Volatility Predictability and Jump Asymmetry

Diep Duong*
Rutgers University

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Abstract

Many recent modelling advances in asset pricing and management are predicated on the importance of jumps, or discontinuous movements in asset returns. In particular, volatility predictability is important in numerous areas of financial econometrics ranging from the pricing of volatility-based derivative products to asset management. In light of this, a number of recent papers have addressed volatility predictability, some from the perspective of the usefulness of jumps in forecasting volatility. Key papers in this area of research include Andersen, Bollerslev, Diebold and Labys (2003), Andersen, Bollerslev and Diebold (2007), Barndoff, Kinnebrock, and Shephard (2010), Corsi (2004), Corsi, Pirino and Reno (2008), Patton and Shephard (2011), and the references cited therein. In this paper, we examine the predictive content of a variety of realized measures of jump power variations, all formed on the basis of power transformations of instantaneous returns (i.e., $|r_t|^q$), as first discussed in Ding, Granger and Engle (1993) and Ding and Granger (1996). More specifically, we consider jump power variations with $0 \leq q \leq 6$, and construct a variety of estimators of jump risk, including upside and downside risk, jump asymmetry (i.e., realized signed jump power variation), and truncated jump measures. Our prediction experiments use high frequency price returns constructed using S&P 500 futures data as well as stocks in the Dow 30, for the period 1993-2009 period; and our empirical implementation involves estimating members of the linear and nonlinear extended Heterogeneous Autoregressive of the Realized Volatility (HAR-RV) class of models. Our findings suggest that past "large" jump power variations help less in the prediction of future realized volatility, than past "small" jump power variations. This in turn suggests the "larger" jumps might help less in the prediction of future realized volatility than "smaller" jumps. Our empirical findings also suggest that past realized signed jump power variations, which have not previously been examined in this literature, are strongly correlated with future volatility, and that past downside jump variations matter in prediction. Finally, incorporation of downside and upside jump power variations does improve predictability, albeit to a limited extent. Overall, our findings are consistent with ABD (2007) in the concluding that continuous components dominate, when predicting volatility.

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* Diep Duong, Department of Economics, Rutgers University, 75 Hamilton Street, New Brunswick, NJ 08901, USA, dduong@econ.rutgers.edu. I am deeply indebted to Norman Swanson for invaluable guidance. I am grateful to Roger Klein and Richard Mclean for encouragement and helpful discussions. I would like to thank John Landon-Lane and Kihwan Kim and other participants at the Econometrics Workshop at Rutgers University for helpful comments. In addition, I owe special thanks to Anh Le for continued support. Finally, research support from Rutgers Graduate School is greatly appreciated. All errors are my own.

1 Introduction

Many recent modelling advances in asset pricing and management are predicated on the importance of jumps, or discontinuous movements in asset returns. In an important paper, Huang and Tauchen (2005) find evidence of discrete large jumps in S&P cash and future (log) returns from 1997 to 2002, in approximately 7% of the trading days. Aït-Sahalia and Jacod (2009b) develop methods to ascertain whether the process describing an asset contains "infinite activity jumps" - those jumps that are tiny and look similar to continuous movements, but whose contribution to the jump risk of the process is not negligible. In an empirical analysis of Intel and Microsoft returns, they find evidence of the presence of infinite active jumps in historical data. In summary, it is now generally accepted that many return processes contain jumps.[1] Once jumps are found, the economic implications of including them in dynamic asset pricing exercises are substantial. For example, the incorporation of jumps lead to break-downs in the typical market completeness condition needed for portfolio replication strategy in derivatives valuation. Additionally, jumps complicate the implementation of the "state of the art" change of risk measure in risk neutral pricing. As a result, asset allocation and risk management, which heavily depend on risk measures and underlying asset return dynamics, are affected. In volatility measurement, it is necessary to separate out the volatility due to jumps or construct variables that appropriately summarize information generated by jumps.

In volatility forecasting, once jumps are detected, understanding the role of variables that capture jump information is potentially important for applied practitioners, especially in the construction of hedging strategies.[2] In general, volatility predictability is important in numerous areas ranging from the pricing of volatility-based derivative products to asset management. In light of this, a number of recent papers have addressed volatility predictability, some from the perspective of the usefulness of jumps in forecasting volatility. However, although there is strong evidence of the importance of jumps in pricing, investment and risk management, there is mixed evidence concerning whether information extracted from jumps is useful for volatility forecasting. In a seminal work, Andersen, Bollerslev and Diebold (ABD: 2007) show that almost all of the predictability in daily, weekly, and monthly return volatilities comes from the non-jump component for DM/$ exchange rate, the S&P500 market index, and the 30-year U.S. Treasury bond yield. Corsi, Pirino and Reno (2008) find that jumps are positively correlated with, and have a significant impact on future volatility of the S&P500 index, various individual stocks and US bond yields. Patton and Shephard (2011) point out that the impact of a jump on future volatility critically depends on the sign of the jump, for both the S&P 500 index, as well as 105 individual stocks. In this paper we add to the empirical literature on this topic by providing results on volatility forecasting using a

[1]For other examples of work in this area, see Aït-Sahalia (2002), Carr et al., (2002), Carr and Wu (2003), Barndorff-Nielsen and Shephard (BNS: 2006), Woerner (2006), Jacod (2008), Jiang and Oomen (2008), Lee and Mykland (2008), Tauchen and Todorov (2009), Aït-Sahalia and Jacod (2009a,b) and the references cited therein.

[2]See Andersen, Bollerslev and Diebold (2007) and Aït-Sahalia and Jacod (2011) for further discussion.

variety of "new" variables that capture information generated by jumps.

