Dominance Testing for “Pro-Poor” Growth with an Application to European Growth

DANIEL SOTELSEK SALEM†,
ISMAEL AHAMDANECH ZARCO‡ and
JOHN. A BISHOP§.

‡ IELAT (Latin American Studies Institute). Av/ Juan Carlos I, 7. 28806. Alcalá de Henares, Spain (e-mail: ismael.ahamdanech@uah.es).
† Department of Economics, University of Alcalá, Alcalá de Henares, Spain (e-mail: daniel.sotelsek@uah.es).
§ Economics Department, East Carolina University, USA.

Abstract

This paper introduces statistical testing procedures to evaluate ‘pro-poor’ growth. Our measure of “pro-poorness” follows Kakwani (2000), Kakwani and Pernia (2000), and Son (2004), who decompose the generalized Lorenz ordinates into a growth effect and an inequality effect. We derive an asymptotic distribution-free covariance matrix for the decomposed generalized Lorenz curves. Using this decomposition (and our standard errors) we test for *pro-poor dominance* in the growth process. We illustrate our test for pro-poor growth in five European countries. For 1993-2006 we find that Spain, France, and Italy enjoyed pro-poor growth, Germany’s growth experience was anti-poor, and the UK pro-poor ranking is ambiguous.

Key Words: Pro-poor Growth, Poverty, Stochastic dominance.
JEL Classification: D31, D63, I32

1. INTRODUCTION

> "My dream has always been to make the poor richer, not to make the rich poorer...A rising tide lifts all boats."
>
> John F. Kennedy

President Kennedy's statement was re-posed as a question by Danziger and Gottschalk in their 1986 American Economic Association conference paper as: "Do rising tides lift all boats?" From the beginning of the US War on Poverty economists have noted that increased growth does not necessarily result in a decline in poverty (see Anderson, 1964 and Ahluwalia, 1974). More recently, this question has been recast to ask which types of economic policies lead to 'pro-poor' growth especially in light of the well-publicized Millennium Development Goals (MDG's). The past decade has produced a great deal of research relating to how to measure "pro-poorness." Essama-Nssah and Lambert (2009) and Duclos (2009) provide thorough reviews of this literature.

The purpose of this paper is introduce statistical testing procedures to evaluate 'pro-poor' growth. Our statistical approach is based on the seminal work of Beach and Davidson (1983) who provide asymptotic distribution-free covariance matrices for Lorenz and generalized Lorenz curves. Our theoretical approach to measuring "pro-poorness" follows Kakwani (2000), Kakwani and Pernia (2000) and Son (2004). The "KPS approach" decomposes changes in the difference in generalized Lorenz ordinates into a growth effect and an inequality effect. Using this decomposition we can test for *pro-poor dominance* in the growth process. Furthermore, we implement the KPS approach directly using the tools of stochastic dominance, as opposed to the more restrictive index number approach often used in this analysis.

We illustrate our test for pro-poor dominance by evaluating the degree of pro-poor growth in five European countries.[1] For the 1993-2006 period we find that Spain, France, and Italy enjoyed pro-poor growth, Germany's growth experience was anti-poor, and the UK pro-poor ranking is ambiguous.

[1] Only one researcher to our knowledge has studied pro-poor growth in Europe (Heinrich, 2003). No researcher has employed dominance methods together with formal statistical tests.

2. THEORETICAL FRAMEWORK

2.1 Second-order dominance (Generalised Lorenz dominance)

As Kakwani's method of evaluating pro-poor growth has its roots in second order stochastic dominance, we briefly introduce this technique. Following Atkinson (1970) we assume that the relationship between the distribution of income and the standard of living is given by a social welfare function, which represents the ethical judgments regarding income distributions. The well-known Atkinson Lorenz Dominance Theorem as extended by Dasgupta, Sen and Starrett (1973) is restricted to comparing distributions with the same mean. However, as Sen (1973) points out, the means of two distributions will rarely be equal. Shorrocks (1983) addresses this problem by introducing generalised Lorenz Dominance.

From Gastwirth (1971), the Lorenz curve can be defined as:

$$L_X(p) := \mu^{-1} \int_0^p X(u)du, \qquad [1]$$

and the generalised Lorenz curve will then be (Shorrocks, 1983):

$$GL_X(p) := \int_0^p X(u)du = \mu_X L_X(p), \quad \forall p \in [0,1] \qquad [2]$$

where $X(u)$ is the inverse of distribution function for incomes. Let W_S be a S-concave (concave and symmetric), increasing welfare function. Then we have the next theorem, demonstrated by Shorrocks (1983):

***Theorem 1—GL Dominance**: $w(X) \geq w(Y), \forall w \in W_S$ iff $GL_X(p) \geq GL_Y(p)$ for all p with at least one inequality prevailing.*

The implications of this approach are straightforward: assuming two widely, though not universally, accepted value judgments (the Pareto principle and the Pigou-Dalton transferences principle) we can rank the economic welfare associated with two different income distributions.

2.2 Truncated second-order dominance and poverty

Foster and Shorrocks (1988) links second order dominance to poverty. As it is well known, income-gap is the weighted sum of the income shortfalls of the poor, that is:

$$P(x;z) = \int_0^z (\frac{z-x}{z}) dF(x) \qquad [3]$$

Being z the poverty line and x individual's income. This criterion implies that income distribution *X* dominates income distribution *Y*, denoted by $X >_{z^*} Y$, if, and only if, $\int_0^{z^*} x dF(x) > \int_0^{z^*} y dF(y)$ for any given z^*. Then[2]:

$$GL_X(p) \geq GL_Y(p) \ \ iff \quad X \geq_{z^*} Y, \forall z < z^*$$

This implies that if the distribution is truncated at any arbitrary poverty line $z < z^*$ and the truncated distribution *X* generalised Lorenz dominates the truncated distribution *Y* at and below that poverty line, then the income-gap poverty in *X* cannot exceed poverty in *Y* using that poverty line, and this is the case for every poverty line $z < z^*$.

