}_{37} , $\tilde{D}_{38,2}$, and $\tilde{D}_{38,3}$ are functions of the undetermined Lagrange multipliers. Both Eq. (44) and the linear restriction from the labor market (i.e., we are using the optimality condition with respect to hour) are two linear equations with two unknowns, the undetermined Lagrange multipliers. With tedious but straightforward mathematical manipulation, it can be shown that there is no singularity issue here and we can solve for the Lagrange multipliers.

There are three more conditions to use:

$$\sum_{t=1}^{\infty} Q_t^1 ({}_0B_t^G) + p_1^* \sum_{t=1}^{\infty} \beta^t ({}_0b_t^G) = D_{45} \quad (45)$$

$$\sum_{t=1}^{\infty} Q_t^1 ({}_0B_t^F) + p_1^* \sum_{t=1}^{\infty} \beta^t ({}_0b_t^F) = D_{46} \quad (46)$$

$$\mu_1^E \sum_{t=1}^{\infty} Q_t^1 ({}_0B_t^F) + \mu_1^G \lambda^* \sum_{t=1}^{\infty} Q_t^1 ({}_0B_t^G) = u_{m1}^* M_0^* - \mu_1^G \lambda^* M_0^*, \quad (47)$$

where

$$D_{45} = p_1^* \sum_{t=1}^{\infty} \beta^{t-1} \left[(z_t h_t^* - g_t) + \frac{u_{ht}^*}{\lambda^*} h_t^* \right] + p_1^* \sum_{t=2}^{\infty} \beta^{t-1} \frac{u_{mt}^*}{\lambda^*} m_t^* - M_0$$

$$D_{46} = p_1^* \sum_{t=1}^{\infty} \beta^{t-1} [z_t h_t^* - c(\lambda^*) - g_t].$$

We use Eq. (45) to solve for $({}_0B_1^G)$, Eq. (47) to solve for $({}_0B_1^F)$, and Eq. (46) to solve for $({}_0b_1^F)$. Further it can be shown that the constructed maturity structure of bonds is consistent with the $t = 0$ government's intertemporal budget constraint and with the $t = 0$ economy's intertemporal budget constraint. Thus, we construct a maturity structure of bonds and a solution to the Lagrange multipliers under arbitrary assumption (39). With the constructed maturity structure of bonds and the solution to the Lagrange multipliers, OMFP is time consistent because the policy continuation satisfies the optimality conditions of the $t = 1$ government. Since assumption (39) is arbitrary, we can change the values in that assumption to construct different maturity structures of bonds and different solutions for the Lagrange multipliers, which will together make the OMFP time consistent. ■

E Proof of Proposition 6

Proof The proof takes three steps. In the first step, we show that Eq. (25) holds if $\Lambda_{t,s}$ is not singular by considering the requirement of TCPC of hours only. In the second step, we show that Eq. (25) holds if $\Lambda_{t,s}$ is singular by considering both the requirement of TCPC of hours and that of real money balances. In the last step, we show that when Eq. (25) holds, then OMFP is time inconsistent.

Step 1: To have TCPC of hours, it must be true that Eq. (36) holds. Following the same argument in the proof of Proposition 2, Eq. (25) must hold if $\Lambda_{t,s}$ is not singular.

Step 2: When $\Lambda_{t,s}$ is singular, we first consider the optimality conditions with respect to real money balances, we then consider the optimality conditions with respect to hours, and we finally conclude that Eq. (25) must hold in this case.

- In the money market, the optimality conditions are:

$$\begin{aligned} u_{mt} &= -\mu_0^G(u_{mmt}m_t + u_{mt}), t \geq 1; \\ u_{mt} &= -\mu_1^G(u_{mmt}m_t + u_{mt}), t \geq 1; \end{aligned}$$

They are Eqs. (33) and (34), respectively. To have TCPC of real money balances, it must be true that

$$\mu_1^G = \mu_0^G = \hat{\mu}^G. \quad (48)$$

To see this, it is sufficient to evaluate Eqs. (33) and (34) at the policy continuation of real money balances and compare two equations.

- In the labor market, when $\Lambda_{t,s}$ is singular, Eq. (36) either reduces to

$$\begin{aligned} & \left[\lambda^* z_t - \frac{(\zeta - 1)(\mu_0^G \lambda^* z_t + \mu_0^E z_t)}{(\zeta - 1)\mu_0^G - 1} \right] \mu_1^G + z_t \mu_1^E \\ &= \left[\lambda^* z_t - \frac{(\zeta - 1)(\mu_0^G \lambda^* z_t + \mu_0^E z_t)}{(\zeta - 1)\mu_0^G - 1} \right] \mu_0^G + z_t \mu_1^E; \end{aligned} \quad (49)$$

if Eq. (26) holds or reduces to

$$(\lambda^* z + u_{hht}^* h_t^* + u_{ht}^*) \mu_1^G + z \mu_1^E = (\lambda^* z + u_{hht}^* h_t^* + u_{ht}^*) \mu_0^G + z \mu_1^E; \quad (50)$$

if Eq. (27) holds.

- Plugging Eq. (48) into either Eq. (49) or Eq. (50), we have $\mu_1^E = \mu_0^E = \hat{\mu}^E$. Put together, Eq. (25) holds when $\Lambda_{t,s}$ is singular.

Step 3: When (25) holds, OMFP is time inconsistent. To show this, we use the optimality condition with respect to λ for the $t = 1$ government:

$$\begin{aligned} \sum_{t=1}^{\infty} \beta^{t-1} u_{ct} \frac{\partial c}{\partial \lambda} &= \mu_1^E \sum_{t=1}^{\infty} \beta^{t-1} \frac{\partial c}{\partial \lambda} - \mu_1^G \sum_{t=1}^{\infty} \beta^{t-1} (z_t h_t - g_t - {}_0 b_t^G) \\ &\quad + \frac{\mu_1^G}{p^{**}} \left[\sum_{t=1}^{\infty} (i^{**})^{-t} ({}_0 B_t^G) + M_{-1} \right]. \end{aligned} \quad (51)$$

OMFP is time inconsistent because both sides of Eq. (51) are pre-determined when (25) holds. (1) β is a structural parameter, and p^{**} , i^{**} , z_t , and g_t are exogenous; (2) u_{ct} , h_t , $\frac{\partial c}{\partial \lambda}$ are evaluated at the policy continuation to assure TC; (3) the sum of $\sum_{t=1}^{\infty} \beta^{t-1} ({}_0 b_t^G)$ and $\frac{1}{p^{**}} \sum_{t=1}^{\infty} (i^{**})^{-t} ({}_0 B_t^G)$ is predetermined by the period budget constraint of the $t = 0$ government, which can be seen from the one-period ahead version of Eq. (31); and (4) Eq. (25) shows that Lagrange multipliers have been uniquely determined by the requirement of TCPC of hours and/or that of real money balances. ■