Public Good Coalitions and Membership Exclusion

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December 2012

Abstract

Many local public goods are provided in coalitions. When coalitions form they may have the power to exclude members. The core applies to such cases. When a coalition cannot exclude members, it allows all who prefer the provided public good to join. The no-exodus equilibrium is proposed for such cases. It is an extension of the Tiebout equilibrium in the long-run when the number of coalitions can vary and membership exclusion is not granted.

1 Introduction

Local public goods are provided in various forms of coalitions. For example, school districts provide public education, and residential communities provide recreation centers. Decisions on public goods are carried out in these coalitions. Cites, towns, labor unions, and political parties are all such examples. Commonly, individuals may have freedom to switch to another coalition or form new ones. Individuals want to join a coalition that offers a preferred public good. For example, people move to other communities for better public schools or amenities. New communities, governments, and international organizations form to provide more public goods.

The stability of public good coalitions is first addressed in the literature of local public finance. Tiebout (1956) models mobile residents freely choosing among local jurisdictions with different packages of public expenditure and tax share. This equilibrium concept which requires that no individual wants to move is then investigated by many in more elaborate models with private markets and political process such as Greenberg (1977), Rose-Ackerman (1979), Epple and Zelenitz (1981), Epple, Filimon and Romer (1984, 1993), Epple and Romer (1990), etc. Similar to Tiebout equilibrium, individual

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based stability notions are also studies in abstract coalition games. Greenberg (1979) proposes "individually stable equilibrium" which requires that when an individual join another coalition, he needs to keep the existing members in the new coalition no worse off. The opposite side of this requirement is that when an individual leaves a coalition, he needs to keep the remaining members in the old coalition no worse off. Dréze and Greenberg (1980) combine the above two concepts into the "individually stable contractual equilibrium;" a switching individual have to keep members in both coalitions, the one he joins and the one he leaves, no worse off. Individuals may also act as a group and form a coalition together. The core is such type of a commonly used notion which defines stability as when no new coalition forms. It is adopted for public goods coalitions by Guesnerie and Oddou (1981) for example. Later studies, such as Greenberg and Weber (1986, 1993), Demange (1994), and Kung (2006), combine the core with Tiebout equilibrium, imposing individual and coalitional stability together.

Coalitions face two types of outside sources of instability. An individual may want to move to another coalition; or a group of individuals may want to form a new coalition offering a more preferred public good. We can specify stability against outside options in two ways: (i) *Individual stability*: there is no individual member who wants to move to another coalition with a more preferred public good. When an individual joins another coalition, there may be some institutional restrictions that require the consent of other players involved. For example, if a coalition can exclude members, a joining member would have to make old members at least no worse off; we call this notion *joiner equilibrium*. On the other hand, if a coalition has inclusion power over members, a member can leave only if the remaining members are no worse off; we call this notion *leaver equilibrium*. (ii) *Coalitional stability*: there is no group of individuals, possibly from different coalitions, that can form a new coalition offering a preferred public good. Membership restrictions may also apply for coalitional stability. If a coalition cannot exclude members, it will allow all individuals who prefer the new public good to join. Equilibrium concepts proposed in the literature differ in these aspects.

Different notions of stability apply to different institutional settings. Individual stability applies in the short-run when the number of total coalitions is fixed, while coalitional stability applies in the long-run when the number of coalitions can vary. Tiebout equilibrium applies when individual can freely enter and exit a coalition, such as local communities in the short-run. One can move in and out a community without permission from other residents. The core applies to situations when new coalitions can exclude members, such as business partnerships. Everyone is supposed to benefit in a joint venture. Joiner equilibrium applies when individual entry is restricted, such as academic departments to professors. One cannot join a department without the faculty's consent. Leaver equilibrium applies when individual exit is restricted, such as gangsters. A gang will not let a member leave unless the departure is beneficial.

It seems that coalitional stability without membership exclusion is not addressed in the literature yet. Therefore, we discuss such type of stability, called the *no-exodus* equilibrium. It captures the idea of free migration where a new community welcomes all immigrants. There is no restriction on who can join; all individuals who prefer the new community to the status quo can move in as long as the composition of population makes it feasible. Stability notions are formally introduced in Section 2.