2.3 *Inequality effect* and *growth effect*

Our approach to measure "pro-poorness" follows Kakwani (2000), Kakwani and Pernia (2000) and Son (2004). The KPS approach decomposes changes in the difference in generalized Lorenz ordinates into a growth effect and an inequality effect. One key point in understanding the evaluation of "pro-poorness" is to clearly identify the

---

[2] This is theorem 2 in Shorrocks´(1983). The proof is given there, along a clear explanation of the relationship of GL dominance to the Pareto Principle and the Principle of Transfers.

reference point. The proposed method uses the underlying income distribution as the reference point.[3]

To evaluate changes in welfare due to economic growth we begin with the generalized Lorenz curve: $GL(p) = \mu L(p)$. Between two periods, its variation will be given as:

$$\Delta GL(p) = GL_2(p) - GL_1(p) = \mu_2 L_2(p) - \mu_1 L_1(p) \quad [6]$$

that is, the total change is due to both the change in the average income and the change in the income distribution measured by the Lorenz curve. So the total effect can be decomposed into *growth effect* and an *inequality effect*. Following Kakwani´s (2000) axiomatic approach, this decomposition can be done as follows:

$$\Delta GL_I = \frac{1}{2}\left\{\mu_1 L_2(p) - \mu_1 L_1(p) + \mu_2 L_2(p) - \mu_2 L_1(p)\right\} \quad [7]$$

$$\Delta GL_g = \frac{1}{2}\left\{\mu_2 L_1(p) - \mu_1 L_1(p) + \mu_2 L_2(p) - \mu_1 L_2(p)\right\} \quad [8]$$

and the sum of [7] and [8] equals the total change:

$$\Delta GL_I + \Delta GL_g = \Delta GL(p) \quad [9]$$

The *inequality effect,* GL_I, shows the variation of inequality, measured by the Lorenz curve, using the income in both the beginning and the final period. The interpretation of the *growth effect* GL_g is analogous.

As it is well known, the generalized Lorenz curve is defined by the pair of coordinates $\{p; \mu L(p)\}$. Then, the generalized Lorenz curve ordinates taking into account only the *inequality effect* will be:

[3] Bishop and Formby (1994) evaluate the "benefits of growth" using generalized concentration curves and like the Kakwani method use the Lorenz curve as the reference point. See Duclos (2009) and Essama-Nssah and Lambert (2009) for approaches to measuring pro-poorness with alternative reference points.

$$GL_I(p) = GL_1(p) + \frac{1}{2}\{\mu_1 L_2(p) - \mu_1 L_1(p) + \mu_2 L_2(p) - \mu_2 L_1(p)\} \qquad [10]$$

On the other hand, it is straightforward that:

$$GL_I(p) + \Delta GL_g = GL_1(p) + \frac{1}{2}\{\mu_1 L_2(p) - \mu_1 L_1(p) + \mu_2 L_2(p) - \mu_2 L_1(p)\} +$$
$$+ \frac{1}{2}\{\mu_2 L_1(p) - \mu_1 L_1(p) + \mu_2 L_2(p) - \mu_1 L_2(p)\} = GL_1(p) + GL_2(p) - GL_1(p) = GL_2(p), \qquad [11]$$
$$\forall p$$

Some interesting insights arise from the expressions above. If the inequality effect generalized Lorenz curve given by [10] dominates the ordinary generalized Lorenz curve GL_1, the inequality effect then reduces poverty as measured by Atkinson (1987). We call this result *pro-poor dominance*. The welfare implications are straightforward: pro-poor dominance implies an increase in economic welfare as measured the welfare functions included in W_S.

The next theorem demonstrates this result:

Theorem 2—Pro-Poor Dominance: If $GL_I(p) \geq GL_1(p)$, $\forall x < z^*$, with at least one inequality holding, then: $X_I \geq_{z^*} X_1$ and $X_I W_2^* X_1$, where X_1 is the initial income distribution and X_I the distribution taking into account only the *inequality effect.*

This theorem implies that if the generalized Lorenz curve taking into account only the *inequality effect* dominates the first period generalized Lorenz curve, the growth will have been pro-poor, not only because the decrease of poverty, but also because the increase of economic welfare of the poor, given the assumptions made above.[4]

The relationship between stochastic dominance and welfare can be extended to the truncated distributions. If the inequality effect generalized Lorenz curve $GL_I(p)$

[4] Note that our result is equivalent to Son's (2004) poverty growth curve. As noted by Son the generalized Lorenz approach is based on second order stochastic dominance while Ravallion and Chen's (2003) growth incidence curve. Son also points the advantages and disadvantages of the two approaches. Son recommends but does not provide formal inference tests.

dominates the ordinary GL curve, $GL_1(p)$, for all incomes up to z^* (defining z^* as the poverty line), then growth is poverty-reducing for all the poverty indexes as defined in Atkinson (1987).

2.4 Inference tests for Pro-Poor Dominance

When comparing generalized Lorenz curves there are three possible, outcomes, equivalence, dominance, or crossing. Bishop, Formby and Thistle (1989) recommend a pair wise statistical inference test procedure to evaluate generalised Lorenz curves, given these three alternatives. We adapt this approach to test for pro-poor dominance.[5] Using a set of k sub-hypotheses to test for the overall hypothesis of equivalence we have:

$$H_{0,i} : GL_1 = GL_I \quad \text{and} \quad H_{A,i} : GL_1 \neq GL_I \qquad [12]$$

If each of the sub-hypotheses is not rejected then the joint null hypotheses is not rejected, and we conclude that the growth process is neither pro or anti poor. On the other hand, if the any of the sub-hypotheses are rejected, then the following are the possible outcomes:

- Weak Pro-Poor dominance: If for some quantiles $GL_I > GL_1$ and for others $GL_I = GL_1$, then we conclude that growth is weakly pro-poor. If $GL_I > GL_1$ for all *I* then we have strong pro-poor growth.
- If for some quantiles $GL_I > GL_1$ and for others $GL_I < GL_1$, then no unambiguous ranking is possible for all z (it will be necessary to analyze truncated dominance).

