Ambiguous Growth and Asset Prices in Production Economies

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January 2011

Preliminary and Incomplete.
Comments are welcome.

Abstract

We propose a dynamic stochastic general equilibrium model with production, where productivity growth follows a Markov-switching process with a hidden state, and households have generalized recursive smooth ambiguity preferences. The utility preferences allow for a three-way separation of risk aversion, ambiguity aversion, and the attitude toward intertemporal substitution. Our calibrated model can match the mean equity premium, the mean risk-free rate, the volatility of equity premium and the volatility of the risk-free rate. Our results also demonstrate that ambiguity about a hidden state governing the regimes of productivity growth is important to generate a variety of dynamic asset pricing phenomena, including predictable equity returns over long horizons, procyclical variation of the price-dividend ratio and countercyclical behavior of equity premia and the volatility of equity returns.

JEL Classification: C61; D81; G11; G12.

Keywords: Ambiguity, Equity premium, Predictability of returns, Production economy, Regime switching,

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1 Introduction

In the decades following the seminal work of Mehra and Prescott (1985), there has been numerous attempts to explain the equity premium puzzle. In particular, reconciling business cycle regularities with equity premium puzzle in a stochastic dynamic general equilibrium model (henceforth, DSGE) with production is viewed as challenging, see Jermann (1998), Boldrin et al. (2001), and Guvenen and Kuruscu (2006), among others. At the heart of all attempts to reconcile asset pricing and business cycle empirical regularities in the standard DSGE framework, it is commonly assumed that a unique probability law governs the dynamics of productivity growth and economic agents have complete confidence in this probability law. In short, all these papers assume that agents have rational expectations, following Muth (1961) and Lucas (1972), among many others. However, this approach may generate several well-documented predictions that are inconsistent with typical asset pricing phenomena found in financial data, among them the risk-free rate puzzle of Weil (1989), equity volatility puzzle of Shiller (1981), and the inability of resulting models to capture empirical dynamic asset pricing features such as pro-cyclical behavior of price-dividend (P/D) ratios, persistence and counter-cyclical behavior of the conditional volatility of equity premium, and other links between aggregate stock market returns and macroeconomic variables.[1]

A promising approach to resolve these puzzles documented in the literature is to allow economic agents to have multiple beliefs about the dynamics of state variables and to assume that agents are ambiguity-averse (see Anderson et al. (2003), Maenhout (2004), Epstein and Schneider (2008), among others). Recently, Ju and Miao (2010) employ generalized recursive smooth ambiguity preferences and successfully address a number of existing asset pricing puzzles in an endowment economy.[2] In this paper, we extend their work to production economies, which enables us to simultaneously examine business cycle and asset pricing implications, and thus resolve major asset pricing puzzles in a general equilibrium framework. Thus, a striking difference from Ju and Miao (2010) is that consumption and dividends are endogenously determined as outcomes of the social planner's problem, rather

[1]See Ju and Miao (2010) for a review of the links between financial data and macroeconomic aggregates.

[2]This utility specification nests Epstein and Zin (1989) utility, smooth ambiguity preferences of Klibanoff et al. (2005, 2009), risk sensitive preferences of Tallarini Jr. (2000), Anderson et al. (2003), Hansen and Sargent (2008), and multiplier preferences of Hansen and Sargent (2001, 2008).

than exogenously given in an endowment economy as in Ju and Miao (2010). Furthermore, asset prices are defined as prices of capital and closely related to Tobin's q.

Our model has three main features. First, we model productivity growth rates as following a Markov-switching process with two different regimes and a hidden state. Economic agents cannot observe the hidden state governing the transition between regimes but learn about the hidden state based on past productivity growth data. The one-step ahead state beliefs together with capital dynamics and productivity shocks summarize time-varying economic uncertainty and are the driving forces of asset prices dynamics. Second, as in Ju and Miao (2010), we assume that agents are ambiguous about the hidden state that governs the regimes of productivity growth and households' preferences are represented by the generalized recursive smooth ambiguity model employed by Ju and Miao (2010). As a result, ambiguity is modeled by fluctuating uncertainty about the hidden state while ambiguity aversion is captured by some concave transformation of the certainty equivalent in the value function. This concave transformation reflects the agents pessimistic view about the future continuation value. Thanks to the adopted preferences, our model can produce a richer set of dynamic asset pricing phenomena than recent works including Kaltenbrunner and Lochstoer (2010) and Croce (2008). Third, we introduce convex capital adjustment costs following Campanale et al. (2010). Capital adjustment costs are introduced to produce barriers to consumption smoothing such that there is a significant amount of aggregate risks to be priced. Ambiguity aversion, on the other hand, increases the price of risk. Our model with the above three ingredients can generate high mean equity premium, low mean risk-free rate, high volatility of equity returns, counter-cyclical variation of equity premium and equity volatility and long-horizon return predictability that are typically found in financial data while our calibration also produces relatively smooth risk-free rates.

Our work contributes to a large body of literature that Cochrane (2008) calls the "macro-finance" literature. An earlier review of this literature is found in Campbell (1999). Two seminal papers by Jermann (1998) and Boldrin et al. (2001), use "habit formation" preferences and introduce capital adjustment costs in a production economy to simultaneously match business cycle and financial moments.[3] They are the first successful attempts in matching the equity premium and macroeconomic moments in the literature, but they can not address the excess volatility in risk-free rates generated by their models. Neither do their models generate a wide range of dynamic asset pricing phenomena. Campanale et al. (2010) use the "Chew-Dekel" class of preferences to show that applying Epstein and Zin (1989) preferences, as a special case of Chew-Dekel class, in a production economy to match business cycle and equity premium moments generates counter intuitively high relative risk aversion coefficient values. They argue that using other members of this class, for example Gul (1991) "disappointment aversion" preferences, are more promising in a production economy framework.[4]

Extending the seminal work of Bansal and Yaron (2004) on "long-run risks" to a production economy framework, Kaltenbrunner and Lochstoer (2010) and Croce (2008) use Epstein and Zin (1989) preferences in conjunction with long-run risks to match equity premium and business cycle moments in production economies.[5] Kaltenbrunner and Lochstoer (2010) calibrate two types of long-run risk models, one with permanent and the other with transitory productivity shocks. They find relatively large trade-offs between matching the business cycle and financial moments especially the volatility of risk-free rate. Even with a discount factor higher than 1, their models cannot address most of major asset pricing puzzles. In addition, their model cannot explain long-horizon predictability in returns and typical dynamic asset pricing phenomena. In a different framework, Guvenen (2009) uses a heterogeneous agents model to solve the excess volatility in risk free rates. He assumes that there are two types of agents in the economy, bond holders who have low elasticity of intertemporal substitution in consumption (henceforth, EIS), and stock holders with high EIS. Thus, separating the SDF (also known as the pricing kernel) for pricing bonds and stock returns. This method addresses the excess volatility of the risk free rate problem. In our research, we show that it is possible to generate comparable results in the standard

[3]In a production economy without capital adjustment costs, volatility of model generated rate of return on capital is low. Using various specifications for preferences in such an economy is not sufficient to boost this volatility.

[4]There is renewed interest in this class of preferences in recent years. Examples include Routledge and Zin (2010), Bonomo et al. (2011), and Feunou et al. (2010) in endowment economies.

[5]Long-run risks are defined as consumption and dividend growth rates that contain a small persistent predictable component; see Bansal and Yaron (2004).

representative agent framework.

The rest of the paper is organized as follows. Section 2 presents the baseline economy and defines the equilibrium. We discuss the data used and the calibration procedure in Section 3. In Section 4 we discuss our findings about the financial and business cycle quantities in this economy. Section 5 concludes.

2 The Model

We consider a one-sector DSGE model as other papers. The economy has two types of agents: the representative household, and firms. An exogenous stochastic process of productivity, which follows a two-state Markov switching process, drives uncertainty in this economy. The agents cannot observe the hidden state governing the regimes of the Markov-switching process but can learn about the hidden state by observing past productivity data. Our economy has one source of real friction, convex capital adjustment costs .

2.1 The Representative Agent

The representative household agent chooses consumption to maximize generalized recursive smooth ambiguity utility, given the budget constraint. The agent receives dividends and capital rents from the firms. There are two means to smooth consumption: purchase of one-period non-state contingent real bonds, and investments. In our model, the supply of labor is inelastic. The representative agent has utility preferences defined over aggregate consumption process:

$$\begin{aligned} U_t(C) &= \left[(1-\beta)C_t^{1-\rho} + \beta\left\{\mathcal{R}_t\left(U_{t+1}\left(C\right)\right)\right\}^{1-\rho}\right]^{\frac{1}{1-\rho}} \\ \mathcal{R}_t\left(U_{t+1}\left(C\right)\right) &= \left(\mathbb{E}_{\zeta_t}\left[\left(\mathbb{E}_{s_{t+1},t}\left[U_{t+1}^{1-\gamma}\left(C\right)\right]\right)^{\frac{1-\eta}{1-\gamma}}\right]\right)^{\frac{1}{1-\eta}} \end{aligned} \tag{2.1}$$

where $\mathcal{R}_t\left(U_{t+1}\left(C\right)\right)$ is a certainty equivalent measure of future utilities under ambiguity aversion, γ is the risk aversion coefficient, η is the ambiguity aversion coefficient and ρ is the inverse of the elasticity of intertemporal substitution.[6] The inner expectation in $\mathcal{R}_t\left(U_{t+1}\left(C\right)\right)$

[6]See Ju and Miao (2010) for a more detailed discussion

is taken with respect to the predictive distribution of the state variable given the next period state. The outer expectation is taken with respect to the one-step-ahead conditional probability distribution of future states given current information. If the representative agent is ambiguity-neutral ($\eta = \gamma$), the certainty equivalent $\mathcal{R}_t$ reduces to the familiar certainty equivalent of future utilities in Epstein and Zin (1989), which implies that compound lotteries can be reduced and expectation of the continuation value is equivalent to a Bayesian probability weighted average of expected future utilities under the predictive distribution conditioning on state s. When the agent is ambiguity-averse ($\eta > \gamma$), compound lotteries cannot be reduced and concave transformation is applied to the certainty equivalent to yield aversion to lower continuation values.

The budget constraint for the representative agent is given by

$$C_t + I_t = Y_t, \tag{2.2}$$

where C_t, I_t, and Y_t represent consumption, investment, and the aggregate output in this economy.