2 Stability in coalitions

The set of players is denoted by $N = \{1, ..., n\}$ and a *coalition* is a subset $S \subset N$. The set of all potential coalitions is 2^N and the set of public goods is X. Each player $i \in N$ has preferences over $X \times 2^N$ that are represented by a utility function $u_i : X \times 2^N \to \mathbb{R}$. For each coalition S, there is a set of feasible public goods $\phi(S)$. Mapping $\phi : 2^N \to 2^X$ is a *feasibility correspondence*. A coalition may have an empty feasible set. To eliminate triviality, there exists $S \subset N$ such that $\phi(S) \neq \varnothing$. The pair (x, S) is a public good coalition. A *local public goods game* $\left(N, X, \phi, (u_i)_{i \in N}\right)$ consists of a set of players, a set of public goods, a feasibility correspondence, and utility functions, where N is finite, X is closed (in its associated topological space), and ϕ is compact-valued. A *coalition partition* $\Pi \subset 2^N$ is a partition of N. An *allocation* $a : N \to X \times 2^N$ assigns a public good coalition pair $a(i)$ to individual i. Allocation a is *feasible* if there is a coalition partition and a list of public goods $\left(\Pi_a, (x_S)_{S \in \Pi_a}\right)$ with $x_S \in \phi(S)$ for all $S \in \Pi_a \subset \Pi$ such that $a(i) = (x_S, S)$ for all $i \in S$ and all $S \in \Pi_a$; we also require that a feasible allocation reaches Pareto efficiency inside coalitions. To simplify notation, we denote $u_i(a(i)) = u_i(x_S, S)$. The feasibility correspondence ϕ is *monotonic* if additional alternatives become feasible to a coalition when it has more members; $\phi(S) \subset \phi(S')$ for all $S, S' \in 2^N$, $S \subset S'$.

(1) Individual stability without membership restriction: A feasible allocation a is a *Tiebout equilibrium* if there is no $i \in S \in \Pi_a$ with $x \in \phi(S)$, and $u_i(x, S) > u_i(a(i))$ for all $i \in S$, and $u_i(a(i)) \geq u(a(j))$ for all $j \neq i$ for all $i, j \in N$. This concept is from Tiebout (1956) "The consumer-voter may be viewed as picking that community which best satisfies his preference pattern for public goods. ... Given these revenue and expenditure patterns, the consumer-voter moves to that community whose local government best satisfies his set of preferences." This describes how a freely mobile individual chooses an existing coalition that offers one of his most preferred alternatives among all offered. In a Tiebout equilibrium, coalitions cannot exclude members. An individual can move to another coalition without considering the effect of his arrival. A

coalition can not reject a new member even if she reduces the existing members' welfare. For example, in a finite model, the arrival of a new member may change a coalition's feasible set and makes the alternative not feasible any more and the existing members worse off.

(2) Individual stability with membership exclusion: A feasible allocation a is a *joiner equilibrium* if there are no $S, T \in \Pi_a$, $i \in S$, and $x \in \phi(T \cup i)$ such that $u_i(x, T \cup i) > u_i(a(i))$ and $u_j(x, T \cup i) \geq u_j(a(j))$ for all for all $j \in T$. In contrast to Tiebout equilibrium, joiner equilibrium requires that an individual makes only a "beneficial entry" to another coalition; the existing members in the new coalition are not made worse off.

(3) Individual stability with membership inclusion: A feasible allocation a in is a *leaver equilibrium* if there are no $S, T \in \Pi_a$, $i \in S$, $x \in \phi(T \cup i)$ and $x' \in \phi(x', S \backslash i)$, such that $u_i(x, T \cup i) > u_i(a(i))$ and $u_j(x', S \backslash i) \geq u_j(a(j))$ for all $j \in S \backslash i$. Leaver equilibrium requires a "beneficial exit" when one joins another coalition; the other members of the original coalition are not made worse off.

(4) Coalitional stability with membership exclusion: The commonly used core is of this type. A feasible allocation a is in the *core* if there is no public good coalition (x, S) such that $x \in \phi(S)$ and $u_i(x) > u_i(a(i))$ for all $i \in S$. A new coalition can form if all members are better off. There may be more individuals who want to join the new coalition, but the coalition may not take all who want to join.