The statistical tests will be:

[5] Xu (1997), Xu and Osberg (1998) and Davidson and Duclos (2000) provide alternative test structures that can be employed with our variance estimator to test for pro-poor dominance. We repeated the GL dominance tests (reported in Section 4) using the Davidson and Duclos test and obtained identical GL rankings.

$$T_{GLi} = \frac{\hat{G}L_I - \hat{G}L_1}{\left[\left(\frac{\hat{\varpi}_{ii}^{I}}{N_I}\right) + \left(\frac{\hat{\varpi}_{ii}^{1}}{N_1}\right)\right]^{1/2}} \text{ where } i=1,2,\ldots,K.^{6} \qquad [13]$$

Beach and Davidson (1983) derive the statistical distribution for $\hat{G}L_1$. However, the distribution for $\hat{G}L_I$ is unknown needs to be derived. Equation [10] implies:

$$GL_I(p) = GL_1(p) + \frac{1}{2}\{\mu_1 L_2(p) - \mu_1 L_1(p) + \mu_2 L_2(p) - \mu_2 L_1(p)\} \qquad [10]'$$

and can be written as:

$$\begin{aligned} GL_I(p) &= GL_1(p) + \frac{1}{2}\{GL_{1,2}(p) - GL_1(p) + GL_2(p) - GL_{2,1}(p)\} = \\ &= \frac{1}{2}\{GL_{1,2}(p) + GL_1(p) + GL_2(p) - GL_{2,1}(p)\} \end{aligned} \qquad [14]$$

where:

$$GL_{1,2}(p) = \mu_1 L_2(p) \qquad [15]$$

That is, $GL_{1,2}(p)$ is the generalized Lorenz curve obtained by scaling the income distribution of the second period by the mean income of the first. In an analogous way, we have:

$$GL_{2,1}(p) = \mu_2 L_1(p) \qquad [16]$$

Taking this into account, the variance of $GL_I(p)$ can be written as:

[6] The critical values for this test are determined by the Student Maximum Modulus distribution. Tables can be obtained from Stoline and Ury (1979).

$$Var(GL_I) = \frac{1}{4}\left\{\begin{array}{l} Var(GL_{1,2}(p)) + Var(GL_1(p)) + Var(GL_2(p)) + Var(GL_{2,1}(p)) + \\ 2Cov\left[GL_{1,2}(p); GL_1(p)\right] + 2Cov\left[GL_{1,2}(p); GL_2(p)\right] - \\ 2Cov\left[GL_{1,2}(p); GL_{2,1}(p)\right] + 2Cov\left[GL_1(p); GL_2(p)\right] - \\ 2Cov\left[GL_1(p); GL_{2,1}(p)\right] - 2Cov\left[GL_2(p); GL_{2,1}(p)\right] \end{array}\right\} \qquad [17]$$

Some items in [17] are equal to zero, since the initial and final distributions are independent:

$$Cov\left[GL_1(p); GL_2(p)\right] = 0 \qquad [18]$$

$Cov\left[GL_{1,2}(p); GL_1(p)\right] = Cov\left[\mu_1 L_2(p); \mu_1 L_1(p)\right] = 0$, since it is the covariance of the initial and final distribution scaled by the same quantity. Moreover:

$$Cov\left[GL_{1,2}(p); GL_{2,1}(p)\right] = Cov\left[\mu_1 L_2(p); \mu_2 L_1(p)\right] = 0 \qquad [19]$$

$$Cov\left[GL_2(p); GL_{2,1}(p)\right] = Cov\left[\mu_2 L_2(p); \mu_2 L_1(p)\right] = 0 \qquad [20]$$

There are still two items to be calculated. The first one is:

$$Cov\left[GL_{1,2}(p); GL_2(p)\right] = Cov\left[\mu_1 L_2(p); \mu_2 L_2(p)\right] \qquad [21]$$

From a very well known property of the variance:

$$\begin{array}{l} Var\left[\mu_1 L_2(p) + \mu_2 L_2(p)\right] = Var\left[(\mu_1 + \mu_2) L_2(p)\right] = \\ = Var(\mu_1 L_2(p)) + Var(\mu_2 L_2(p)) + 2Cov\left[\mu_1 L_2(p); \mu_2 L_2(p)\right] \end{array} \qquad [22]$$

from where:

$$2Cov\left[\mu_1 L_2(p); \mu_2 L_2(p)\right] = Var(\mu_1 L_2(p)) + Var(\mu_2 L_2(p)) - Var\left[(\mu_1 + \mu_2) L_2(p)\right] \qquad [23]$$

Analogously, we have:

$$2Cov\left[\mu_1 L_1(p); \mu_2 L_1(p)\right] = Var(\mu_1 L_1(p)) + Var(\mu_2 L_1(p)) - Var\left[(\mu_1 + \mu_2) L_1(p)\right] \qquad [24]$$

Using all these results in [17]:

$$Var(GL_I) = \frac{1}{4}\begin{Bmatrix} Var(GL_{1,2}(p)) + Var(GL_1(p)) + Var(GL_2(p)) + Var(GL_{2,1}(p)) + \\ + 2Cov\left[GL_{1,2}(p);GL_2(p)\right] - 2Cov\left[GL_2(p);GL_{2,1}(p)\right] \end{Bmatrix} \qquad [25]$$

And after some manipulations, the next expression is reached:

$$Var(GL_I) = \frac{1}{4}\{2Var(\mu_2 L_2(p)) + 2Var(\mu_1 L_2(p)) - Var[(\mu_1 + \mu_2)L_2(p)] + Var[(\mu_1 + \mu_2)L_1(p)]\} \qquad [26]$$

As it can be seen in [26], the variance of the generalized Lorenz curve that only has into account the *inequality effect* is computed using the variances of four different income distributions.

Now, it is possible to calculate both the variance of GL_1 and GL_I. The variance of GL_1 is given by Beach and Davidson (1983). For GL_I, it will be given, for *i=j*, by the expression[7]:

$$Var(GL_I) = \varpi_{ii}^{I} = \frac{1}{4}\left\{2\varpi_{ij}^{2,2} + 2\varpi_{ij}^{1,2} - \varpi_{ij}^{1,2^*} + \varpi_{ij}^{1^*,2}\right\} \qquad [27]$$

being $\varpi_{ij}^{a,a}$ the variance of the generalized Lorenz curve of the distributions seen in [26].

3. RECENT TRENDS IN EUROPEAN POVERTY AND INEQUALITY

According to the OECD (2008) economic inequality in Europe is on the rise. For our sample of countries the Gini coefficients in mid-2000s vary between 0.28 for France and 0.35 for Italy. Germany shows a Gini coefficient of 0.3, Spain of 0.32 and Great Britain of 0.34. While the differences are not wide at all, it is interesting to note that

[7] The final expression of the variances are developed in the appendix.

there has been changes in the inequality trends among these countries in the last two decades.