2.2 The Firm

The consumption good is produced according to a constant return-to-scale Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} \left(A_t N_t\right)^{1-\alpha} \tag{2.3}$$

where Y_t is the output, K_t is the capital stock, N_t is the amount of labor hours, and A_t is the aggregate productivity shock.[7] Labor input is assumed to be exogenous and equal to $\bar{N}$.[8] Uncertainty in the economy is driven by productivity growth dynamics. The productivity growth rate $\Delta a_{t+1} \equiv \log\left(\frac{A_{t+1}}{A_t}\right)$ follows a Markov-switching process with two regimes

$$\Delta a_t = \mu\left(s_t\right) + \sigma \epsilon_t, \qquad \epsilon_t \sim N\left(0, 1\right) \tag{2.4}$$

where s_t is unobservable and can take one of two different values representing possible states of the economy. We denote the good or high productivity growth state as $s_t = 1$, and the

[7] A_t can also be viewed as an exogenous, labor-enhancing technology level. Thus, Δa_t is the productivity growth rate.

[8] We do not consider leisure in the utility function.

low growth or bad state as $s_t = 2$. The transition probability matrix is given by

$$\mathbf{P} = \begin{bmatrix} P_{11} & 1 - P_{11} \\ 1 - P_{22} & P_{22} \end{bmatrix}. \tag{2.5}$$

Here, the one-step-ahead conditional probability (or equivalently, state probability) of the hidden state ζ_t denotes $\zeta_t = \Pr(s_{t+1} = 1 \mid \mathcal{I}_t)$, where '1' represents the good regime. The conditional probabilities ζ_t are updated over time in Bayesian fashion given the prior belief ζ_0 :

$$\zeta_{t+1} = \frac{P_{11} f(\Delta a_{t+1}|1)\,\zeta_t + (1 - P_{22})\, f(\Delta a_{t+1}|2)\,(1 - \zeta_t)}{f(\Delta a_{t+1}|1)\,\zeta_t + f(\Delta a_{t+1}|2)\,(1 - \zeta_t)}$$

where $f(z|i) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-(z - \mu_i)^2 / (2\sigma^2)\right]$ is the density function of the normal distribution with mean μ_i and variance σ^2.

The capital stock evolves according to

$$K_{t+1} = (1 - \delta_k) K_t + I_t - G(K_t, K_{t+1}) \tag{2.6}$$

$$G(K_t, K_{t+1}) = \left|\left(\frac{K_{t+1}}{K_t} - \omega\right)\right|^{\iota} K_t, \quad \iota > 1, \omega > 0 \tag{2.7}$$

where function G is the convex capital adjustment cost function which introduces frictions. The functional form of the capital adjustment cost follows the formulation of Campanale et al. (2010). Given the constant returns to scale assumption about production technology and the assumption that net investment is incorporated in the capital stock within one period, the production economy presented here is a standard real business cycle economy.

Firms are fully owned by the representative household. Each firm issues a single stock which is traded. Firms choose labor inputs (which are exogenously given), the amount of investment expenditure and capital stock to maximize the present value of all future profits. The first-order conditions for the firms' optimization problem are standard and have the usual interpretation. For brevity, the first-order conditions are not shown here.

Following the DSGE literature, for example Jermann (1998), Boldrin et al. (2001), and Campanale et al. (2010) among many others, we view return on the net capital stock to be the return on equity. We represent this return as R_t^e and treat it as an unlevered equity claim, given the production technology in the model; see Kaltenbrunner and Lochstoer (2010). It can be shown, for example in Campanale et al. (2010) and in the Appendix to this paper, that

$$R^e_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

$$R^e_{t+1} = \frac{D_{t+1} + [1 + G_{K_{t+2}}(K_{t+1}, K_{t+2})]K_{t+2}}{[1 + G_{K_{t+1}}(K_t, K_{t+1})]K_{t+1}}, \tag{2.8}$$

where $1 + G_{K_{t+i}}(K_{t+i-1}, K_{t+i})$ specifies Tobin's q in this model and dividends are given by $D_t = \alpha Y_t - I_t$.

2.3 The Stochastic Discount Factor

In our model, we consider that the representative agent prices two assets, a one-period non-state contingent real bond, and equity that is contingent on aggregate uncertainty.[9] The representative agent prices assets using the stochastic discount factor (SDF) defined over aggregate consumption. In this section, we outline the construction of the SDF in our DSGE model.

First, we construct and solve the social planner's problem. Equilibrium allocations are derived as the outcomes of the optimal solutions.[10] To do so, we first define the following stationary variables:

$$\{c_t, i_t, y_t, k_t, u_t\} = \left\{\frac{C_t}{A_{t-1}}, \frac{I_t}{A_{t-1}}, \frac{Y_t}{A_{t-1}}, \frac{K_t}{A_{t-1}}, \frac{U_t}{A_{t-1}}\right\} \tag{2.9}$$

where A_{t-1} denotes period $t-1$ productivity, and C_t, I_t, Y_t, K_t, and U_t are consumption, investment, output, capital stock, and continuation value at time t, respectively. As shown in Section 2.2, the process $\{A_t\}_{t=0}^{\infty}$ is not *i.i.d.* distributed. This is in contrast with construction of the above stationary series in Campanale et al. (2010), who build these series using *i.i.d.* cumulative productivity shocks. In constructing the series in (2.9), we follow Croce (2008), who model productivity growth as having a persistent component and a GARCH component.

[9] Our model can be extended to study cross-sectional asset returns and other contingent claims such as options.

[10] See Cagetti et al. (2002) and Campanale et al. (2010) for two recent examples in continuous and discrete time, respectively.

The social planner's problem, in recursive form, is given by

$$u\left(k_t, \Delta a_t, \zeta_t\right) = \max_{c_t, k_{t+1}} \Big\{ (1-\beta)\, c_t^{1-\rho} + \beta e^{(1-\rho)\Delta a_t} \left(\mathbb{E}_{\zeta_t}\left[\left(\mathbb{E}_{s_{t+1},t}\left[v_{t+1}^{1-\gamma}\left(k_{t+1}, \Delta a_{t+1}, \zeta_{t+1}\right)\right]\right)^{\frac{1-\eta}{1-\gamma}}\right]\right)^{\frac{1-\rho}{1-\eta}} \Big\}^{\frac{1}{1-\rho}} \tag{2.10}$$

subject to the following constraints:

$$y_t = \exp[(1-\alpha)\,\Delta a_t] k_t^{\alpha} \bar{n}^{1-\alpha} \tag{2.11}$$

$$\exp[\Delta a_t] k_{t+1} = (1-\delta_k)\, k_t + i_t - \left|\left(\frac{e^{\Delta a_t} k_{t+1}}{k_t} - \omega\right)\right|^{\iota} k_t \tag{2.12}$$

$$c_t \geq 0, k_{t+1} \geq 0,$$

where y_t, i_t, and k_t are as defined in (2.9), and Δa_t follows the process specified in Equation (2.4).

Ju and Miao (2010) show that the pricing kernel (or SDF) for generalized recursive ambiguity preferences is given by

$$M_{s_{t+1},t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left(\frac{U_{t+1}}{\mathcal{R}_t\left(U_{t+1}\right)}\right)^{\rho-\gamma} \left(\frac{\left(\mathbb{E}_{s_{t+1},t}\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_t\left(U_{t+1}\right)}\right)^{-(\eta-\gamma)}, \tag{2.13}$$

with $s_{t+1} = 1, 2$, which is equivalent to

$$M_{s_{t+1},t+1} = \beta e^{-\rho \Delta a_t} \left(\frac{c_{t+1}}{c_t}\right)^{-\rho} \left(\frac{u_{t+1}}{\mathcal{R}_t\left(u_{t+1}\right)}\right)^{\rho-\gamma} \left(\frac{\left(\mathbb{E}_{s_{t+1},t}\left[u_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_t\left(u_{t+1}\right)}\right)^{-(\eta-\gamma)}, \tag{2.14}$$

where $s_{t+1} = 1, 2$. As usual, the Euler condition must be satisfied, as a summarization of the representative firm's optimality condition of investment. That is,

$$\mathbb{E}_t\left[M_{s_{t+1},t+1} R_{t+1}^{e}\right] = 1. \tag{2.15}$$

3 Model Calibration

We first describe stylized facts about the financial data in the US. We then calibrate our model, and study unconditional and conditional moments of returns, risk-free rate, and business cycle quantities generated by our model. Due to nonlinearities, our model does not admit an explicit analytical solution. Thus, we solve the model numerically, using the value function iteration method and then run Monte Carlo simulations to compute the desired model moments.[11]

We consider the full information model as our baseline model where the state of the economy is fully observable and capital dynamics and productivity shocks are the only two state variables driving aggregate uncertainty. This corresponds to a special case of our ambiguous growth model if neither ambiguity nor ambiguity aversion exists. The full information model serves as a benchmark to shed light on the role of learning and is similar to the rational expectation models with Epstein-Zin preferences considered by Croce (2008) and Kaltenbrunner and Lochstoer (2010). After introducing learning and ambiguity aversion, we are able to compare the implications of the ambiguous growth model with Epstein-Zin preferences to those of the ambiguous growth model with generalized recursive smooth ambiguity preferences to highlight the impacts of ambiguity aversion.

3.1 Data

Our data on asset returns are drawn from CRSP (Center for Research in Security Prices) and FRED II data bank at the Federal Reserve Bank of St. Louis. Data on consumption, outputs, and interest rates are all from FRED II. The nominal risk-free rates correspond to the 3-month Treasury bill rates. We define outputs as the sum of investment, exports, consumption of perishables, non-perishables, and services. We construct price-dividend ratio series following the methodology in Campanale et al. (2010). All values are deflated using the CPI data from FRED.

We use a newly available data set on total factor productivity (TFP) from Federal Reserve Bank of San Francisco. This data set contains a quarterly, non-utilization adjusted series for TFP.[12]. We apply the EM (expectation maximization) algorithm in Hamilton (1989) to changes in TFP data to estimate transition probabilities in the two-state Markov-switching model where parameter values of μ_1, μ_2 and σ are specified exogenously. As will be shown later, this is done to highlight the effects of ambiguity and ambiguity aversion. This choice of calibration is without loss of generality because the estimated transition probabilities strongly suggest two distinct regimes with the high productivity state regime being persistent and

[11]The code is written in Compaq Visual FORTRAN and available upon request from the authors.

[12]See Fernald (2009) for a detailed discussion

the low productivity regime transitory.

3.2 Calibration of model parameters

Table 1 reports the parameter values which are held constant across all models in this paper. Coefficient of risk aversion, γ, is set to 5 throughout the paper. This value is in the middle of the range viewed as reasonable by Mehra and Prescott (1985) and very modest compared to Croce (2008) and Tallarini Jr. (2000). We set capital share (α) to 0.35, following Boldrin et al. (2001) and Kaltenbrunner and Lochstoer (2010). The discount factor β is set to $\beta = 0.9935$ instead of being larger than 1 as in Kaltenbrunner and Lochstoer (2010). Exogenous labor supply, $\bar{N}$, is set equal to 0.20, which is very close to the value considered by Croce (2008). We set the exponent of the adjustment costs function, ι, equal to 1.082, to generate a significant amount of frictions and thus volatile enough asset returns. Quarterly depreciation rate of capital is set at $\delta = 0.015$, which implies a 6% annualized depreciation rate, following Croce (2008). We choose parameter values of μ_1, μ_2, and σ such that given estimated transition probabilities P_{11} and P_{22}, we can match the mean and the standard deviation of the total factor productivity growth series. As shown in Equation (2.4), subscripts 1 and 2 stand for "good/high" and "bad/low" productivity growth regimes. Figure 1 plots the filtered probabilities based on total factor productivity data. The figure shows that a Markov-switching model can effectively identify two distinct regimes where the good regime is very persistent. The parameter ω in Equation (2.7) is set to $\exp(\Delta\bar{a}) = P_{SS}\mu_1 + (1 - P_{SS})\mu_2$ where P_{SS} is the steady-state probability of the high productivity regime.