When undertaking empirical research using volatility, a key issue involves the choice of the volatility estimator. One approach involves "backing out" volatility from parametric from ARCH, GARCH, Stochastic Volatility, or Option pricing models. The approach that we adopt involves using recently developed "model free" estimators (see the influential work of Andersen, Bollerslev, Diebold and Laby (2001)), including realized volatility (RV), and variants thereof such as bipower variation, tripower variation, multipower variation, semivariance, and various others.[3] One key reason for the use of these "model free" realized measures (RMs), is that they allow us to treat volatility as if it is observed, when we fit regressions in order to assess jump predictability. Modeling and forecasting RMs are important not only because RMs are a natural proxy for volatility, but also because of the many practical applications and uses of RMs in constructing synthetic measures of risk in the financial markets. For example, since shortly after the inception in 1993 of the VIX (index of implied volatility), a variety of volatility-based derivative products have been engineered using RV as an input. These include variance swaps, caps on variance swaps, corridor variance swaps, covariance swaps, options on RV overshooters, and up and downcrossers. The key here is that investors worry about future volatility risk, and hence often opt for this type of contract in order to hedge against risk, adding to the traditional volatility "Vega".[4] In light of the above uses of RV, several authors have advocated forecasting RV (and more generally RMs) using extensions of ARMA models (see e.g., Andersen, Bollerslev, Diebold and Labys (2003), Corsi (2004), and ABD (2007)). In related work, Corradi, Distaso and Swanson (2012) develop model -free conditional predictive density estimators and confidence intervals for integrated volatility.

Given the availability of volatility estimators, as discussed above, it remains to choose variables that capture information generated by jumps. In this paper, we examine four realized measures of jump power variations, all formed on the basis of power transformation of the instantaneous return (i.e., $|r_t|^q$). The analysis of power transformations of returns is not new. Ding, Granger and Engle (1993) and Ding and Granger (1996) develop long memory Asymmetric Power ARCH models based on power transformations of daily absolute returns. They find that the autocorrelations of power transformations of S&P 500 returns are the strongest for $q < 1$. In the context of high frequency data, Liu and Maheu (LH: 2005) and Ghysels and Sohn (GS: 2009) study the predictability of future realized volatility using past absolute power variations and multipower variations. GS (2009) find that the optimal value of q is approximately unity. However, their empirical evidence considers the continuous class of models, and does not account for jumps. Andersen, Bollerslev and

[3]See e.g., Barndorff-Nielsen and Shephard (2004), Aït-Sahalia, Mykland and Zhang, (2005), Zhang (2006), Barndorf-Nielsen, Hansen, Lunde, and Shephard (2006,2008), Jacod (2008), Barndoff, Kinnebrock, Shephard (2010), and the references cited therein.

[4]Volatility and variance swaps are newer hedging instruments, adding to the traditional volatility "Vega", which is derived from options data. See Hull and White (1997, pp. 328) for a definition of Vega. For example, as noted in Carr and Lee (2008), the UBS book was short many millions of vega in 1993, and they were the first to use variance swaps and options on realized volatility to hedge against volatility risk. See Duong and Swanson (2011) for further discussion.

Diebold (ABD: 2007), on the other hand, develop an interesting framework for separating jump and continuous components of RV, and carry out predictability experiments indicating that jumps play a small but notable role in forecasting volatility. In related recent work, Barndoff, Kinnebrock, and Shephard (BKS: 2010) construct new estimators of downside (and upside) risk (i.e., so-called realized semivariances), using square transformations of positive and negative intra-daily return, and find that downside risk measures are important when attempting to model and understand risk. They note, as quoted from Granger (2008), that: *'It was understood that risk relates to an unfortunate event occurring, so for an investment this corresponds to a low, or even negative, return. Thus getting returns in the lower tail of the return distribution constitutes this "downside risk." However, it is not easy to get a simple measure of this risk.'* This point is noteworthy, since it is argued in the literature (see e.g., Ang, Chen and Xing (2006)), that investors treat downside losses differently than upside gains. As a result, agents who put higher weight on downside risk demand additional compensation for holding stocks with high sensitivity to downside market movements. Most authors in this literature pay attention to co-skewness as a measure of downside risk, and use daily data for estimation thereof. Patton and Shephard (2011) build on these ideas and use semivariance estimators to forecast volatility.

Building on the work of above authors, and in particular BKS (2010), we contribute to the volatility prediction literature by examining recently proposed realized measures of (downside) jump power variations. The measures are constructed using power transformations of absolute intra-daily returns, based on recent limit theory advances due to Jacod (2008) and BKS (2010). Theoretically, the measures do not require the use of a jump test in order to "pre-test" for jumps. Although construction of the measures is closely related to the work of Ghysels and Sohn (2009), our approach differs in that we focus on jump power variations with $q > 2$. Furthermore, the limit theory that we adopt allows us to construct estimators of downside and upside jump power variations using intra-daily positive and negative returns. These estimators are suggested by BKS (2010) as alternatives to the semivariances implemented in Patton and Shephard (2011). We also examine jump asymmetry (i.e., realized signed jump power variation) in realized volatility prediction experiments. Of note is that the role of the size of jumps that are most useful for forecasting can be inferred through examination of the order of q. For this reason, we consider jump power variations with $0 \leq q \leq 6$. While previous authors have focused on $q \leq 2$,allowing for a wider range of values for q is sensible, given that convergence to jump power variation occurs only when $q > 2$ (see e.g. Todorov and Tauchen (2009) and BKS (2010)). [5]We also use an approach recommended in Duong and Swanson (2010) for constructing truncated jump measures, in order to assess whether jumps of a particular range of magnitudes are more useful than measures based upon the use of all jumps, or of signed jumps. Our dataset includes high frequency price returns constructed using S&P futures index data as well as stocks in the Dow 30, for the period 1993-

[5]In our implementation, for $q > 6$, the prediction results are almost the same as the case $q = 6$ and therefore are not presented.