Two periods can be distinguished in the evolution of income distribution, mid-1980's to mid 1990's and the mid-1990's to mid 2000's. In the first period inequality fell in Spain and France, while it increased in Germany, Italy and UK. In the second period, inequality remained steady in Spain and France, increased in Germany and in Italy, and decreased in the UK. From the mid 1990's to mid 2000's Germany's inequality continues to increase, France and Spain enjoy a decrease in inequality, while there was no significant change in inequality in Italy and the UK.[8]

If we turn our attention to headcount poverty evolution in the last decade (with 50% of median income as poverty line) France, Italy and the UK experienced decreases in poverty while poverty in Germany and Spain increased. The analysis of relative poverty is interesting, but also it is to know what has happened in absolute terms. One way to do this is to fix a relative poverty line for a period t and, adjusted for inflation, to use it to compare poverty in, say, period $t+n$[9]. Following this approach, *absolute* poverty decreased in all countries analyzed between mid 1990's and mid 2000's with the exception of Germany, where absolute poverty increased.

4. DATA AND EMPIRICAL ANALYSIS

4.1 Data

To identify the 'pro-poor' aspects of economic growth we need data of a sufficient time span to capture the effects of distributional changes imbedded in the growth process. To this aim we have used data from two different, although consistent, datasets: The European Community Household Panel (ECHP) and the Survey on Income and Living Conditions (SILC), both developed by Eurostat. We have drawn data for the years 1993 and 2000 from the ECHP (waves first and eight of the survey) while for 2006 we use data from the SILC (the 2007 wave).

Our income measure is *per capita* household disposable income including total market income, adding transfers, and deducting taxes and Social Security contributions, adjusted by the modified OECD equivalence scale. The data are weighted using the

[8] These conclusions are based on the Eurostat database.
[9] This approach is comparable to the empirical analysis carried out in this paper.

weights given by the sample design and the number of individuals in the households, so we convert household data into individual data, assuming the same income for each member of the household. Finally, data have been deflated to 1993 units using the HIPC from Eurostat.

4.2 Results

In this section we apply formal inference tests to address two questions. First, did welfare improve (i.e., generalized Lorenz dominance) in each of the five European counties selected between 1993 and 2006? Secondly, and of more direct interest to us, was economic growth during this period 'pro-poor'? In this second case we compare the ordinary generalized Lorenz curve of the initial period to the inequality effect generalized Lorenz curve of the second period. Tables 1a-1e provide the results of the GL dominance tests and the pro-poor tests for the five European countries considered.

Table 1a shows the results for Germany, 1993-2006. Columns 2 and 3 provide the 1993 and 2006 generalized Lorenz ordinates while the fourth column provides the test statistics for ordinary GL dominance. As each of the test statistics is greater than the five percent SMM critical value of 2.80 (see Stoline and Ury, 1979) we conclude 2000 generalized Lorenz dominates 1993.

To test for pro-poor dominance we compare the GL ordinate for 1993 (column 2) with the G_I ordinate (column 5). The test statistics for deciles 4 to 9 are negative and significant at the five percent level. From these results we conclude that growth in Germany between 1993 and 2000 was unambiguously 'anti-poor'. [10]

Tables 1b to 1e provide the GL dominance and pro-poor test for France, Italy, Spain and the UK (1993-2006). In each case 2006 GL dominates 1993. France, Italy, and Spain all show pro-poor growth over this time period. For France and Spain the test statistics are positive and significant at all deciles. For Italy, deciles 1-6 are positive and significant while deciles 7 to 9 show no statistical difference. Examining the UK carefully (Table 1e) we find that deciles 1-3 are negative and significant while deciles 5-9 are positive and significant. The crossing of the GL curve with the GL_I curve implies

[10] In the case of Germany 1993 to 2006 if we define the poverty line the 30th percentile (truncated dominance) we can conclude that the time period 1993 to 2006 was neither pro-poor or anti-poor (no significant difference).

that no unambiguous conclusion regarding pro-poor growth is possible for the 1993 to 2006 UK comparison.

Table 2 provides a summary of the numerical and statistical comparisons for all five Europe countries examined. In this table we repeat our findings from Tables 1a-1e and provide a summary of additional comparisons between 2000 and 2006. To understand how the table works consider Germany, 1993 to 2006. Under GL dominance we report '+ +' which means that both the numerical and statistical comparisons ranked 2006 over 1993. However, under pro-poor dominance we report 'x -' which implies that the numerical crossing was replaced by a statistical ranking of anti-poor. This table demonstrates the errors in rankings that can occur in the absence of statistical tests like those developed in this paper. In fact, five out of 20 or 25 percent of the comparisons in Table 4 are mis-ranked using numerical comparisons.

The comparison of Germany 2000 to 2006 highlights the distinction between GL dominance and pro-poor dominance. First, we note that the ordinary GL curves cross (see Figure 1)--no unambiguous welfare conclusion can be drawn from the German growth experience of 2000 to 2006. However, we are able to rank this time period as unfavourable to the poor. These results are illustrated Figure 2 with GL_I lying below GL_{00}, indicating that the German poor lost ground over this time period. In sum, for 2000 to 2006, France and Spain experienced pro-poor growth, while growth was anti-poor in Germany, Italy, and the UK.

5. CONCLUSIONS

This paper introduces statistical testing procedures to evaluate 'pro-poor' growth. Our theoretical approach to measuring "pro-poorness" follows Kakwani (2000), Kakwani and Pernia (2000), and Son (2004). The "KPS approach" decomposes changes in the difference in generalized Lorenz ordinates into a growth effect and an inequality effect. Using this decomposition we can test for *pro-poor dominance* in the growth process. Furthermore, we test for pro-poor dominance directly using the tools of stochastic dominance, as opposed to the more restrictive index number approach often used in this analysis. We derive an asymptotic distribution-free covariance matrix for the decomposed generalized Lorenz curves on which pro-poor dominance testing is based.

We illustrate our test for pro-poor dominance by evaluating the degree of pro-poor growth in five European countries. We consider two time periods, 2000 to 2006 and 1993 to 2006. For 2000 to 2006, France and Spain experienced pro-poor growth, while growth was anti-poor in Germany, Italy, and the UK. For the combined 1993-2006 period we find that Spain, France, and Italy enjoyed pro-poor growth, Germany's growth experience was anti-poor, and the UK pro-poor ranking is ambiguous.