4 Discussion of the Main Results

The purpose of this section is the following. First, we want to understand the impact of ambiguity aversion and learning about productivity growth states on asset prices, by varying the level of ambiguity aversion, η. We also compare the performance of our ambiguous growth model to the full information model. In addition, we present comparative statics results for different values of the EIS parameter, ψ. Second, we attempt to show the existence of some parameter sets (ψ, γ, η) that lead to sensible implications regarding the risk-free rate and the equity returns, as well as the market price of risk. Third, we want to demonstrate the ability of our ambiguous growth model to replicate some well-known dynamic asset pricing phenomena documented in financial data, such as long-term predictability of equity returns and cyclical variation in equity premium and volatility. Interestingly, we also uncover some new patterns and results regarding the impact of ambiguity aversion and learning about productivity growth states on the conditional price of risk, $\sigma(M)/\mathbb{E}(M)$, conditional equity premium and the volatility of the equity premium, and conditional mean and volatility of the model-generated price-dividend ratios. Fourth, we examine the ability of our model to reproduce a set of business-cycle moments that are commonly considered in the literature.

4.1 Asset Returns and Equity Premium

We present some stylized facts about annualized CRSP value-weighted real index returns, three-month U.S. Treasury Bill rates, equity premium, Sharpe and price-dividend ratios for the sample period 1947-2006. The results are shown in Table 2. As seen from the table, the annualized equity premium is approximately 8%, a well-documented stylized fact dating back to Mehra and Prescott (1985). This level is significantly higher than the level considered by Ju and Miao (2010). The annualized standard deviation of excess equity returns is approximately 16% a year. The data also suggest a low mean ($\mathbb{E}(r_f) = 0.0126$) and low volatility ($\sigma(r_f) = 0.0126$) of risk-free rates. The mean of price-dividend ratios in our sample is larger than that reported by Croce (2008). The Sharpe ratio implied by our sample is large compared to the one used by Kaltenbrunner and Lochstoer (2010). This is because their sample contains observations from 1926 to 1998 while our sample includes exclusively post-WWII data.

Figure 2 plots the impulse response function of both business cycle quantities and asset prices after a temporary shock to productivity growth. The graphs show the percentage deviation from the trend after a single positive pulse shock to productivity growth with a size of 4σ. We assume that the economy starts at its deterministic steady state and the shock realizes at the second period. Given a one-time positive shock to productivity growth, the one-step-ahead conditional probability ζ will be revised upward and then revert back to its steady-state level. The agent believes that the economy is more likely to be in the high productivity growth regime, which in fact reinforce the effects of the shock. At the same time, it is optimal for the representative agent to increase consumption and investment. Consumption increases steadily to reflect consumption smoothing given that investment increases, and capital stock also accumulates at a higher rate relative to its trend without shocks. It is also worth noting that the risk-free rate responds positively to the shock. This is because expected consumption growth increases in response to the shock, and as a result, the representative agent is less willing to save, which results in a rise in the risk-free rate.

Table 3 summarizes our main findings regarding the first and second unconditional moments of the risk-free rate and equity returns. Our benchmark calibration ($\psi = 1.5, \gamma = 5, \eta = 53$) of the ambiguous growth model can reproduce most of salient features found in the data. We choose the ambiguity aversion parameter, η, such that our model can closely match the equity premium and Sharpe ratio. With a modest risk aversion ($\gamma = 5$) and reasonable discount factor ($\beta < 1$), our model can generate a high equity premium (8%), volatile equity returns ($\sigma(r_{ep}) = 16\%$), and a high Sharpe ratio (0.5) while the mean risk-free rate is kept low (1.74%), and more important, the risk-free rate is not excessively volatile (0.5%). Thus, our benchmark calibration represents a significant improvement relative to previous works such as Kaltenbrunner and Lochstoer (2010), Croce (2008), Jermann (1998) and Boldrin et al. (2001).[13] The ability of our model to generate such a high equity premium is mainly attributed to the effect of ambiguity aversion on the price of risk ($\sigma(M)/\mathbb{E}(M)$). To understand the rationale, we present the following general relationship between equity premium and the price of risk:

$$\mathbb{E}_t (R_{t+1}) - R_{f,t} = -\frac{\sigma_t (M_{t+1})}{\mathbb{E}_t (M_{t+1})} \sigma_t (R_{t+1}) \rho_t (M_{t+1}, R_{t+1})$$

Note that in (2.13) ambiguity aversion introduces an extra multiplicative term. When the size of ambiguity aversion is large, this term significantly magnifies the volatility of the pricing kernel and thus the price of risk. However, different from the endowment economy case considered by Ju and Miao (2010), higher ambiguity aversion also reduces the amount of risk (this is quantified by the term $-\sigma_t (R_{t+1}) \rho_t (M_{t+1}, R_{t+1})$) in our production economy. This is because the variation in the price of capital decreases under ambiguity aversion. Even so, the effect of ambiguity aversion on the price of risk dominates over in most cases. As a result, the resulting equity premium is high due to the aversion to ambiguity about the underlying

[13] Kaltenbrunner and Lochstoer (2010) and Croce (2008) cannot reproduce high equity premium and/or Sharpe ratio with modest values of γ and β. Kaltenbrunner and Lochstoer (2010), Jermann (1998) and Boldrin et al. (2001) generate excessively volatile risk-free rates.

state governing productivity growth. Intuitively, when the agent is more ambiguity averse, he assigns more weight to the lower continuation value state (this corresponds to the low productivity regime) and is less willing to invest into the technology. Thus, capital must pay off considerably, leading to high equity premium. The calibrated ambiguity aversion ($\eta = 53$) in our benchmark model is much higher than that in Ju and Miao (2010). Two reasons drive this result. First, our targeted equity premium, 8%, is much higher than the level considered by Ju and Miao (2010). A high η is therefore needed. Second, consumption and dividends are endogenously determined in our production economy model. Consumption smoothing offsets, to some extent, the effect of ambiguity aversion on equity premium.

To maximize our model's ability of reproducing stylized facts about financial data, we choose the parameter in the adjustment costs function, ι, such that the volatility of excess equity returns can be closely matched. As will be clear, this comes at a cost: the model implied consumption growth becomes excessively volatile given high marginal adjustment costs. Our model can generate low mean risk-free rate through the precautionary savings channel. As the agent becomes more ambiguity averse, he is more willing to use the risk-free rate as a tool for consumption smoothing. Thus, the mean of the risk-free rate becomes low. Unlike Boldrin et al. (2001) and Kaltenbrunner and Lochstoer (2010), among others, our model does not generate excessive volatility in the risk-free rate. The success is attributed to the persistence in state probabilities. As a consequence, we do not see a large amount of variation in the conditional mean of the pricing kernel. The low volatility in the conditional mean of the pricing kernel then translates into low volatility in the risk-free rate.

For the purpose of comparison, we also present simulated results for the full information model where the regime-switching state is completely observable (the first line of Panel B, C and D) and for various values of the ambiguity aversion parameter. None of these parameter configurations are as successful as the benchmark calibration in reproducing the salient features of asset prices. However, it is worth noting that our full information model with Epstein-Zin preferences and $\gamma = 5$ can still generate 4.7% equity premium, which is higher than the level in Croce (2008)'s long-run production risk model with a risk aversion of 30. The first two lines of results in Panel D of Table 3 correspond to the expected utility case, where risk aversion and intertemporal substitution are inversely related. The second line summarizes results for expected utility with Bayesian learning. The next two cases

(η = 20 and 40) allow for separation between risk aversion and ambiguity aversion but not between risk aversion and intertemporal substitution. Although in Panel D, the implied equity premium can reach high levels, the mean risk-free rate and equity volatility are largely counterfactual.

Figure 3 shows that the price of risk displays an asymmetric and hump-shaped response to the state probability as ambiguity aversion increases, where capital stock is set to its mean and productivity growth to its steady-state level in the simulations. The feature that the price of risk displays hump-shape when plotted against the state probability is typical in the asset pricing literature (see, for example, Veronesi (1999) and Cagetti et al. (2002)). When the agent is unsure about the current state of the economy (ζ takes values around 0.5), aggregate uncertainty is high. Here, the effect of ambiguity aversion is to turn the usual hump shape into a highly skewed one. Furthermore, the skewness is toward to the upper boundary of state probabilities. An important implication is that when state probability is persistent and takes values near around 1, a small decline in the state probability, as a result of Bayesian updating following a negative shock to productivity growth, will lead to much aggregate uncertainty. Thus, ambiguity aversion not only increases the level of the price of risk for all possible values of the state probability but also changes the way of aggregate uncertainty in response to variation in beliefs. In Panel I of Figure 4, we observe that the level of the conditional equity premium also displays an asymmetric hump-shaped response with respect to the state probability ζ. This response is strongly right-skewed for high values of η. In Panel II of Figure 4, we observe that the volatility of the conditional equity premium displays a hump-shaped, but generally symmetric, response to the state probability. In addition, the volatility of the conditional equity premium is smaller for higher η values. This finding is in contrast to those of Ju and Miao (2010).

Our model can also generate countercyclical variation in equity premium and conditional volatility of equity premium that are typically found in the data. Since we study production economies, we simulate productivity growth rates and output growth rates as indicators of business cycles, where output growth rates are endogenously determined from equilibrium allocations. We observe in Figure 7 that the full information model with Epstein-Zin utility implies a correlation of -0.46 between equity premium and output growth and a correlation of -0.23 between conditional volatility of equity premium and output growth. In Figure

8, the Bayesian learning model with Epstein-Zin utility performs marginally better than the full information model, with a correlation of -0.56 between equity premium and output growth and a correlation of -0.33 between conditional volatility of equity premium and output growth. However, as shown in Figure 9, our benchmark model generates strong correlations of equity premium and conditional volatility of equity premium with output growth.

The upper left hand side panel in Figure 5 shows that with ambiguity aversion parameter, η, equal to 5, which is equivalent to Kreps and Porteus utility and hence ambiguity neutral, model generated expected price-dividend ratios have an inverses and almost linear relationship with ζ_t. That is, the higher the value of the probability of being in a good state, the expected P/D ratio is smaller.