2009; and our empirical implementation involves estimating members of the linear and nonlinear extended Heterogeneous Autoregressive of the Realized Volatility (HAR-RV) class of models. Our findings suggest that past "large" jump power variations help less in the prediction of future realized volatility, than past "small" jump power variations. This in turn suggests the "larger" jumps might help less in the prediction of future realized volatility than "smaller" jumps. Our empirical findings also suggest that past realized signed jump power variations, which have not previously been examined in this literature, are strongly correlated with future volatility, and that past downside jump variations matter in prediction. Moreover, our results include various experimental setups in which the (forecast) best values of q are larger than 2 for S&P 500 futures. Interestingly, whether or nor jump tests are implemented prior to the construction of jump power variations also affects the choice of q, in a variety of in-sample and out-of-sample forecasting contexts. Finally, incorporation of downside and upside jump power variations does improve predictability, albeit to a limited extent. Overall, our findings are consistent with ABD (2007) in the concluding that continuous components dominate, when predicting volatility.

The rest of the paper is organized as follows: Section 2 discusses volatility and price jump variation, and Section 3 discusses the various realized measures of price jump variation that we examine. Section 4 outlines our experimental setup, and Section 5 gathers our empirical findings. Concluding remarks are contained Section 6.

2 Volatility and Price Jump Variations

We adopt a general semi-parametric specification for asset prices. Following Todorov and Tauchen (2009), the log-price of asset, $p_t = \log(P_t)$, is assumed to be an Itô semimartingale process,

$$p_t = p_0 + \int_0^t b_s ds + \int_0^t \sigma_s dB_s + J_t, \tag{1}$$

where $p_0 + \int_0^t \mu_s ds + \int_0^t \sigma_{s-} dB_s$ is a Brownian semi-martingale and J_t is a pure jump process which is the sum of all "discontinuous" price movements up to time t,

$$J_t = \sum_{s \leq t} \Delta p_s.$$

J_t is assumed to be finite[6] and a jump at time s is defined as $\Delta p_s = p_s - p_{s-}$.

When the jump component is a Compound Poisson Process (CPP) - i.e. a finite activity jump process - then,

$$J_t = \sum_{i=1}^{N_t} Y_i, \tag{2}$$

[6] See, for example Jacod (2008) or Todorov and Tauchen (2009) for the conditions for the finiteness of jump.

where N_t is number of jumps in $[0, t]$. N_t follows a Poisson process, and the $Y_i's$ are i.i.d. and are the sizes of the jumps. The CCP assumption has been widely used in the literature on modeling, forecasting, and testing for jumps. However, jumps could have more general specifications, which contain so called - infinite activity jumps as in Todorov and Tauchen (2009).

The empirical evidence discussed in this paper involves examining the variation of the log-price jump component using an equally spaced path of a historically observed price sample, i.e. $\{p_0, p_{1\Delta_n}, p_{2\Delta_n} ..., p_{n\Delta_n}\}$, where the sampling frequency $\Delta_n = \frac{t}{n}$ is deterministic[7]. The intra-daily return or increment of p_t is

$$r_{i,n} = p_{i\Delta_n} - p_{(i-1)\Delta_n}.$$

Returns are observed at various frequencies. However, volatility of log-price is often treated as an unobserved variable. The "true" value of variance of price (risk) is defined in the literature by so-called quadratic variation of the process p_t, i.e.,

$$V_t = [p, p]_t = \int_0^t \sigma_s^2 ds + QJ_t,$$

where the variation of continuous component is $\int_0^t \sigma_s^2 ds$ (integrated volatility) and the variation of jump component is $QJ = \sum_{s \leq t} (\Delta p_s)^2$.

The realized volatility (RV), constructed by simply summing up all successive intra-daily squared returns, converges to the quadratic variation of the process as sampling frequency $n \to \infty$. Andersen, Bollerslev, Diebold and Labys (2001) use realized volatility as an estimator of variation or volatility of the process,

$$RV_t = \sum_{i=1}^{n} r_{i,n}^2 \xrightarrow{ucp} V_t, \quad (3)$$

where ucp denotes uniform convergence in probability. RV is useful, in particular in volatility modeling and forecasting.

As jumps are often linked to abnormal or tailed behaviors of returns, the assessment of different RMs of jump variations is important. One way is to decompose price jumps Δp_s as in Duong and Swanson (DS: 2010) and Ait-Sahalia and Jacod (2011) using pre-fixed truncation level γ, $\gamma \geq 0$,

$$JT_{t,\gamma} = \sum_{0<s\leq t} (\Delta p_s)^2 I_{\Delta p_s > \gamma} + \sum_{0<s\leq t} (\Delta p_s)^2 I_{\Delta p_s < -\gamma}, \quad (4)$$

where I is an indicator taking 1 if jump size is larger than γ (upside truncated jumps) or less than $-\gamma$ (downside truncated jumps). Intuitively, $JT_{t,\gamma}$ keeps all jumps with absolute magnitude larger than γ.

[7] For instance, if we use 5 minute sampling frequency to calculate daily measure in our application. Then $t = 1$ and $n = 78$ and $\Delta_n = \frac{1}{78}$.