Figure 1: Generalized Lorenz Crossing, Germany, 2000-2006

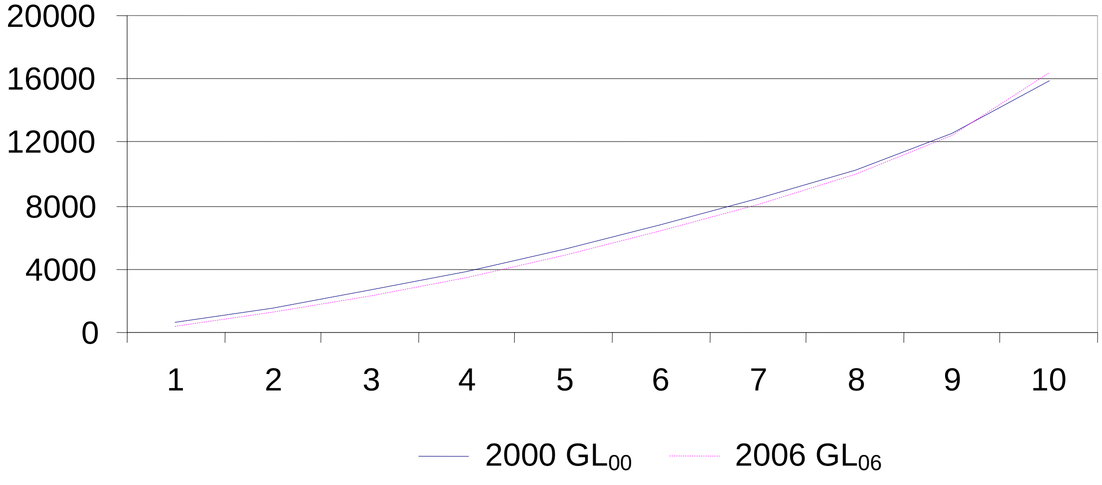

Figure 2: Pro-Poor Dominance, Germany 2000-2006

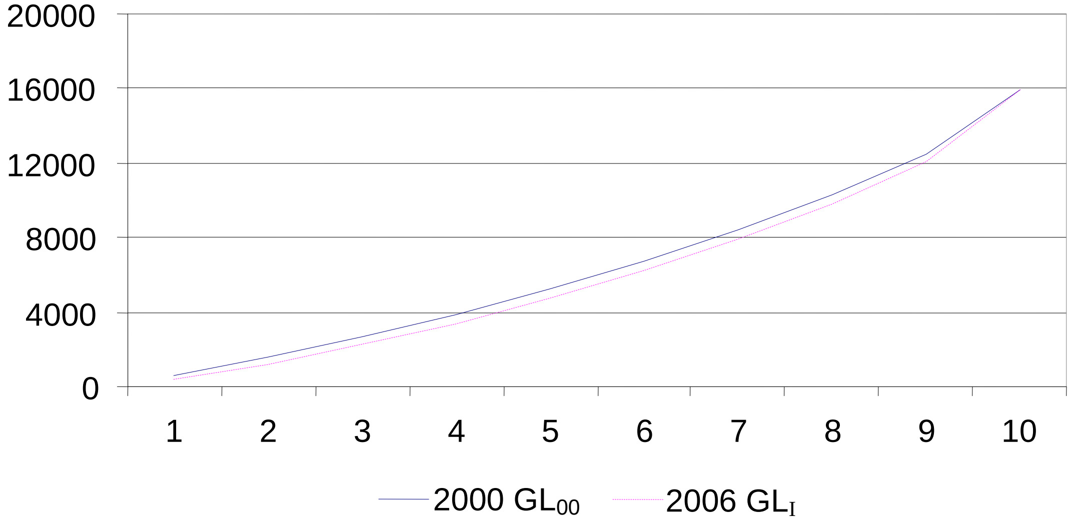

Table 1.a
Generalized Lorenz and Pro-Poor Dominance,
Germany, 1993-2006

Generalized Lorenz Dominance			Pro-Poor Dominance		
Decile (1)	$\hat{GL}_{93}$ (2)	$\hat{GL}_{06}$ (3)	T_{GL1} (4)	$\hat{GL}_{1}$ (5)	T_{GL1} (6)
1	384.63	414.11	1.92	-389.41	0.33
2	1209.68	1257.81	2.38	-1181.50	-1.35
3	2220.50	2298.13	3.20*	-2158.52	-2.43
4	3375.42	3496.15	4.20*	-3283.92	-2.98*
5	4668.51	4849.66	5.47*	-4554.82	-3.21*
6	6119.51	6370.29	6.48*	-5983.65	-3.24*
7	7775.97	8084.48	6.80*	-7594.70	-3.66*
8	9661.71	10055.50	7.49*	-9445.93	-3.76*
9	11918.11	12410.92	7.84*	-11658.78	-3.78*
10	15384.11	16364.51	9.97*	-15384.11	0.00

Five percent critical value for SMM=2.80

Table 1.b
Generalized Lorenz and Pro-Poor Dominance,
France: 1993-2000

Generalized Lorenz Dominance			Pro-Poor Dominance		
Decile (1)	$\hat{GL}_{93}$ (2)	$\hat{GL}_{06}$ (3)	T_{GL1} (4)	$\hat{GL}_{1}$ (5)	T_{GL1} (6)
1	372.02	522.97	17.17*	511.09	13.36*
2	1045.46	1308.89	18.71*	1277.30	14.69*
3	1865.43	2260.32	20.69*	2205.33	16.40*
4	2820.18	3353.11	21.52*	3271.51	17.25*
5	3919.35	4594.20	21.77*	4480.15	17.53*
6	5169.11	5985.89	21.79*	5839.07	17.66*
7	6591.56	7550.41	21.60*	7362.30	17.44*
8	8232.49	9325.17	20.61*	9090.65	16.53*
9	10218.05	11447.91	19.03*	11158.29	15.07*
10	14212.63	14597.57	2.87*	14212.63	0.00

Table 1.c
Generalized Lorenz and Pro-Poor Dominance,
Italy: 1993-2006

Generalized Lorenz Dominance			Pro-Poor Dominance		
Decile (1)	$\hat{GL}_{93}$ (2)	$\hat{GL}_{06}$ (3)	T_{GL1} (4)	$\hat{GL}_{1}$ (5)	T_{GL1} (6)
1	169.69	316.97	29.58*	229.77	10.95*
2	555.66	897.15	42.86*	633.50	8.34*
3	1053.13	1623.59	52.73*	1135.87	6.59*
4	1646.63	2489.92	59.73*	1734.74	5.44*
5	2344.06	3505.80	65.94*	2437.02	4.60*
6	3155.91	4668.91	70.75*	3237.82	3.35*
7	4101.95	6003.82	73.38*	4153.86	1.74
8	5208.52	7552.96	77.13*	5215.24	0.19
9	6521.78	9424.02	80.21*	6501.20	-0.51
10	8610.04	12473.11	70.70*	8610.04	0.00