The upper right hand side panel in Figure 5 demonstrates the beginning of one the intriguing results we have found in our research. As is seen in this panel, once the agent moves from ambiguity neutral preferences, $\eta = 5$, to ambiguity aversion in preferences, $\eta = 20$, the linear and downward sloping pattern observed in the previous panel changes dramatically. First, we observe a slightly convex relationship between expected equity premium and ζ_t. This implies that as values of ζ_t increase, which implies confidence in higher productivity in the next period, the agent assigns a higher expected price-dividend ratio. In our opinion, this observations means that with a constant level of dividend payments, the agent is willing to pay a higher price for the asset, since the agent is confident that higher future productivity justifies this premium.

This pattern is more pronounced in the lower left hand side panel of Figure 5. We notice an increase in the convexity of this relationship as the agent becomes more ambiguity averse (η increases from 20 to 40). In fact, there seems to be a minimum expected price-dividend ration in the neighborhood of ζ_t between 0.20 and 0.40. We interpret this result as such: with ζ_t to the right hand side of the minimum expected price-dividend ratio neighborhood, we believe that the agent is willing to pay a premium for the asset (manifested in higher prices) since expected higher future productivity justifies this premium. Notice that even for lower levels of expected paid dividends, as long as the premium that the agent is willing to pay is high enough, this pattern of increasing expected price-dividend ratio holds. We also notice higher expected price-dividend ratios for values of ζ_t to the left hand side of the minimum expected price-dividend ratio neighborhood. In this case, we believe that the agent is not paying a premium for expected lower productivity growth, but rather the agent expects lower expected dividends, or dividend payments that fall more than the fall in the expected price levels.

The lower right hand side panel of Figure 5, which depicts the case with the highest level of ambiguity aversion ($\eta = 53$), the convexity of expected price-dividend ratio with respect to the probability of the economy being in the good state in the next period is even more pronounced than what is seen in the previous panels. In this panel, we observe an almost parabolic relationship between expected price-dividend ratios and ζ_t. The lowest observed point in this function roughly coincides with $\zeta = 0.50$ neighborhood. This means that when the agent is undecided about the state of the economy in the next period, he expects the lowest price-dividend ratio. As the agent becomes more convinced, either in positive or negative direction, about the future state of the economy, expected price-dividend ratios rise. Similar to the previous case, these changes can be due to higher premium paid for holding an asset due to higher probability of the economy to be in the good state in the next period, or due to expectations of lower future paid dividends.

We call the observed patters in panels of Figure 5 that pertain to ambiguity aversion as expected price-dividend ratio "smirk" (for $\eta = 20$ and $\eta = 40$) and "smile" (for $\eta = 53$).

Another interesting pattern that emerges in Figure 5, is the magnitude of expected price-dividend ratios. As is seen in the Figure, starting in the upper left hand side panel, and moving in rows, an increase in ambiguity aversion is accompanied by a drop in the magnitude of the expected price-dividend ratio values. Consider the case of upper left hand side and lower right hand side panels in this figure. These two panels display the ambiguity neutrality ($\eta = 5$) and the highest level of ambiguity aversion ($\eta = 53$) in our model. It is immediately obvious that first, the level of expected price-dividend ratio in ambiguity neutral case is much higher than what is observed in the other panel; and second, the repones of expected price-dividend ratios to changes in ζ_t is much smaller, the higher the level of ambiguity aversion is. We believe that this muted response in the presence of ambiguity aversion is due to the fact that ambiguity aversion implies a high degree of pessimism for the agent; see Hansen and Sargent (2008) or Epstein and Schneider (2007). In this case, the agent is already very pessimistic and any change in ζ_t, while changing the level of expected P/D ratio, does not change the magnitude of this variable. On the other hand, with $\eta = 5$, the agents do not have ambiguity aversion in their preferences. Thus, changes in expectations of the future state of the economy have a more significant impact on the magnitude of expected P/D ratios.

4.2 Return predictability

A large body of literature has documented predictability of equity returns with respect to variations in the price-dividend ratio or dividend yield.[14] In particular, Cecchetti et al. (2000) document increasing values for slope parameters and R^2 for regressing the k-year ahead equity returns on the current P/D ratio. Ju and Miao (2010) show that in endowment economies, their baseline model with Bayesian learning and ambiguity, the full-information model and the Bayesian model with Epstein-Zin preferences can reproduce the increasing regression slopes but cannot capture the pattern of increasing R^2s. Instead, their baseline model generates the pattern of decreasing R^2s with respect to return horizons.

In Table 4, we report results from regressing equity returns and excess returns onto log P/D ratio. Results are generated from 20,000 Monte Carlo simulations for different return horizons, i.e., 1 quarter, 1 year, 2, 3, 5 and 7 years. We present results of our benchmark model ($\psi = 1.5, \gamma = 5, \eta = 53$) in Panel A of Table 4, results of Epstein-Zin model with Bayesian learning in Panel B and results of our ambiguous growth model with ($\psi = 0.6, \gamma = 5, \eta = 53$) in Panel C. We show regression results for both equity returns being the dependent variable and excess returns being the dependent variable in order to demonstrate that variation in risk-free rates almost plays no role in return predictability. This is in contrast to Campanale et al. (2010), where predictability in excess returns is mostly driven by variation in risk-free rates.

It can be seen that our benchmark model can not only capture the pattern of increasing slopes (in magnitude) but also the pattern of increasing R^2s, though the overall predictability is still weak. This holds true for both equity returns as the dependent variable and excess returns as the dependent variable. The sign of slopes is negative because the independent variable is the current log P/D ratio. Furthermore, based on arguments elaborated by Lewellen et al. (2010), our model-generated R^2s may lie within the 95% confidence intervals

[14] Examples include Campbell (1987), Campbell and Shiller (1988a,b), Cecchetti et al. (2000), Welch and Goyal (2008), Campbell and Yogo (2006) among many others.

of R^2 constructed for a variety of pricing ratios, including D/P and dividend yields. Our benchmark model's ability to generate predictability is due to two reasons. First, our model generates negatively serial correlation in equity returns and excess returns, which implies that the variance ratio is declining in return horizon. The average first-order autocorrelation of excess returns implied by our benchmark model is about -0.13. Second, our model generates highly persistent price-dividend ratios, with the average first-order autocorrelation being about 0.97. The persistence in the price of capital is in fact inherited from the persistence in state probabilities. For Epstein-Zin model with Bayesian learning, we find the pattern of increasing R^2s, but the sign of the slopes for long horizons (i.e., 5 and 7 years) is counter-intuitive. In addition, predictability appears to be weaker than the benchmark case. Similar results can be obtained for the ambiguous growth model with a lower EIS parameter value ($\psi = 0.6$) (see Panel C of Table 4).

4.3 Business Cycle Dynamics

Under our benchmark parametrization specified in Table 1, we generate unconditional moments of the usual business cycle quantities and compare them with the documented moments in the DSGE literature. These results are summarized in Table 5. In this table, we present results when adjustment costs specified in Equation (2.7) are present. As is seen, volatility of investment generated by our model is lower than what is observed in the data. On the other hand, the model generates volatility of consumption growth which is significantly larger than what is observed in the data. However, the size of this discrepancy is within the range of moments generated by one and two sector models studied in Boldrin et al. (2001). Moreover, the data considered by us only covers the post-WWII period, during which consumption growth volatility is small. Kaltenbrunner and Lochstoer (2010) considers the pre-WWII period and a much higher level of consumption growth volatility. Our model produces a similar level compared to theirs. Excessive volatility in consumption growth is mainly caused by high marginal adjustment costs (low values of ι). Reducing marginal adjustment costs can alleviate this problem but may not be able to generate enough volatility in equity returns. Our model generates values for volatility of output, $\sigma(Y)$, which are reasonably close to the data. The same is true for the volatility of investment to output ratio, $\sigma(I/Y)$, and for the cross-correlation between investment growth and output growth,

$Corr(\Delta I, \Delta Y)$. For the latter value, an η of 53 generates values which are 0.03 less from those observed in the data.

To see the effects of adjustment costs, we calibrate the model by eliminating the adjustment costs in Equation (2.7). The results are reported in Table 6. It is immediately obvious that this model generates business cycle moments that are closer to the data. In particular, there is a significant improvement in matching the ratio of the volatility of consumption growth to the volatility of output, $\sigma(\Delta C)/\sigma(\Delta Y)$. However, this improvement comes at a steep price. As Jermann (1998) and Boldrin et al. (2001) show, in the absence of adjustment costs, supply of capital is perfectly elastic. This in turn implies a perfect opportunity for smoothing the consumption intertemporally, which implies a smooth stochastic discount factor and hence, inability of the model in generating reasonable levels of equity premium. Thus, without capital adjustment costs, we fail to generate equity premium levels in line with what is observed in the data. Thus, we detect a trade-off between matching the financial moments and matching the business cycle moments. Keep in mind that our results in the presence of capital adjustment costs are comparable with what is reported by Jermann (1998) and Boldrin et al. (2001). Kaltenbrunner and Lochstoer (2010) report fewer business cycle moments in their study, but they are more successful in matching the volatility of consumption and the ratio of the volatility of consumption growth to the volatility of output. However, their model generates counterfactuals in the financial side, namely, excess volatility of risk free rate.

5 Conclusion

In this paper, we show that in a production economy where productivity growth follows a hidden Markov-switching process and adjustment costs are present, ambiguity about the hidden state underlying the productivity regimes is important in generating a number of salient asset pricing phenomena that are well-documented in the literature. Our calibrated model can match the mean equity premium, the volatility of equity returns and the mean risk-free rate observed in the data. In addition, our model can reproduce the pattern of long-horizon return predictability, counter-cyclical behavior of equity premium and conditional volatility of equity premium. Nevertheless, our model generates excessively volatile consumption growth.

The improvement of our model relative to the existing production-based asset pricing literature is due to adoption of generalized recursive smooth ambiguity preferences, which allows for a three-way separation of risk aversion, ambiguity aversion, and the attitude toward intertemporal substitution and also for the role of learning under incomplete information. Ambiguity about the mean productivity growth regimes separate the state of the economy into good and bad regimes. Learning about the hidden state plays an important role in generating time-varying aggregate uncertainty. Ambiguity aversion greatly enhances the price of risk by effectively distorting the pricing kernel in a pessimistic way. However, our model requires a much higher ambiguity aversion parameter than in Ju and Miao (2010) in order to match the mean equity premium because in production economies, consumption and dividends are endogenously determined from equilibrium allocations.

The main shortcoming of our model seems to be the trade-off between matching the financial moments and matching business cycle moments. In other words, we are close to reproducing asset pricing features observed in the data at the expense of precisely matching business cycle moments. For example, our model-predicted consumption growth is very volatile compared to the post-WWII level in the data.