In this paper, we assess jump variations using different measures, jump power variations formulated by power transformation of absolute log-price jumps ($|\Delta p_s|^q$),

$$JP_{q,t} = \sum_{0<s\le t} |\Delta p_s|^q, \tag{5}$$

and "upside" jump power variation measure, defined as

$$JPV^+_{q,t} = \sum_{0<s\le t} |\Delta p_s|^q I_{\Delta p_s > 0}. \tag{6}$$

$JPV^+_{q,t}$ retains the "upside" jump movements. Similarly, we could consider the "downside" jump power variation which keeps all the "downside" jump movements, i.e.,

$$JPV^-_{q,t} = \sum_{0<s\le t} |\Delta p_s|^q I_{\Delta p_s < 0}, \tag{7}$$

Finally, jump asymmetry could be measured by the so-called signed jump power variation, defined as

$$JA_{q,t} = \sum_{0<s\le t} |\Delta p_s|^q I_{\Delta p_s > 0} - \sum_{0<s\le t} |\Delta p_s|^q I_{\Delta p_s < 0}. \tag{8}$$

In the above expression, we are particularly interested in the case where q is larger or equal to 2. Note that for a large value of q, $JP_{q,t}, JPV^+_{q,t}, JPV^-_{q,t}$, $JA_{q,t}$ are dominated by large jumps. For $q < 2$, the jump variations are not always guaranteed to be finite. The natural estimators for the above jump variations are based on power transformation of intra-daily return, $|r_{i,n}|^q$, which we will discuss in the next section.

3 Realized Measures of Price Jump Variations

Our interest in this paper is to construct and examine the realized measures (RMs) of jump power variations such as $JP_{q,t}$, $JPV^+_{q,t}$, $JPV^-_{q,t}$, $JA_{q,t}$, for a wide range of values of q, and then use them for various prediction experiments. In this line of research, note that for the case $q = 2$, BKS (2010) develop the so-called realized semivariances which are the estimators of $JPV^+_{q,t}$, $JPV^-_{q,t}$. PS (2011) build on these results and make use of realized semivariances to forecast volatility. For the variations with $q \neq 2$, GS (2009) study the predictability of future RV using realized power variations. Realized power variations are formed on the basis of the power transformation of absolute return. They look for the optimal predictors of this type in the forecast. In their set-up, the log-price process is a continuous semimartingale. In the following sections, we briefly review the estimators used in GS (2009), BKS (2010) and PS (2011) and then present the RMs of jump power variations $JP_{q,t}$, $JPV^+_{q,t}$, $JPV^-_{q,t}$, $JA_{q,t}$ used in our paper.

3.1 Semivariances and Realized Power Variations

We start by reviewing the estimators used in BKS (2010) and PS (2011). BKS (2010) construct realized semivariances on the basis of square transformation of intra-daily return, $r_{i,n}^2$, defined as follows:

$$RS^- = \sum_{i=1}^{n} (r_{i,n})^2 \, I_{\{r_{i,n}<0\}},$$

and

$$RS^+ = \sum_{i=1}^{n} (r_{i,n})^2 \, I_{\{r_{i,n}>0\}}.$$

RS^- (RS^+) retains only negative (positive) intra-daily returns and could serve as a measure of downside (upside) risks as pointed out in BKS (2010). They show that RS^+ and RS^- converge uniformly in probability to semi-variances,

$$RS^+ \to \frac{1}{2}\int_0^t \sigma_s^2 ds + \sum (\Delta p_s)^2 I_{\Delta p_s>0}, \tag{9}$$

and

$$RS^- \to \frac{1}{2}\int_0^t \sigma_s^2 ds + \sum (\Delta p_s)^2 I_{\Delta p_s<0}.$$

With the above limit results, realized measure of "downside" jump variation is obtained by replacing $\int_0^t \sigma_s^2 ds$ with it's estimator $\widehat{IV}$,

$$\sum_{i=1}^{n} r_{i,n}^2 I_{\{r_{i,n}<0\}} - \frac{1}{2}\widehat{IV} \to \sum (\Delta p_s)^2 I_{\Delta p_s \leq 0}. \tag{10}$$

In volatility forecasting experiments, PS (2011) use bipower variation for $\widehat{IV}$ [8]. In addition, they construct the so-called "signed" jump variation variable, $\Delta RJ = RS^+ - RS^-$ that captures jump variation asymmetry,

$$\Delta RJ \to \sum (\Delta p_s)^2 I_{\Delta p_s>0} - \sum (\Delta p_s)^2 I_{\Delta p_s<0}.$$

When jumps are not present, ΔRJ converges to 0 and there is no asymmetry in volatility of (log) price process. When the process has jumps, ΔRJ could be a proxy for jump variation asymmetry.

Turning to the discussion of variations with $q \neq 2$, to our knowledge, very few papers empirically examined realized power variations for forecasting. GS (2009) examine the optimal realized power variation, $n^{-1+q/2} \sum_{i=1}^{n} |r_{i,n}|^q$ (optimal q) in forecasting future RV. They build their estimators on the assumption that the price process follows Brownian Semi-martingale. Their implications

[8] See BNS (2004) for the dicussion on bipower variation and integrated volatility.

are therefore restricted to the higher order variation of log-price continuous component, $\int_0^t \sigma_s^q ds$, involving no jumps. In such case, Ait-Sahalia and Jacod (2011) point out that for all $q > 0$,

$$n^{-1+q/2} \sum_{i=1}^{n} |r_{i,n}|^q \rightarrow \mu_q \int_0^t \sigma_s^q ds, \tag{11}$$

where $\mu_q = E(|u|^q)$ and u is a standard normal random variable.

3.2 Realized Downside and Signed Jump Power Variation

Understanding the role of variables that capture jump information is potentially important for applied practitioners. In this section, we first study recently proposed realized measures of jump power variations $JP_{q,t}$. The measures are constructed using power transformations of absolute intra-daily returns, based on recent limit theory advances due to Jacod (2008) and BKS (2010). Furthermore, the limit theory that we adopt then allows us to construct estimators of downside and upside jump power variations, $JPV_{q,t}^+$, $JPV_{q,t}^-$ for $q > 2$, using intra-daily positive and negative returns. These estimators are suggested by BKS (2010) as alternatives to the semivariances implemented in PS (2011). Finally. making use of the RMs of $JPV_{q,t}^+$, $JPV_{q,t}^-$, we develop a novel proxy for jump asymmetry (i.e., realized signed jump power variation). The RMs of jump power variations are defined as:

$$RPV_{q,t} = \sum_{i=1}^{n} |r_{i,n}|^q,$$

for $q > 0$.