Table 1.d

Generalized Lorenz and Pro-Poor Dominance,
Spain: 1993-2006

Generalized Lorenz Dominance			**Pro-Poor Dominance**		
Decile (1)	$\hat{G}L_{93}$ (2)	$\hat{G}L_{06}$ (3)	T_{GL1} (4)	$\hat{G}L_{1}$ (5)	T_{GL1} (6)
1	181.29	226.51	11.75*	197.86	4.18*
2	517.02	637.98	19.69*	556.66	6.25*
3	941.04	1152.44	24.80*	1005.22	7.25*
4	1443.75	1764.82	28.73*	1539.18	8.37*
5	2027.44	2483.06	32.06*	2165.81	9.51*
6	2702.35	3312.91	35.11*	2889.87	10.51*
7	3488.95	4266.83	36.47*	3721.40	10.55*
8	4437.11	5379.23	36.08*	4688.34	9.09*
9	5633.65	6720.78	33.86*	5851.40	6.33*
10	7579.51	8728.52	25.45*	7579.51	0.00

Table 1.e
Generalized Lorenz and Pro-Poor Dominance,
UK: 1993-2006

Generalized Lorenz Dominance			**Pro-Poor Dominance**		
Decile (1)	$\hat{G}L_{93}$ (2)	$\hat{G}L_{06}$ (3)	T_{GL1} (4)	$\hat{G}L_{1}$ (5)	T_{GL1} (6)
1	187.85	553.98	-52.2*	265.27	17.76*
2	568.86	1473.18	-79.9*	657.94	12.02*
3	1066.60	2616.76	-95.6*	1130.21	6.17*
4	1657.95	3964.10	-108.6*	1682.99	1.87
5	2346.55	5534.26	-116.7*	2328.20	-1.09
6	3140.00	7343.69	-125.7*	3070.97	-3.32*
7	4056.93	9412.87	-132.9*	3914.64	-5.67*
8	5123.81	11828.96	-137.3*	4902.13	-7.30*
9	6431.44	14796.43	-140.0*	6116.26	-9.05*
10	8381.06	19859.12	-99.4*	8381.06	0.00

Table 2
Summary of Generalized Lorenz and Pro-Poor Dominance[1]

Country	2000-2006 GL Dominance	2000-2006 Pro-Poor Dominance	1993-2006 GL Dominance	1993-2006 Pro-Poor Dominance
Germany	XX	– –	+ +	X –
France	X +	X +	+ +	+ +
Italy	+ +	– –	+ +	X +
Spain	+ +	X +	+ +	+ +
UK	+ +	– –	+ +	XX

[1]First entry is numerical comparison, second entry is statistical comparison.
"+" denotes dominance of 2nd year over first, " – " first year over second, and "X" denotes a crossing.

REFERENCES

1. Ahluwalia, M.S.(1974): “Income Inequality: some dimensions of the problem”, in H. Chenery et. al., Redistribution with Growth. Oxford: Oxford University Press.
2. Anderson, W. (1964): “Trickling Down: The Relationship Between Economic Growth and the Extent of Poverty Among American Families”, *Quarterly Journal of Economics*, 78: 511-524.
3. Atkinson, A. B. (1970): “On the Measurement of inequality”, *Journal of Economic Theory*, 2: 244-263.
4. Atkinson, A. B. (1987): “On the Measurement of Poverty”, *Econometrica*, 55: 749-764.
5. Beach, C. and R. Davidson (1983): “Distribution-Free Statistical Inference with Lorenz Curves and Income Shares”, *Review of Economic Studies*, 50: 723-735.
6. Bishop, J.A., J.P. Formby and P.D. Thistle (1989), “Statistical inference, income Distributions and social welfare”, en D.J. Slotje (ed.), *Research on Economic Inequality*, Vol.1, Greenwich, CN: JAI Press.
7. Danziger, Sheldon and Gottschalk, Peter, 1986. “Do Rising Tides Lift All Boats? The Impact of Secular and Cyclical Changes on Poverty”, *American Economic Review*, American Economic Association, 76(2): 405-10.
8. Dasgupta, P., Sen, A., and Starrett, D. (1973): “Notes on the measurement of inequality”, *Journal of Economic Theory*, 6: 180-187.
9. Davidson, R. and Duclos, J.-Y. (2000): Statistical inference for stochastic dominance and for the measurement of poverty and inequality, *Econometrica*, 68(6) 1435-1464
10. Duclos, J.-Y. (2009): “What is “Pro-poor”, *Social Choice and Welfare*, vol. 32, 37-58

11. Essama-Nssah, B. and Lambert, Peter J. (2009): "Measuring Pro-Poorness : A Unifying Approach With New Results" *Review of Income and Wealth*, 55, 3: 752-778

12. Foster, J. E., and A. F. Shorrocks (1988): "Poverty orderings and welfare dominance", *Social Choice and Welfare*, 5: 179-198.

13. Gatswirth, J.L. (1971): "A General Definition of the Lorenz Curve", *Econometrica*, 39: 1037-1039.

14. Heinrich, G. (2003): "More is not Necessarily Better: An Empirical Analysis of the Inequality-Growth Tradeoff using the Luxembourg Income Study", LIS Working Paper Nº 344.

15. Kakwani, N. and Pernía, M. (2000): "What is Pro-poor Growth?", *Asian Development Review*, 18(1): 1-16.

16. Kakwani, N. (2000): "On Measuring Growth and Inequality Components of Poverty with Application to Thailand", *Journal of Quantitative Economics*, 16(1): 67-80.