(5) Coalitional stability without membership exclusion: A feasible allocation a is a *no-exodus equilibrium* if there is no public good coalition (x, S) such that $S = \{i \mid u_i(x, S) > u_i(a(i))\}$ and $x \in \phi(S)$. In this situation, coalitions cannot exclude members. Public good x attracts an "exit coalition" containing all individuals who want to join. Although, some members may have negative impacts on a coalition and make others worse off. This is an extension of the Tiebout equilibrium in the long-run when the number of coalitions can vary and exclusion power is not granted. The additional requirement of the no-exodus equilibrium is that individuals consider only credible moves; a new coalition form when the public good is feasible.

Example 1. The following illustrates how an exit coalition works. A no-exodus equilibrium does not exist when individuals have a preference cycle. Consider $N = \{1, 2, 3\}$, $X = \{x, y, z\}$, and $\phi(1) = \{z\}$, $\phi(2) = \{x\}$, $\phi(3) = \{y\}$, $\phi(S) = X$ if $|S| \geq 2$. Utility functions are

$$u_1(x) = 3, u_1(y) = 2, u_1(z) = 1,$$
$$u_2(y) = 3, u_2(z) = 2, u_2(x) = 1,$$
$$u_3(z) = 3, u_3(x) = 2, u_3(y) = 1.$$

First, y will attract exit coalition $\{1\ 2\}$ from one-person partition $(\{1,2,3\},(z,x,y))$. Second, z (x and y respectively) will attract an exit coalition from the grand coalition $\{1\ 2\ 3\}$ with public good x (y and z respectively). Third, z, x and x can respectively attract exit coalitions from allocations $(\{1\ 2,3\},(x,y))$, $(\{1\ 2,3\},(y,y))$, and $(\{1\ 2,3\},(z,y))$. By symmetry, none of the allocations with coalition partitions $\{1,2\ 3\}$ or $\{1\ 3,2\}$ is no-exodus equilibrium.∎

It is obvious that *if a is a core allocation, then it is a no-exodus equilibrium.* Since an exit coalition is also a blocking coalition to the core. The inverse holds under a mild condition; *when ϕ is monotonic, a no-exodus equilibrium is a core allocation.* Suppose a is not a core allocation, then there is a coalition (x,S) that blocks a and $S \subseteq \{i \mid u_i(x,S) > u_i(a(i))\}$. Take coalition $(x,\{i \mid u_i(x,S) > u_i(a(i))\})$ and $x \in \phi(\{i \mid u_i(x,S) > u_i(a(i))\})$ by monotonicity.

Example 2. The following illustrates the difference between these two concepts in a game whose feasibility correspondence is not monotonic. The core is empty but a no-exodus equilibrium exists. Consider $N = \{1,2,3\}$, $X = \{x,y,z,w\}$, and $\phi(N) = \phi(1) = \phi(2) = \phi(3) = \{w\}$, $\phi(1\ 2) = \{y\}$, $\phi(2\ 3) = \{z\}$, $\phi(1\ 3) = \{x\}$. Utility functions are $u_i(w) = 0$ for all i, $u_1 = u_2$, and

$$u_1(x) = 3, u_1(y) = 2, u_1(z) = 1,$$
$$u_3(z) = 3, u_3(x) = 2, u_3(y) = 1.$$

Coalition partitions $\{1\ 2\ 3\}$ and $\{1,2,3\}$ can only provide y, and hence are blocked by $(y,\{1\ 2\})$. Allocation $(\{1\ 2,3\},(y,w))$ is blocked by $(x,\{1\ 3\})$. Allocation $(\{1\ 3,2\},(x,w))$ is blocked by $(z,\{2\ 3\})$ with z. Allocation $(\{1,2\ 3\},(w,z))$ is blocked by $(y,\{1,2\})$. The core is empty. However, allocations $(\{1,2,3\},w)$, and $(\{1\ 2,3\},(y,w))$ are no-exodus equilibria.∎

3 Conclusion

In this paper, we study how local public goods are provided in coalitions. Notions of stability can be classified by individual stability and coalitional stability, and whether a coalition can exclude members. The core applies to the case when coalitions can exclude members. A new notion of coalitional stability without membership exclusion is discussed. When a coalition cannot exclude members, it allows all who prefer the provided public good to join. The no-exodus equilibrium is of such type. It is an extension of the Tiebout equilibrium in the long-run when the number of coalitions can vary and membership exclusion is not granted.

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