The realized downside and upside power variations are defined as:

$$RJ_{q,t}^+ = \sum_{i=1}^{n} |r_{i,n}^+|^q,$$

and

$$RJ_{q,t}^- = \sum_{i=1}^{n} |r_{i,n}^-|^q,$$

for $q > 2$.

For a brief discussion of the above realized measures, the convergences of the above RMs to jump power variations occur when $q > 2$. Therefore, in the prediction experiments, different from previous work, we are particularly interested a range of q from 2 to 6 and allow for price process to contain jumps. Regarding $RPV_{q,t}$, we also look at at the range of q from 0 to 2 by applying a jump robust limit result of Jacod (2008).

Regarding the limiting behavior of $RPV_{q,t}$, Todorov and Tauchen (2009) summarize selected results from Barndorff-Nielsen et. al. (2005) and Jacod (2008). In their set-up, the log-price process contains continuous martingale, jump and drift components. The value of q directly affects the limiting behavior of $RPV_{q,t}$. For instance, for $q < 2$, the limit of $RPV_{q,t}$ is determined by the continuous martingale. For $q > 2$, the limit is driven by jump component. When $q = 2$, both continuous and jump components contribute to the limit of $RPV_{q,t}$. The results are as follows:

$$\begin{cases} \Delta_n^{1-q/2} RPV_{q,t} \xrightarrow{ucp} \mu_q \int_0^t \sigma_s^q ds \text{ , if } 0 < q < 2, \\ RPV_{q,t} \xrightarrow{ucp} V \text{ if } q = 2, \\ RPV_{q,t} \xrightarrow{ucp} JP_{q,t} \text{ if } q > 2. \end{cases} \tag{12}$$

BKS (2010) point out that we can go one step further to decompose jump power variations into upside movements and downside movements, i.e.

$$\begin{cases} RJ_{q,t}^{+} \xrightarrow{ucp} JPV_{q,t}^{+} \\ RJ_{q,t}^{-} \xrightarrow{ucp} JPV_{q,t}^{-} \end{cases} \text{if } q > 2 \tag{13}$$

As earlier mentioned, for $q < 2$, the scaled $RPV_{q,t}$ converges to power variations of the continuous component, involving no jumps. Intuitively, with $q > 2$, the scaled $RPV_{q,t}, RJ_{q,t}^{+}, RJ_{q,t}^{-}$ eliminate all variations due to the continuous component and keep all the large jumps. In addition, the realized measures are more dominated by large jumps for the high value of q. Conversely, for the case $q < 2$, all jumps are eliminated asymptotically.

Building on (13), we could construct the novel RMs of jump power variation asymmetry, so-called "signed" jump power variation. It is straightforward to verify that:

$$RJA_{q,t} = RJ_{q,t}^{+} - RJ_{q,t}^{-} \xrightarrow{ucp} JA_{q,t}$$

Note that this variable has not been studied in volatility forecasting literature. PS (2011) study the predictability of the similar estimator, ΔRJ, constructed on the basis of realized semivariance. In our forecasting experiments, we examine the usefulness of this new jump asymmetry variable, $RJA_{q,t}$ with a wide range of values of $q > 2$, in future volatility forecasting.

As the last remark in our discussion of RMs of variations, in the predictive comparison of variables that capture information generated by jumps and the continuous component, we need to select variables that measure the variation of the continuous movements of price process. In this paper we use multi-power variations, which are estimators of $\int_0^t \sigma_s^q ds$. Those estimators are robust to the existence of jumps. We also utilize these estimators for the jump test implementation highlighted in the next section. The multipower variations discard the impact of jumps by multiplying power transformations of successive absolute intra-daily returns, i.e.,

$$V_{m_1, m_2 ..., m_j} = \sum_{i=2}^{n} |r_{i,n}|^{m_1} |r_{i-1,n}|^{m_2} .... |r_{i-j,n}|^{m_j},$$

where $m_1, m_2, ..., m_j$ are positive, such that $\sum_1^j m_i = q$.

3.3 Testing for Jumps

As discussed in the previous section, realized measures of jump power variations $RPV_{q,t}$, $RJ^+_{q,t}$, $RJ^-_{q,t}$, $RJA_{q,t}$ converge asymptotically to jump power variations $JP_{q,t}$, $JPV^+_{q,t}$, $JPV^-_{q,t}$, $JA_{q,t}$ of the price process. Theoretically, this result also holds for price process without jumps, yielding the limits of zeros. However, in finite sample, it might be useful to implement a pre-testing step to determine whether the log-price process has jumps. The pre-testing approach is developed by ABD (2007) and is empirically examined in DS (2010) for the construction of RMs of truncated jump quadratic variation. We follow this approach in our construction of variables that capture information generated by jumps, in particular we use the jump test statistics developed by BNS (2006) and Huang and Tauchen (2005).

Firstly, we review some theoretical results relating to testing for jumps, namely testing whether $J_t \neq 0$.

In pioneering work, BNS (2006) propose a robust and simple test for a class of Brownian Itô Semimartingales plus Compound Poisson jumps[9]. In recent work, Aït-Sahalia and Jacod (2009a) among others develop a different test which applies to a large class of Itô-semimartingales, and allows the log price process to contain infinite activity jumps - small jumps with infinite concentrations around 0.