17. Kakwani, N. and Son, H. (2008): "Poverty Equivalent Growth Rate", *Review of Income and Wealth*, 54, 4: 643-655.

18. OCDE (2008): "Growing Unequal? Income Distribution and Poverty in OECD Countries", Paris.

19. Ravallion, M. and Chen, S. (2003): "Measuring Pro-Poor Growth", *Economic Letters*, 78: 93-99.

20. Sen, A.K. (1973): On Economic Inequality, Oxford University Press.

21. Shorrocks, A. F. (1983): "Ranking income distributions", *Economica*, 50: 3-17.

22. Son, H. (2004): "A note on Pro-Poor Growth", *Economic Letters*, 82: 307-314.

23. Stoline, M.R., and H.K. Ury (1979): "Tables of the Studentized Maximum Modulus Distributions and an application to multiple comparisons among means", *Technometrics*, 21: 87-93.

24. Wilks, S. S. (1962): Mathematical statistics, Nueva York: John Wiley & Sons.

25. Xu, K. (1997): Asymptotically distribution-free statistical test for the generalized Lorenz curve, Journal of Income Distribution, 7(1), 45-62.

26. Xu, K. and Osberg, L. (1998): A distribution-free test for deprivation dominance, Econometric Reviews, 17(4) 415-429.

APPENDIX

The variance of the ordinates of the generalized Lorenz curve taking into account only the *inequality effect* is as follows:

$$Var(GL_I) = \varpi_{ii}^{I} = \frac{1}{4}\left\{2\varpi_{ij}^{2,2} + 2\varpi_{ij}^{1,2} - \varpi_{ij}^{1,2^*} + \varpi_{ij}^{1^*,2}\right\} = \\ = \frac{1}{4}\left\{2Var(\mu_2 L_2(p)) + 2Var(\mu_1 L_2(p)) - Var\left[(\mu_1 + \mu_2)L_2(p)\right] + Var\left[(\mu_1 + \mu_2)L_1(p)\right]\right\} \quad \text{[A1]}$$

To be able to compute this expression we need to calculate each of its parts. To do this we can begin with $Var(\mu_1 L_2(p))$. This is the variance of the generalized Lorenz curve multiplying the Lorenz curve of the second period by the average income of period one.

On the other hand, the variance of the generalized Lorenz curve of the second period is (Beach and Davidson, 1983):

$$\varpi_{ij} = p_i\left[\lambda_i^2 + (1 - p_j)(\xi_{pi} - \gamma_i)^2\right] \quad \text{[A2]}$$

where, since *i=j*, $p_i = p_j$ is the quintile considered (0.1, 0.2,…); λ_i^2 is the conditioned variance of each quintile, and ξ_{pi}, γ_i are the maximum and the conditioned average of each quintile.

To calculate the generalized Lorenz curve $GL_{1,2}$, we scale the Lorenz curve of the second period by the income mean of the first period, or begin from the generalized Lorenz curve of the second period and multiply by μ_1 and divide by μ_2. Since this is only a change in scale, this will not affect the inequality as measured by the Lorenz curve, since this is a relative inequality measure. Then, if [A2] represents the variance of the generalized Lorenz curve of the second period, the variance of $Var(\mu_1 L_2(p))$ will be:

$$\varpi_{ij}^{1,2} = p_i\left[\left(\frac{\mu_1}{\mu_2}\right)^2 \lambda_{i,2}^2 + (1-p_j)\left(\frac{\mu_1}{\mu_2}\xi_{pi,2} - \frac{\mu_1}{\mu_2}\gamma_{i,2}\right)^2\right] \qquad \text{[A3]}$$

It is important to note that we are changing the scale of the distribution of the second period, so it will be the sample of this second period that is relevant to compute the statistical test.

Reasoning in a similar way, we have:

$$\varpi_{ij}^{1,2^*} = p_i\left[\left(\frac{\mu_1+\mu_2}{\mu_2}\right)^2 \lambda_{i,2}^2 + (1-p_j)\left(\frac{\mu_1+\mu_2}{\mu_2}\xi_{pi,2} - \frac{\mu_1+\mu_2}{\mu_2}\gamma_{i,2}\right)^2\right] \qquad \text{[A4]}$$

$$\varpi_{ij}^{1^*,2} = p_i\left[\left(\frac{\mu_1+\mu_2}{\mu_1}\right)^2 \lambda_{i,1}^2 + (1-p_j)\left(\frac{\mu_1+\mu_2}{\mu_1}\xi_{pi,1} - \frac{\mu_1+\mu_2}{\mu_1}\gamma_{i,1}\right)^2\right] \qquad \text{[A5]}$$

Then the variance of each ordinate of the generalized Lorenz curve that takes into account only the *inequality effect* will be:

$$
Var(GL_I)=\varpi_{ii}^{I}=\frac{1}{4}\left\{\begin{array}{l}
2\left(p_i\left[\lambda_{i,2}^2+\left(1-p_j\right)\left(\xi_{pi,2}-\gamma_{i,2}\right)^2\right]\right)+\\
2\left(p_i\left[\left(\frac{\mu_1}{\mu_2}\right)^2\lambda_{i,2}^2+\left(1-p_j\right)\left(\frac{\mu_1}{\mu_2}\xi_{pi,2}-\frac{\mu_1}{\mu_2}\gamma_{i,2}\right)^2\right]\right)\\
-\left(p_i\left[\left(\frac{\mu_1+\mu_2}{\mu_2}\right)^2\lambda_{i,2}^2+\left(1-p_j\right)\left(\frac{\mu_1+\mu_2}{\mu_2}\xi_{pi,2}-\frac{\mu_1+\mu_2}{\mu_2}\gamma_{i,2}\right)^2\right]\right)\\
+\left(p_i\left[\left(\frac{\mu_1+\mu_2}{\mu_1}\right)^2\lambda_{i,1}^2+\left(1-p_j\right)\left(\frac{\mu_1+\mu_2}{\mu_1}\xi_{pi,1}-\frac{\mu_1+\mu_2}{\mu_1}\gamma_{i,1}\right)^2\right]\right)
\end{array}\right\}
$$

[A6]