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Table 1: **Calibration values of model parameters.**

Parameter	Description	Value
γ	Coefficient of risk aversion	5.00
α	Capital share	0.35
β	Time discount parameter	0.9935
$\bar{N}$	Exogenous labor input	0.20
ι	Exponent of adjustment costs function	1.082
δ	Depreciation rate of capital	0.015
μ_1	Mean TFP growth rate (good regime)	0.010
μ_2	Mean TFP growth rate (bad regime)	-0.015
σ	Standard deviation of TFP growth rate	0.010
P_{11}	Transition probability (good regime to good regime)	0.931
P_{22}	Transition probability (bad regime to bad regime)	0.427

This Table reports values of the model parameters that are invariant in the calibration unless otherwise stated. The mean TFP growth rates in good and bad regimes, μ_1 and μ_2 respectively, and the volatility of TFP growth, σ, are quarterly values. The estimates of transition probabilities, P_{11} and P_{22}, are obtained by applying Hamilton (1989) EM algorithm to TFP data in Fernald (2009).

Table 2: **Stylized Facts of Equity and Short-Term Bond Returns, Annualized Observations 1947-2006.**

U.S. Data, Q1:1947-Q4:2006			
$\mathbb{E}(r_f)$	0.0126	Sharpe Ratio	0.5220
$\sigma(r_f)$	0.0158	$\mathbb{E}(p_t - d_t)$	4.7499
$\mathbb{E}(r_e)$	0.1173	$\sigma(p_t - d_t)$	0.3862
$\sigma(r_e)$	0.1519		
$\mathbb{E}(r_{ep})$	0.0802		
$\sigma(r_{ep})$	0.1571		

This Table reports annualized unconditional financial moments for CRSP value-weighted index data and U.S. 3-month Treasury Bill rate from the first quarter of 1947 to the fourth quarter of 2006. All values reported are deflated by changes in CPI and expressed in real terms. r_f, r_e, and r_{ep} represent the risk-free rate, equity return, and the equity premium, respectively. The Sharpe ratio is defined by $SR = \frac{\mathbb{E}(r_e - r_f)}{\sigma_e}$. We also report the unconditional expected value and the standard deviation of the price-dividend ratio, where we calculate the ratio using the method in Campanale et al. (2010).

Table 3: **Model Generated Unconditional Financial Moments**

η	$\mathbb{E}[r_f]$	$\sigma(r_f)$	$\mathbb{E}[r_e]$	$\sigma(r_e)$	$\mathbb{E}(r_{ep})$	$\sigma(r_{ep})$	$\frac{\sigma(M)}{\mathbb{E}(M)}$	SR
Panel A (benchmark calibration): $\psi = 1.5$								
53	0.0174	0.0048	0.0978	0.1601	0.0804	0.1602	1.0850	0.5022
Panel B: $\psi = 1.5$								
n.a.	0.0431	0.0039	0.0901	0.2744	0.0470	0.2744	0.0716	0.1713
5	0.0431	0.0031	0.0869	0.2699	0.0439	0.2698	0.0719	0.1626
20	0.0380	0.0041	0.0869	0.2278	0.0489	0.2278	0.2575	0.2145
40	0.0270	0.0050	0.0941	0.1885	0.0672	0.1886	0.6942	0.3562
Panel C: $\psi = 0.6$								
n.a.	0.0720	0.0085	0.1146	0.2752	0.0425	0.2750	0.0639	0.1546
5	0.0719	0.0068	0.1160	0.2748	0.0441	0.2746	0.0643	0.1606
20	0.0666	0.0079	0.1058	0.2439	0.0393	0.2438	0.2345	0.1610
40	0.0563	0.0091	0.1115	0.2064	0.0551	0.2063	0.6470	0.2673
Panel D: $\psi = 0.2$								
n.a.	0.1705	0.0239	0.2540	0.4365	0.0835	0.4361	0.0491	0.1914
5	0.1703	0.0197	0.2608	0.4366	0.0906	0.4364	0.0491	0.2076
20	0.1635	0.0202	0.2525	0.4341	0.0890	0.4337	0.1809	0.2052
40	0.1540	0.0210	0.2374	0.4112	0.0834	0.4108	0.4964	0.2030

This table reports annualized financial moments generated by both the full information model and the ambiguous growth model with adjustment costs as in Equation (2.7). The moments are calculated based on 20,000 Monte Carlo simulations. The first line of Panel B, C and D reports results for the full information model. The notation "SR" stands for Sharpe ratio.

Table 4: **Predictability in Returns and Equity Premium**

Panel A: $\psi = 1.5, \gamma = 5, \eta = 53$												
Dependent variables	$r^e_{t,t+s}$						$r^e_{t,t+s} - r^f_{t,t+s}$					
	Model		Data (1947-2009)		Data (1947-2006)		Model		Data (1947-2009)		Data (1947-2006)	
Horizon (s)	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2
1 quarter	-0.0201	0.0136	-0.0336	0.0275	-0.0423	0.0226	-0.0199	0.0130	-0.0281	0.0188	-0.0256	0.0162
1 year	-0.0301	0.0289	-0.0355	0.0305	-0.0744	0.0360	-0.0330	0.0300	-0.0323	0.0247	-0.0527	0.0182
2 years	-0.0417	0.0468	-0.0264	0.0170	-0.0951	0.0534	-0.0494	0.0510	-0.0237	0.0134	-0.0595	0.0217
3 years	-0.0513	0.0622	-0.0268	0.0173	-0.0991	0.0657	-0.0638	0.0687	-0.0248	0.0145	-0.0658	0.0215
5 years	-0.0661	0.0875	-0.0334	0.0256	-0.1209	0.0630	-0.0875	0.0963	-0.0344	0.0266	-0.0865	0.0505
7 years	-0.0762	0.1080	-0.0309	0.0207	-0.1558	0.0731	-0.1056	0.1175	-0.0343	0.0249	-0.1399	0.0714
Panel B: $\psi = 1.5, \gamma = 5, \eta = 5$												
Dependent variables	$r^e_{t,t+s}$						$r^e_{t,t+s} - r^f_{t,t+s}$					
	Model		Data (1947-2009)		Data (1947-2006)		Model		Data (1947-2009)		Data (1947-2006)	
Horizon (s)	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2
1 quarter	-0.0487	0.0254	-0.0336	0.0275	-0.0423	0.0226	-0.0486	0.0254	-0.0281	0.0188	-0.0256	0.0162
1 year	-0.0378	0.0280	-0.0355	0.0305	-0.0744	0.0360	-0.0372	0.0285	-0.0323	0.0247	-0.0527	0.0182
2 years	-0.0204	0.0342	-0.0264	0.0170	-0.0951	0.0534	-0.0196	0.0349	-0.0237	0.0134	-0.0595	0.0217
3 years	-0.0021	0.0418	-0.0268	0.0173	-0.0991	0.0657	-0.0012	0.0426	-0.0248	0.0145	-0.0658	0.0215
5 years	0.0348	0.0588	-0.0334	0.0256	-0.1209	0.0630	0.0356	0.0598	-0.0344	0.0266	-0.0865	0.0505
7 years	0.0709	0.0765	-0.0309	0.0207	-0.1558	0.0731	0.0716	0.0777	-0.0343	0.0249	-0.1399	0.0714
Panel C: $\psi = 0.6, \gamma = 5, \eta = 53$												
Dependent variables	$r^e_{t,t+s}$						$r^e_{t,t+s} - r^f_{t,t+s}$					
	Model		Data (1947-2009)		Data (1947-2006)		Model		Data (1947-2009)		Data (1947-2006)	
Horizon (s)	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2
1 quarter	-0.0730	0.0402	-0.0336	0.0275	-0.0423	0.0226	-0.0682	0.0391	-0.0281	0.0188	-0.0256	0.0162
1 year	-0.0696	0.0508	-0.0355	0.0305	-0.0744	0.0360	-0.0722	0.0516	-0.0323	0.0247	-0.0527	0.0182
2 years	-0.0553	0.0621	-0.0264	0.0170	-0.0951	0.0534	-0.0735	0.0650	-0.0237	0.0134	-0.0595	0.0217
3 years	-0.0386	0.0723	-0.0268	0.0173	-0.0991	0.0657	-0.0720	0.0764	-0.0248	0.0145	-0.0658	0.0215
5 years	-0.0005	0.0904	-0.0334	0.0256	-0.1209	0.0630	-0.0621	0.0947	-0.0344	0.0266	-0.0865	0.0505
7 years	0.0409	0.1065	-0.0309	0.0207	-0.1558	0.0731	-0.0464	0.1094	-0.0343	0.0249	-0.1399	0.0714

The regression slopes and R^2s are for regressions of the k-holding period ($k = 1Q, 1, 2, 3, 5, 7Y$) returns and excess returns on the current log price-dividend ratio. We present both results generated from our model and results estimated from the data. All estimates are obtained using OLS regressions. Results are obtained based on 20,000 Monte Carlo simulations.

Table 5: **Model Generated Unconditional Business-Cycle Moments**

	$\psi = 1.5, \gamma = 5$				
	Data	Full info.	$\eta = 5$	$\eta = 20$	$\eta = 53$
$\sigma(\Delta I)$	0.0546	0.0071	0.0071	0.0063	0.0056
$\sigma(\Delta C)$	0.0111	0.0216	0.0215	0.0214	0.0210
$\sigma(\Delta Y)$	0.0118	0.0163	0.0163	0.0163	0.0163
$\sigma(\Delta C)/\sigma(\Delta Y)$	0.4711	1.3268	1.3235	1.3131	1.2888
$\sigma(I/Y)$	0.028	0.0160	0.0160	0.0164	0.0169
$Corr(\Delta I, \Delta Y)$	0.805	0.7020	0.7111	0.7412	0.7773
$Corr(\Delta C_{t+1}, \Delta C_t)$	0.2259	0.1389	0.1413	0.1432	0.1440