Regardless of the estimator that is used, the appropriate test hypotheses are:

$$H_0 : p_t \text{ is a continuous process in the interval } [0, t]$$
$$H_1 : \text{the negation of } H_0 \text{ (there are jumps)}$$

If we use multipower variation, under the null hypothesis the test statistic which directly follows from the CLT mentioned above is:

$$LS_{jump} = \frac{\sqrt{\frac{t}{n}} \left(\sum_{i=1}^{n} (r_{i,n})^2 - \widehat{IV} \right)}{\sqrt{\vartheta \widehat{IQ}}} \xrightarrow{D} N(0,1)$$

and the so-called jump ratio test statistic is:

$$RS_{jump} = \frac{\sqrt{\frac{t}{n}}}{\sqrt{\vartheta \widehat{IQ}/(\widehat{IV})^2}} \left(1 - \frac{\widehat{IV}}{\sum_{i=1}^{n} (r_{i,n})^2} \right) \xrightarrow{D} N(0,1).$$

[9] A simplified version of the results of the above authors applied to (1) for the one-dimensional case is as follows. If the process X is continuous, let $f(x) = x^n$ (exponential growth), let ρ_{σ_s} be the law $N(0, \sigma_s^2)$, and let $\rho_{\sigma_s}(f)$ be the integral of f with respect to this law. Then:

$$\sqrt{\frac{1}{\Delta_n}} \left(\Delta_n \sum_{i=1}^{n} f(\frac{\Delta_i^n X}{\sqrt{\Delta_n}})^2 - \int_0^t \rho_{\sigma_s}(f) ds \right) \longrightarrow \int_0^t \sqrt{\rho_{\sigma_s}(f^2) - \rho^2_{\sigma_s}(f)} dB_s \quad (14)$$

where $\widehat{IV}$ and $\widehat{IQ}$ are multipower variation estimators of integrated volatility $\int_0^t \sigma_s^2 ds$ and $\int_0^t \sigma_s^4 ds$. BNS (2006) use $V_{1,1}$ (bipower variation) and $V_{1,1,1,1}$. In jump test implementation with multipower estimators, ABD (2007) suggest the use $V_{\frac{2}{3},\frac{2}{3},\frac{2}{3}}$ (tripower variation) and $V_{\frac{4}{5},\frac{4}{5},\frac{4}{5}}$. The reason we use tripower variation, $V_{\frac{2}{3},\frac{2}{3},\frac{2}{3}}$, instead of bipower variation, $V_{1,1}$, is that it is more robust to clustered jumps and note that:

$$\widehat{IV} = V_{\frac{2}{3},\frac{2}{3},\frac{2}{3}} \mu_{\frac{2}{3}}^{-3} \tag{15}$$

and

$$\widehat{IQ} = \frac{n}{t} V_{\frac{4}{3},\frac{4}{3},\frac{4}{3}} \mu_{\frac{4}{5}}^{-5} \tag{16}$$

where $\mu_r = E(|Z|^r)$ and Z is a $N(0,1)$ random variable.

Andersen, Dobrev, Schaumburg (2008) suggest a different estimator that could handle the case of consecutive jumps. This estimator is also more robust to occurrence of zero-return. This robust jump measure is as follows:

$$\widehat{IV} = MedRV_n = \frac{\pi}{6 - 4\sqrt{3} + \pi} \left(\frac{n}{n-2} \right) \sum_{i=2}^{n-1} med \left(|r_{i-1,n}| \, |r_{i-2,n}| \, |r_{i-3,n}| \right)^2$$

Of note is that an adjusted jump ratio statistic has been shown by extensive Monte Carlo experimentation in Huang and Tauchen (2005), in the case of CCP jumps, to perform better than the two above statistics, being more robust to jump over-detection. This adjusted jump ratio statistic is:

$$AJ_{jump} = \frac{\sqrt{\frac{n}{t}}}{\sqrt{\vartheta max(t^{-1}, \widehat{IQ}/(\widehat{IV})^2)}} \left(1 - \frac{\widehat{IV}}{\sum_{i=1}^{n} (r_{i,n})^2} \right) \xrightarrow{D} N(0,1)$$

In general if we denote the daily test statistics to be $Z_{t,n}(\alpha)$, where n is the number of observations per day and α is the test significance level [10], then we reject the null hypothesis if $Z_{t,n}(\alpha)$ is in excess of the critical value Φ_α, leading to a conclusion that there are jumps. The converse holds if $Z_{t,n}(\alpha)$ is less than Φ_α. In our empirical application, $Z_{t,n}(\alpha)$ is the adjusted jump ratio statistic.

3.4 Realized Measures of Daily Variations

With the availability of the RMs such as $RPV_{q,t}, RJ_{q,t}^{+}, RJ_{q,t}^{-}$, $RJA_{q,t}$ and the jump tests, in this section, we elaborate further on how to construct daily time series of variables that captures information generated by the variations of log-price process for forecast experiments.

For each day, we calculate the realized measures of jump power variations using a high frequency price path. To mitigate the effect of microstructure noises,[11] we sample data at five-minute

[10] i.e., $\Delta_n = 1/n$

[11] The main drawback of realized measures constructed on the basis of high frequency data is that they are contaminated by mictrostructure noises. See Aït-Sahalia, Mykland and Zhang (2005) for further dicussion.

frequency as suggested in Aït-Sahalia, Mykland and Zhang (2005). The first group of predictors is constructed without jump tests. The second group of predictors utilizes jump test adjusted technique by ABD (2007). We set $n = 78$, the number of five-minute observations within a day, and consider the range of q from 0.1 to 6, i.e. $q = \{0.1, 0.2, ..., 5.9, 6\}$.

3.4.1 Predictors with No Jump Test

The daily times series of realized measures of jump power variations are formed at a particular day t as follows:

$RPV_{q,t}$ = Realized qth order power variation at day $t = \sum_{i=1}^{78} |r_{i,78}|^q$ with $q > 0$,

$RJ^+_{q,t}$ = Realized Measure of qth order upside jump power variation at day $t = \sum_{i=1}^{78} \left(|r^+_{i,78}|^q\right)$, $q > 2$,

$RJ^-_{q,t}$ = Realized Measure of qth order downside jump power variation at day $t = \sum_{i=1}^{n} \left(|r^-_{i,78}|^q\right)$, $q > 2$,

$RJA_{q,t}$ = Realized Measure of qth order signed jumps power variation at day $t = RJ^+_{q,t} - RJ^-_{q,t}$, $q > 2$,

As noted before, realized qth order power variation with $q < 2$ does not involve jumps.