And the statistical test:

$$
T_{GLi}=\frac{\hat{G}L_I-\hat{G}L_1}{\left[\left(\frac{1}{4}\left\{\begin{array}{l}
2\left(p_i\left[\lambda_{i,2}^2+\left(1-p_j\right)\left(\xi_{pi,2}-\gamma_{i,2}\right)^2\right]\right)/N_2+\\
2\left(p_i\left[\left(\frac{\mu_1}{\mu_2}\right)^2\lambda_{i,2}^2+\left(1-p_j\right)\left(\frac{\mu_1}{\mu_2}\xi_{pi,2}-\frac{\mu_1}{\mu_2}\gamma_{i,2}\right)^2\right]\right)/N_2\\
-\left(p_i\left[\left(\frac{\mu_1+\mu_2}{\mu_2}\right)^2\lambda_{i,2}^2+\left(1-p_j\right)\left(\frac{\mu_1+\mu_2}{\mu_2}\xi_{pi,2}-\frac{\mu_1+\mu_2}{\mu_2}\gamma_{i,2}\right)^2\right]\right)/N_2\\
+\left(p_i\left[\left(\frac{\mu_1+\mu_2}{\mu_1}\right)^2\lambda_{i,1}^2+\left(1-p_j\right)\left(\frac{\mu_1+\mu_2}{\mu_1}\xi_{pi,1}-\frac{\mu_1+\mu_2}{\mu_1}\gamma_{i,1}\right)^2\right]\right)/N_1
\end{array}\right\}\right)+\left(\frac{\hat{\varpi}_{ii}^{1}}{N_1}\right)\right]^{1/2}}
$$

para i=1,2,…,K. [A7]

In this expression, all the variables re known or can be calculated.

Table A1.a
Generalized Lorenz and Pro-Poor Dominance,
Germany, 2000-2006

Generalized Lorenz Dominance			Pro-Poor Dominance		
Decile (1)	**$\hat{G}L_{00}$** (2)	**$\hat{G}L_{06}$** (3)	**T_{GL1}** (4)	**$\hat{G}L_1$** (5)	**T_{GL1}** (6)
1	632.26	414.11	-15.44*	400.17	-19.44*
2	1560.87	1257.81	-16.58*	1220.63	-19.73*
3	2660.08	2298.13	-16.02*	2230.68	-19.03*
4	3909.28	3496.15	-15.24*	3399.74	-18.22*
5	5274.97	4849.66	-13.64*	4719.85	-17.32*
6	6757.93	6370.29	-10.72*	6200.31	-14.84*
7	8418.21	8084.48	-7.77*	7873.24	-11.98*
8	10298.79	10055.50	-4.85*	9794.47	-9.47*
9	12515.64	12410.92	-1.73	12088.61	-6.57*
10	15950.20	16364.51	3.91*	15950.20	0.00

Five percent critical value for SMM=2.80

Table A1.b
Generalized Lorenz and Pro-Poor Dominance,
France: 2000-2006

Generalized Lorenz Dominance			Pro-Poor Dominance		
Decile (1)	**$\hat{G}L_{00}$** (2)	**$\hat{G}L_{06}$** (3)	**T_{GL1}** (4)	**$\hat{G}L_1$** (5)	**T_{GL1}** (6)
1	522.97	548.45	3.17*	554.10	3.61*
2	1308.89	1355.04	3.56*	1369.83	4.21*
3	2260.32	2315.71	3.14*	2340.85	4.07*
4	3353.11	3404.27	2.25*	3440.46	3.40*
5	4594.20	4624.49	1.06	4674.87	2.48
6	5985.89	5985.61	-0.01	6049.76	163
7	7550.41	7510.77	-0.97	7593.16	0.93
8	9325.17	9247.72	-1.59	9349.50	0.44
9	11447.91	11308.83	-2.34	11434.02	-0.21
10	14597.57	14438.36	-1.80	14597.57	0.00

Table A1.c
Generalized Lorenz and Pro-Poor Dominance,
Italy: 2000-2006

Generalized Lorenz Dominance			Pro-Poor Dominance		
Decile (1)	**$\hat{G}L_{00}$** (2)	**$\hat{G}L_{06}$** (3)	**T_{GL1}** (4)	**$\hat{G}L_1$** (5)	**T_{GL1}** (6)
1	256.17	316.97	11.68*	237.00	-3.32*
2	716.75	897.15	20.94*	672.46	-4.31*
3	1295.76	1623.59	27.61*	1216.84	-5.58*
4	1987.44	2489.92	31.41*	1865.98	-6.33*
5	2799.61	3505.80	35.15*	2627.95	-7.13*
6	3727.09	4668.91	38.80*	3499.91	-7.87*
7	4770.11	6003.82	43.16*	4503.68	-7.94*
8	5959.27	7552.96	47.99*	5673.34	-7.45*
9	7343.68	9424.02	53.12*	7093.55	-5.64*
10	9434.52	12473.11	52.08*	9434.52	0.00

Table A1.d
Generalized Lorenz and Pro-Poor Dominance,
Spain: 2000-2006

Generalized Lorenz Dominance			**Pro-Poor Dominance**		
Decile (1)	$\hat{GL}_{00}$ (2)	$\hat{GL}_{06}$ (3)	T_{GL1} (4)	$\hat{GL}_{1}$ (5)	T_{GL1} (6)
1	223.85	226.51	0.60	216.46	-1.50
2	607.86	637.98	4.10*	610.19	0.28
3	1090.93	1152.44	5.99*	1102.24	0.97
4	1660.52	1764.82	7.79*	1688.09	1.84
5	2317.65	2483.06	9.63*	2375.53	2.99*
6	3080.01	3312.91	11.10*	3170.31	3.83*
7	3956.15	4266.83	11.97*	4083.09	4.32*
8	4994.75	5379.23	12.52*	5147.87	4.44*
9	6261.29	6720.78	12.12*	6430.36	3.94*
10	8347.71	8728.52	6.60	8347.71	0.00

Table A1.e
Generalized Lorenz and Pro-Poor Dominance,
UK: 2000-2006

Generalized Lorenz Dominance			**Pro-Poor Dominance**		
Decile (1)	$\hat{GL}_{00}$ (2)	$\hat{GL}_{06}$ (3)	T_{GL1} (4)	$\hat{GL}_{1}$ (5)	T_{GL1} (6)
1	333.39	553.98	23.15*	296.86	-3.18*
2	875.35	1473.18	41.68*	794.57	-5.08*
3	1538.85	2616.76	57.44*	1417.77	-6.89*
4	2321.43	3964.10	69.90*	2152.71	-8.75*
5	3218.60	5534.26	78.03*	3013.63	-9.11*
6	4241.97	7343.69	77.00*	4010.56	-5.69*
7	5410.08	9412.87	82.42*	5151.55	-5.29*
8	6768.90	11828.96	85.79*	6484.45	-4.80*
9	8405.38	14796.43	88.94*	8137.39	-3.81*
10	11025.18	19859.12	64.21*	11025.18	0.00