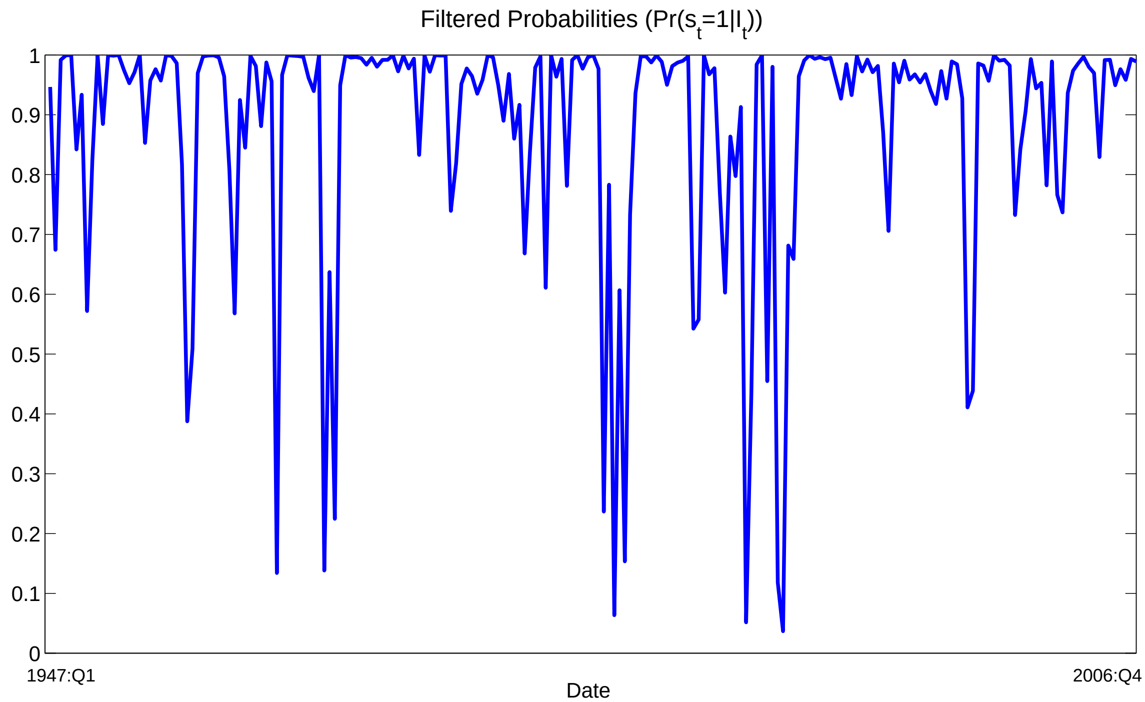
This table reports annualized business-cycle moments generated from the full-information model, incomplete information-ambiguity neutral model ($\eta = 5$) and incomplete information-ambiguity aversion models ($\eta = 20, 50$) where adjustment costs are present. The value of $\sigma(I/Y)$ is taken from Croce (2008). The moments are calculated based on 20,000 Monte Carlo simulations.

Table 6: **Model Generated Unconditional Business-Cycle Moments, No Adjustment Costs**

$\psi = 1.5, \gamma = 5$				
	Data (1947-2006)	Full information	$\eta = 5$	$\eta = 20$
$\sigma(\Delta I)$	0.0546	0.0366	0.0358	0.0335
$\sigma(\Delta C)$	0.0111	0.0086	0.0086	0.0097
$\sigma(\Delta Y)$	0.0118	0.0163	0.0163	0.0163
$\sigma(\Delta C)/\sigma(\Delta Y)$	0.4711	0.5268	0.5274	0.5959
$\sigma(I/Y)$	0.0280	0.0158	0.0158	0.0154
$Corr(\Delta I, \Delta Y)$	0.8050	0.9758	0.9826	0.9692
$Corr(\Delta C_t, \Delta C_{t-1})$	0.2259	0.1414	0.1413	0.0190

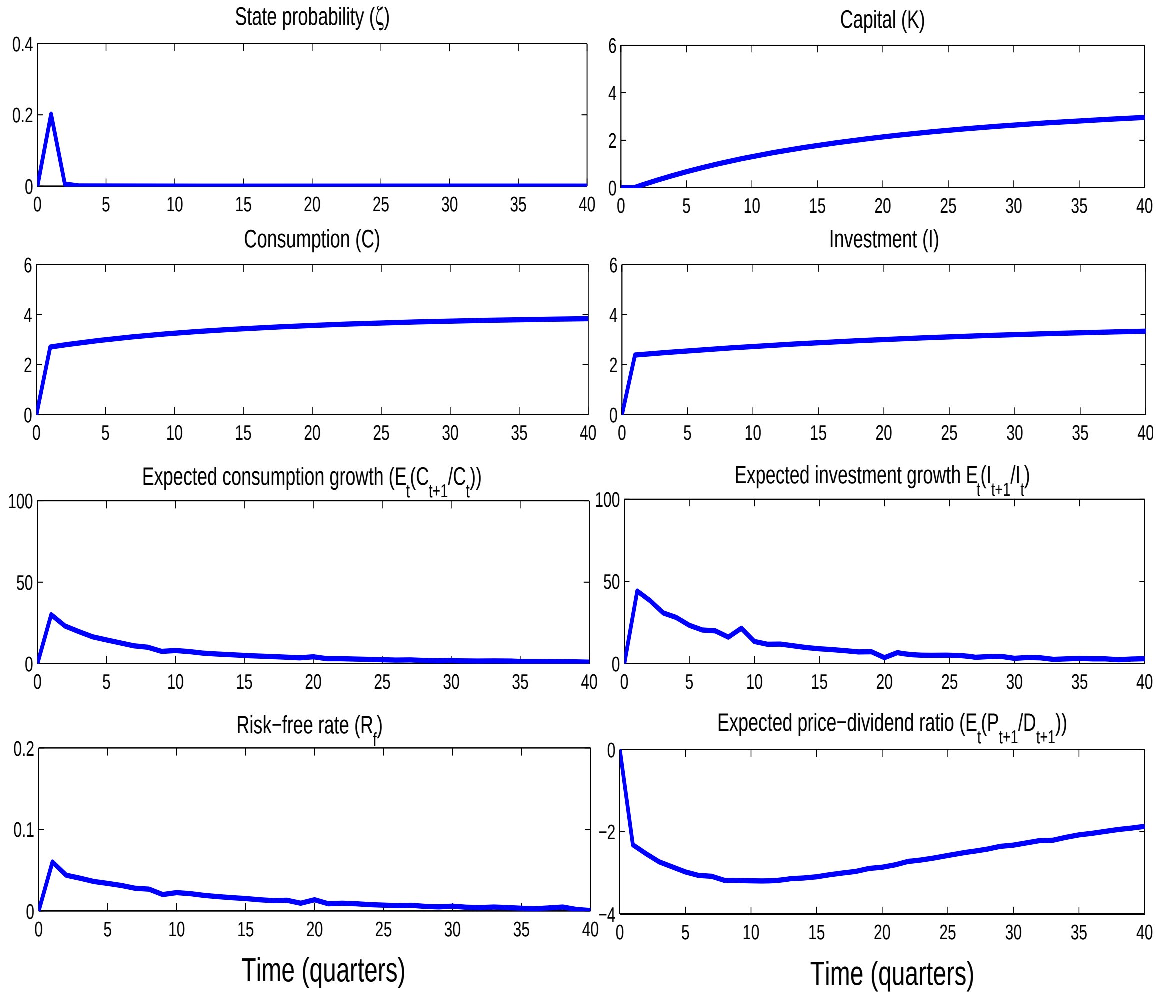
This table reports annualized business-cycle moments generated from the full-information model, incomplete information-ambiguity neutral model ($\eta = 5$) and incomplete information-ambiguity aversion model ($\eta = 20$) where adjustment costs are present. The value of $\sigma(I/Y)$ is taken from Croce (2008). The moments are calculated based on 20,000 Monte Carlo simulations.

Figure 1: Filtered probabilities



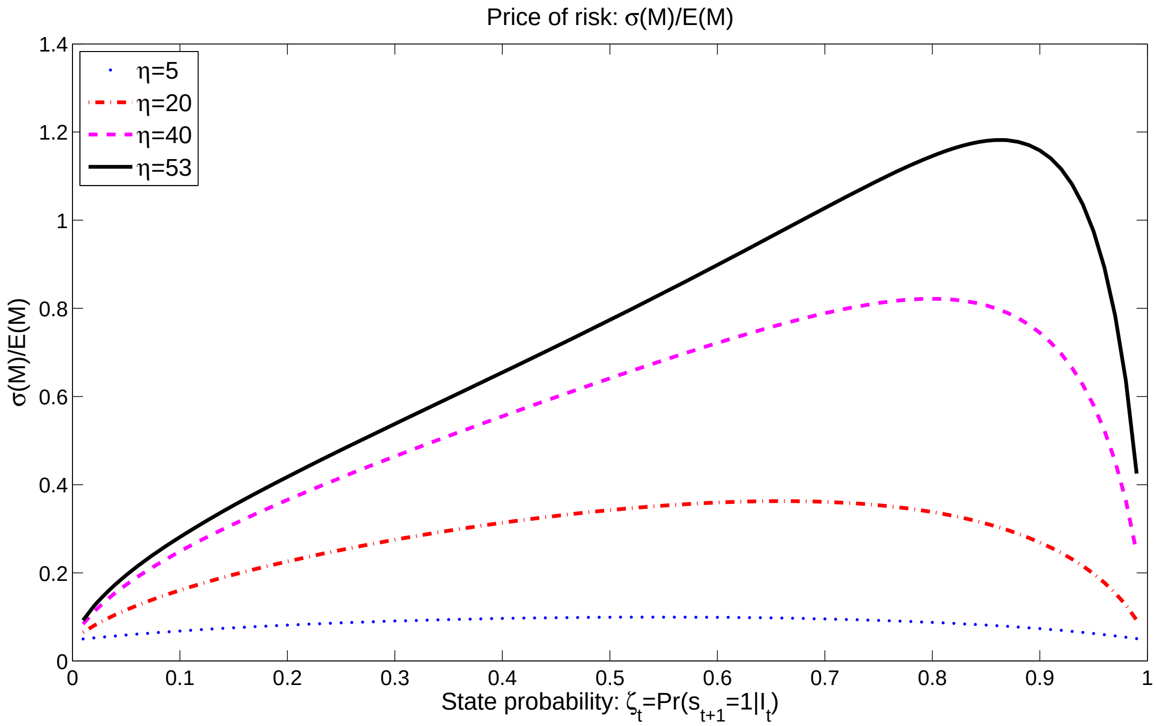
This figure depicts the filtered probabilities obtained from applying Hamilton (1989) EM algorithm to quarterly total factor productivity data described in Fernald (2009). The data spans from 1947:Q1 to 2006:Q4.

Figure 2: Impulse response functions: $\eta = 53, \gamma = 5, \psi = 1.5$.