3.4.2 Predictors with Jump Test

First, the predictors are calculated as in section 3.4.1. Jump tests are then implemented on daily basis and the predictors are adjusted accordingly. Specifically, RMs of jump power variations at day t are positive if jumps are detected and 0 otherwise. This simple approach is first studied by ABD (2007) in the construction of time series of RMs of quadratic variations of jump component. Let $I_{jump,t}$ be the indicator of jumps, i.e. $I_{jump,t} = 1$ if jumps occur at day t and $I_{jump,t} = 0$ otherwise. Then the adjusted realized measure of jump power variations are expressed as,

$RPV_{q,t}$ = Realized $q-th$ order power variation $= I_{jump,t} * \{\sum_{i=1}^{78} |r_{i,78}|^q\}$,

$RJ^+_{q,t}$ = Realized $q-th$ order upside jump power variation $= I_{jump,t} * \{\sum_{i=1}^{78} \left(|r^+_{i,78}|^q\right)\}$,

$RJ^-_{q,t}$ = Realized $q-th$ order downside jump power variation $= I_{jump,t} * \{\sum_{i=1}^{n} \left(|r^-_{i,78}|^q\right)\}$,

$RJA_{q,t}$ = Realized $q-th$ order signed jumps power variation $= I_{jump,t} * \{RJ^+_{q,t} - RJ^-_{q,t}\}$.

3.4.3 Benchmark Realized Variations of the Continuous and Jump Components

The RMs of quadratic variation (RV) and variation of continuous component are formalized as in ABD (2007),

$RVJ_t = Variation\ of\ the\ jump\ component = max\{0, RV_t - \widehat{IV_t}\} * I_{jump,t}$,

$RVC_t = Variation\ of\ continuous\ component = RV_t - RVJ_t$,

where $\widehat{IV_t}$ is an estimator of variation of continuous component $\int_0^t \sigma_s^2 ds$. One could use Tripower Variation or Truncated Power Variation. In the paper, we use Tripower Variation:

$RVJ_t = Variation\ of\ the\ jump\ component = max\{0, RV_t - \widehat{IV_t}\} * I_{jump,t}$,

RVC_t = *Variation of continuous component* = $RV_t - RVJ_t$.

As the above measures in section 3.4.1 and 3.4.2 depend on q, we take into account the fact that larger q magnifies larger jump in the sum. $RPV_{q,t}$, $RJ^{+}_{q,t}$, $RJ^{-}_{q,t}$ and $RJA_{q,t}$ are assessed for a wide range of values of q, from 2.1 to 6, i.e. $q = \{2.1, 2.2, ..., 5.9, 60\}$. We end up with 40 sub-models (predictors) for each case. For the realized power variation RPV, we set q from 0.1 to 6.

4 Models and Forecast Evaluations

4.1 Model Specifications

In a classical paper, Ding, Granger and Engle (DGE:1993) find that the auto-correlation of power transformation of daily return of S&P 500 is strongest when $q = 1$, as opposed to the value $q = 2$ widely used in the literature. This leads them to generalize ARCH type model to the class of so-called Asymmetric Power ARCH (APARCH) model. The APARCH specification allows for the flexibility of q in the power qth transformation of absolute returns. GS (2009) point out that this class of models ends up working with volatility that is not measured by squared returns, which researchers and practitioners care the most. Using the five-minute intra-daily returns of the Down Jones Composite over the period 1993-2000, GS (2009) make a thorough empirical correlation analysis of daily RV and realized power variations, with the forecasting horizon from one to four weeks. They conclude that realized power variation with $q = 1$ and future RV display the strongest cross-correlation over the first 10 lags. Beyond this first 10 lags, the cross-correlation holds for $q = 0.5$. This suggests that the prediction of RVs using variables such as realized power variation might be a better approach compared to the lag of RVs. GS (2009) use the Mixed Data Sampling Regression (MIDAS) models to investigate the predictive power of realized power variation for $q < 2$.

We add to the empirical research on this topic by providing results on volatility forecasting using a variety of "new" variables that capture information generated by jumps. In particular, we utilize RMs of jump power variations discussed in the previous section. We estimate an extended Heterogeneous Autoregressive of the Realized Volatility (HAR-RV) class of models. The HAR-RV model, initially developed in Corsi (2009), has been implemented with success in a number of recent contributions. These models are formulated on the basis of the so-called Heterogeneous ARCH, or HARCH, a class of models analyzed by Müller et al. (1997), in which the conditional variance of the discretely sampled returns is parameterized as a linear function of the lagged squared returns over the identical return horizon together with the squared returns over shorter return horizons. Intuitively, different groups of investors have different investment horizons, and consequently behave differently. The genuine HAR-RV model is formally a constrained AR(22) model and is convenient in application as volatility is treated as if it is observed, when we fit regressions in order to assess predictability. In the following, we describe the set-up of HAR-RV and present the specifications that extends HAR-RV to incorporate our new jump variables.

Define the multi-period normalized realized measures for jump and continuous components as the average of the corresponding one-period measures. Namely for daily time series Y_t, we construct $Y_{t,t+h}$ such that

$$Y_{t,t+h} = h^{-1}[Y_{t+1} + Y_{t+2} + ... + Y_{t+h}], \tag{17}$$

where h is an integer. $Y_{t,t+h}$ aggregates information between time $t+1$ and $t+h$. The daily time series Y_t could be the RMs such as $RV_t, RVJ_t, RVC_t, RPV_{q,t}, RJ^+_{q,t}, RJ^-_{q,t}, RJA_{q,t}$ and $q = \{0.1 + 0.1k\}_{k=0}^{k=59}$.