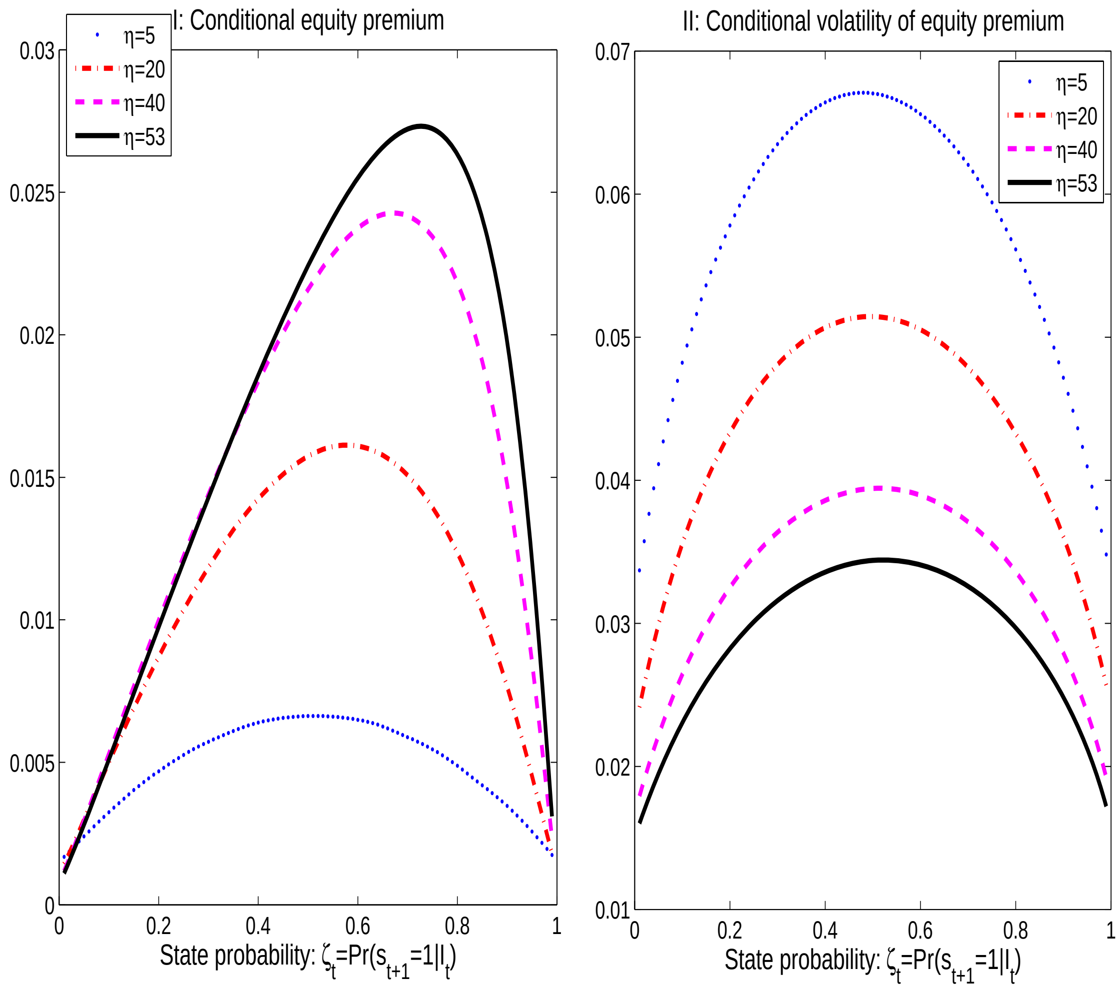
This figure plots the impulse response functions of state probability, capital, consumption, investment, expected consumption growth, expected investment growth, risk-free rate and expected price-dividend ratio to a positive shock to productivity growth. We consider 40 quarters response time.

Figure 3: Price of risk, given state probabilities and η levels.



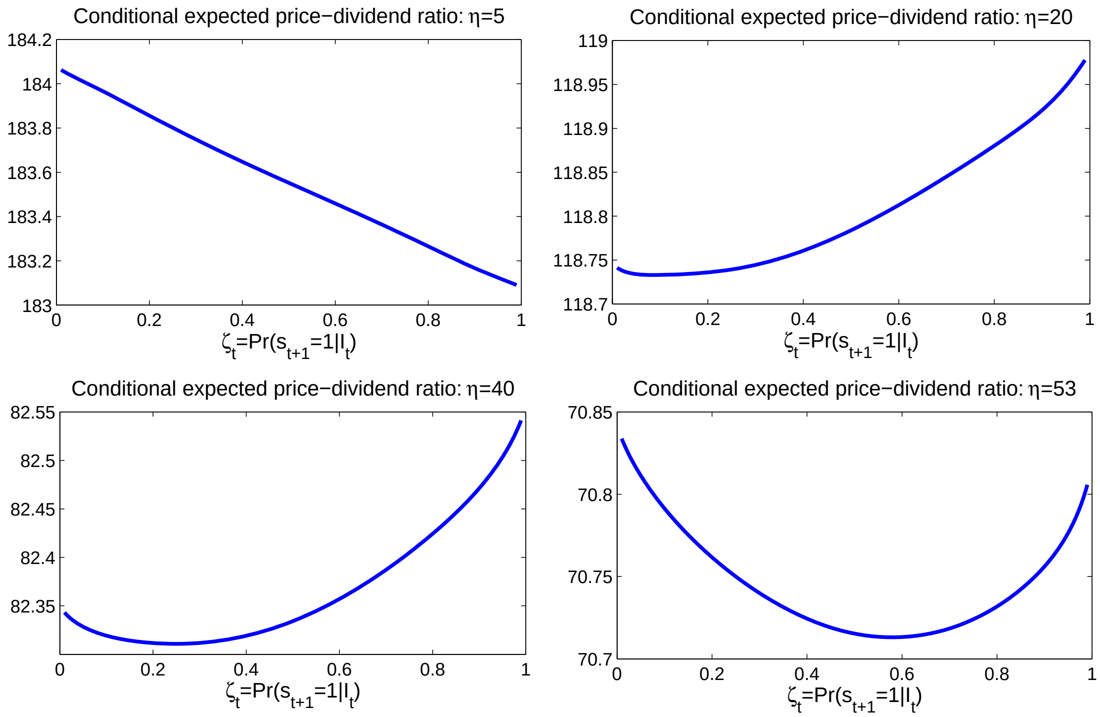
This figure plots the price of risk, defined as $\sigma(M)/\mathbb{E}(M)$, as a function of one-step ahead conditional probability (ζ_t) of the high productivity regime for different levels of ambiguity aversion. The level of capital stock is set to its mean in the Monte Carlo simulations, and productivity growth is set to its steady-state level.

Figure 4: Conditional equity premium and conditional volatility of equity premium, given state probabilities and η levels.



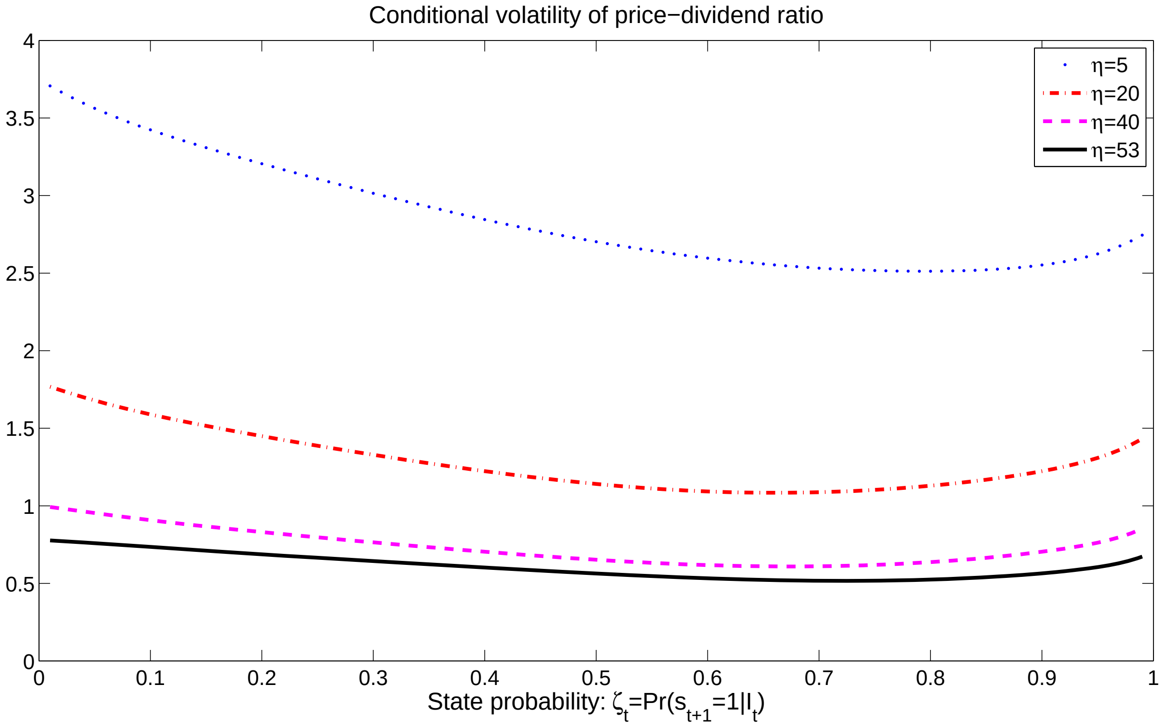
This figure plots the equity premium and volatility of equity premium as functions of one-step ahead conditional probability (ζ_t) of the high productivity regime for different levels of ambiguity aversion. The level of capital stock is set to its mean in the Monte Carlo simulations, and productivity growth is set to its steady-state level.

Figure 5: Conditional price-dividend ratios, given state probabilities and η levels.



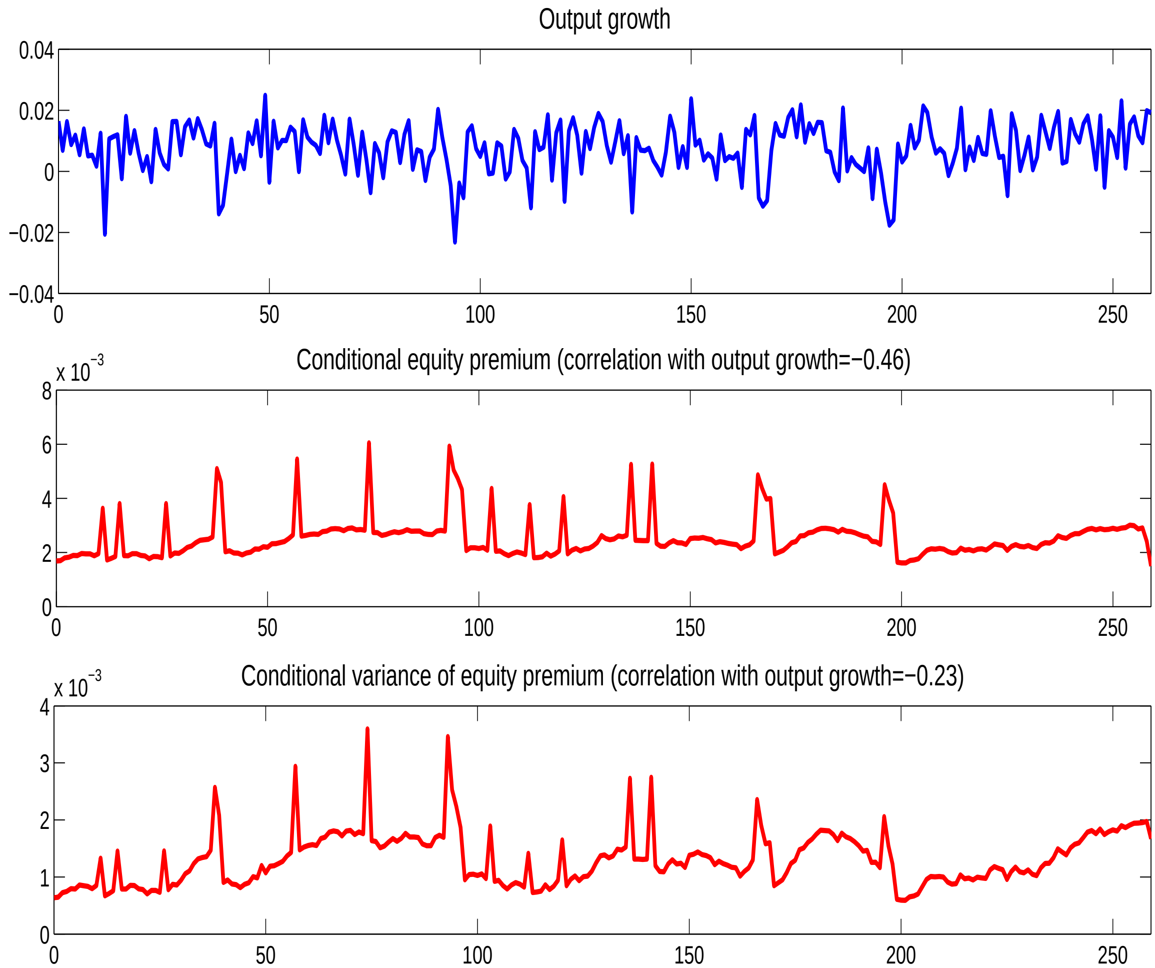
This figure plots the price-dividend ratio as a function of one-step ahead conditional probability (ζ_t) of the high productivity regime for different levels of ambiguity aversion. The level of capital stock is set to its mean in the Monte Carlo simulations, and productivity growth is set to its steady-state level.

Figure 6: Conditional volatility of price-dividend ratios, given state probabilities and η levels.



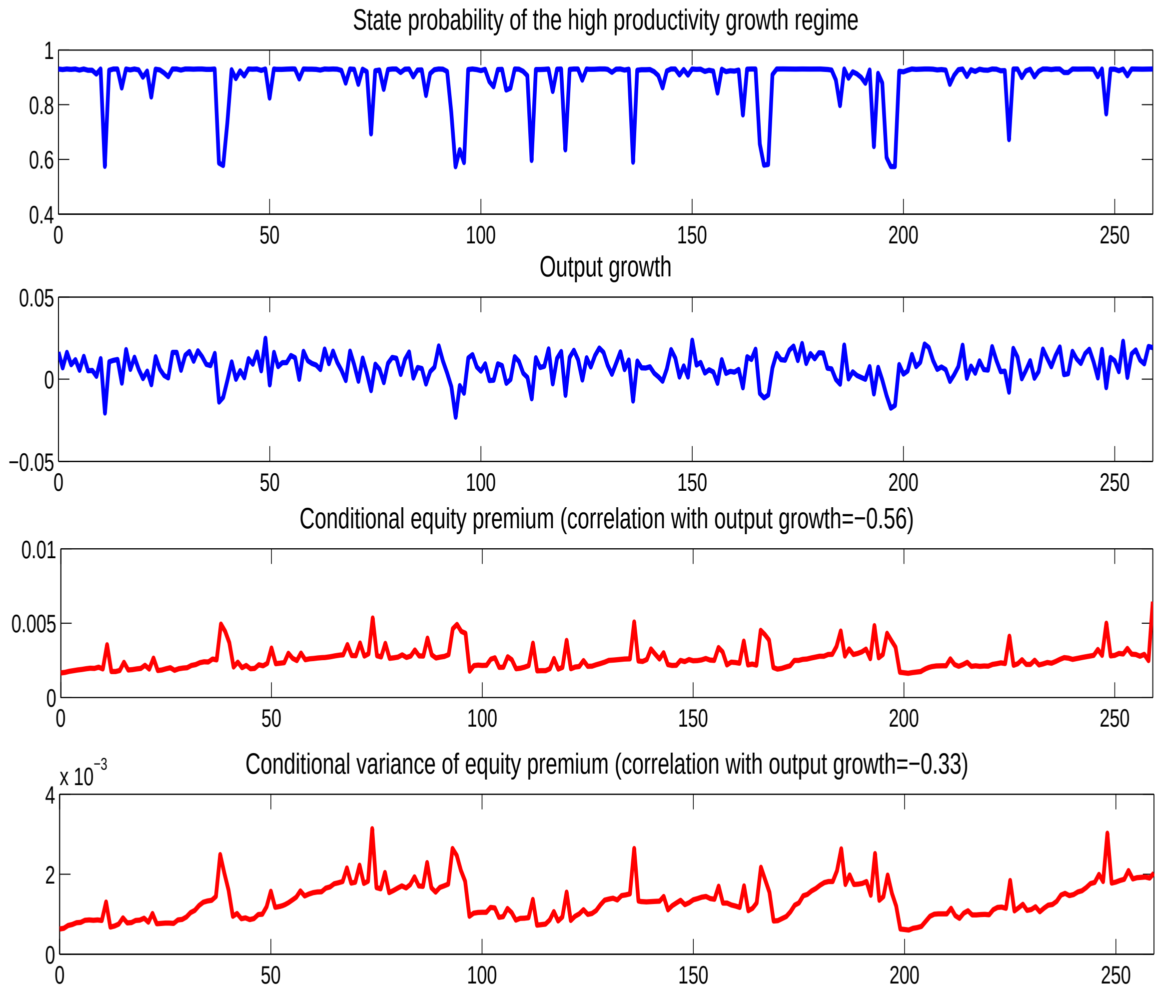
This figure plots the volatility of price-dividend ratios as a function of one-step ahead conditional probability (ζ_t) of the high productivity regime for different levels of ambiguity aversion. The level of capital stock is set to its mean in the Monte Carlo simulations, and productivity growth is set to its steady-state level.

Figure 7: Simulated output growth, conditional equity premium and conditional variance of equity premium: full information.



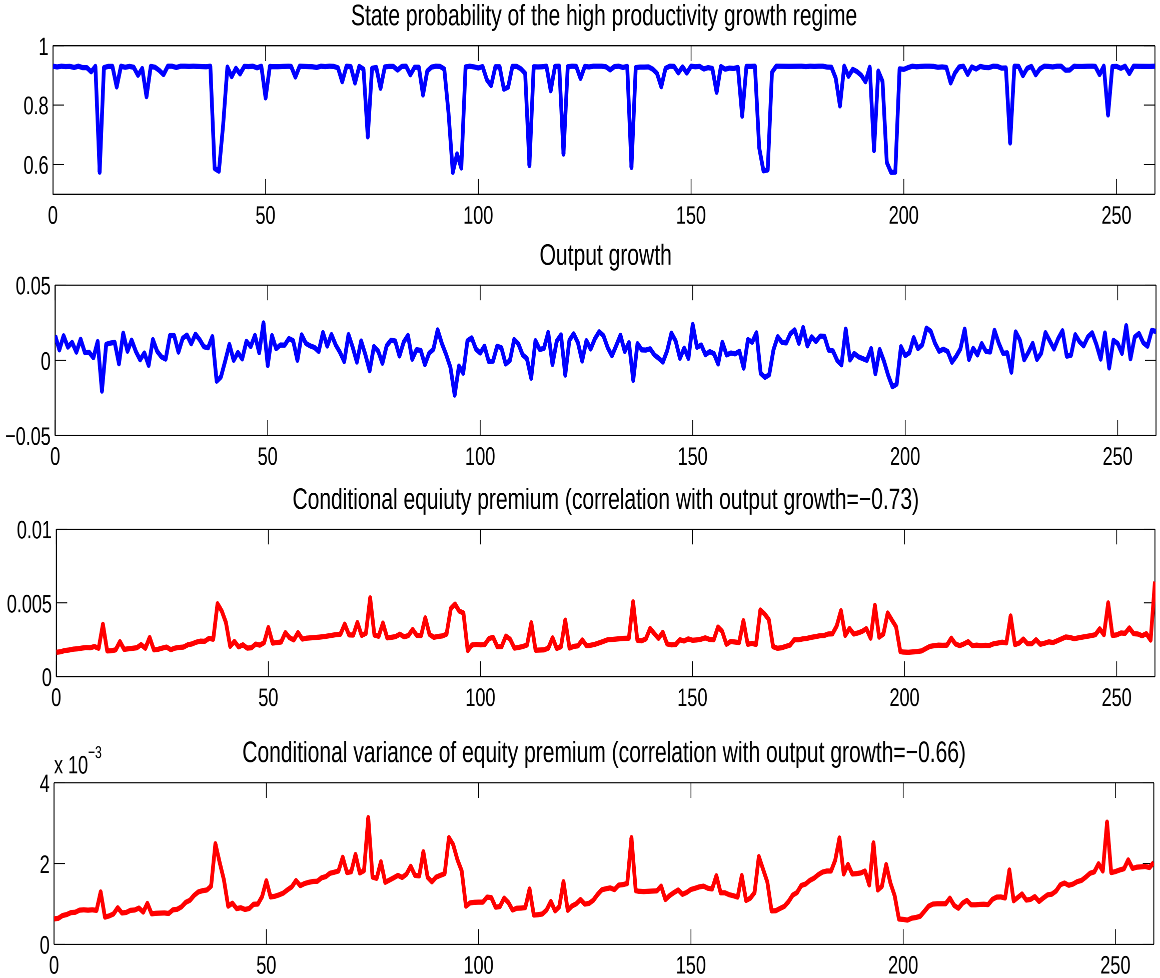
This figure depicts simulated output growth rates, conditional equity premium and conditional variance of equity premium for the full information model. Results are based on simulations over 260 quarters.

Figure 8: Simulated state probabilities, output growth, conditional equity premium and conditional variance of equity premium: $\eta = \gamma = 5$.



This figure depicts state probabilities of the high productivity growth regime, $\zeta_t = Pr[s_{t+1} = 1 | \mathcal{I}_t]$, output growth rates, conditional equity premium and conditional variance of equity premium when $\eta = \gamma = 5$. Results are based on simulations over 260 quarters.

Figure 9: Simulated state probabilities, output growth, conditional equity premium and conditional variance of equity premium: $\eta = 53, \gamma = 5$.



This figure depicts state probabilities of the high productivity growth regime, $\zeta_t = Pr[s_{t+1} = 1|\mathcal{I}_t]$, output growth rates, conditional equity premium and conditional variance of equity premium when $\eta = 53, \gamma = 5$. Results are based on simulations over 260 quarters.