In standard linear and nonlinear HAR-RV models, future RV depends on the past of RV,

$$\phi(RV_{t,t+h}) = \beta_0 + \beta_d \phi(RV_t) + \beta_w \phi(RV_{t-5,t}) + \beta_m \phi(RV_{t-22,t}) + \epsilon_{t+h}, \tag{18}$$

where ϕ is a linear, square root or log function.

The incorporations of RMs of jump variations, RVJ_t could be done as in ABD (2007), using the HAR-RV-J,

$$\phi(RV_{t,t+h}) = \beta_0 + \beta_d \phi(RV_t) + \beta_w \phi(RV_{t-5,t}) + \beta_m \phi(RV_{t-22,t}) + \beta_j \phi(RVJ_t) + \epsilon_{t+h},$$

or HAR-RV-CJ,

$$\begin{aligned}\phi(RV_{t+h}) = \beta_0 &+ \beta_d \phi(RVC_t) + \beta_w \phi(RVC_{t-5,t}) + \beta_m \phi(RVC_{t-22,t}) + \beta_{jd} \phi(RVJ_t),\\ &+ \beta_{jw} \phi(RVJ_{t-5,t}) + \beta_{jm} \phi(RVJ_{t-22,t}) + \epsilon_{t+h}.\end{aligned}$$

ABD (2007) find that the class of log HAR-RV, log HAR-RV-J and log HAR-RV-CJ models performs the best for several market indexes. DS (2010) revisit this class of models but focus on the predictive performance of the models applied to Dow 30 individual stock returns. PS (2011) extend this class of models to assess different predictors, the realized semivariance and realized signed jump measure. Their extended HAR-RV model is,

$$\begin{aligned}\phi(RV_{t,t+h}) = \beta_0 &+ \beta_1^+ \phi(RS_t^+) + \beta_1^- \phi(RS_t^-) + \beta_5^+ \phi(RS^+_{t-5,t}) + \beta_5^- \phi(RS^-_{t-5,t})\\ &+ \beta_{22}^+ \phi(RS^+_{t-22,t}) + \beta_{22}^- \phi(RS^-_{t-22,t}) + \varepsilon_{t+h}.\end{aligned}$$

Building on Corsi (2004), ABD (2007) and PS (2011), we extend the HAR- RV to incorporate time series of RMs of jump power variations. In addition, we modify the forecast set-up by examining the forecast of RV_{t+h}, rather than $RV_{t,t+h}$. The specifications are presented as follows:

Specification 1: Class of standard HAR-RV-C Model (Benchmark Model),

$$\phi(RV_{t+h}) = \beta_0 + \beta_{cd} \phi(RVC_t) + \beta_{cw} \phi(RVC_{t-5,t}) + \beta_{cm} \phi(RVC_{t-22,t}) + \epsilon_{t+h}. \tag{19}$$

In this benchmark case, future RVs depend on lags of the variation of the continuous component of the process.

Specification 2: Class of HAR-RV-C-PV(q) Model,

$$\begin{aligned}\phi(RV_{t+h}) &= \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta_{jd}\phi(RPV_{q,t}) + \beta_{jw}\phi(RPV_{q,t-5,t}) + \beta_{jm}\phi(RPV_{q,t-22,t}) + \epsilon_{t+h},\end{aligned} \tag{20}$$

where $RPV_{q,t}$ is qth order variation of the jump component. $RPV_{q,t-5,t}$ and $RPV_{q,t-22,t}$ are calculated using (17). As discussed in the previous section, we allow for a wide range of values of q from 0.1 to 6. Note that when $q > 2$, the implication of this variable is jump power variations. With $q < 2$, actually the limit is robust to jumps as discussed in Section 2.

Specification 3: HAR-RV-C-UJ(q) Model (upside jump) is defined as,

$$\begin{aligned}\phi(RV_{t+h}) &= \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta^+_{jd}\phi(RJ^+_{q,t}) + \beta^+_{jw}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jm}\phi(RJ^+_{q,t-22,t}) + \epsilon_{t+h}.\end{aligned} \tag{21}$$

This specification incorporates the RMs of qth order upside jump power variations as explanatory variables to forecast future RV. Specifically, $RJ^+_{q,t}, RJ^+_{q,t-5,t}, RJ^+_{q,t-22,t}$ measure the qth order power variation of positive jumps of today, previous week, and previous month, respectively. $RJ^+_{q,t-5,t}, RJ^+_{q,t-22,t}$ are calculated using (17). The range of q varies from 2.1 to 6.

Specification 4: HAR-RV-C-DJ(q) Model (downside jump),

$$\begin{aligned}\phi(RV_{t+h}) &= \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta^-_{jd}\phi(RJ^-_{q,t}) + \beta^-_{jw}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jm}\phi(RJ^-_{q,t-22,t}) + \epsilon_{t+h}.\end{aligned} \tag{22}$$

This specification incorporates the RMs of qth order downside jump variations as explanatory variables. Specifically, $RJ^-_{q,t}, RJ^-_{q,t-5,t}, RJ^-_{q,t-22,t}$ are the RMs of the qth order power variations of negative jumps of today, previous week, and previous month, respectively. $RJ^-_{q,t-5,t}, RJ^-_{q,t-22,t}$ are calculated using (17). The range of q varies from 2.1 to 6.

Specification 5: HAR-RV-C-UDJ(q) Model (Full decomposition),

$$\begin{aligned}\phi(RV_{t+h}) &= \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta^+_{jd}\phi(RJ^+_{q,t}) + \beta^+_{jw}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jm}\phi(RJ^+_{q,t-22,t}) \\ &+ \beta^-_{jd}\phi(RJ^-_{q,t}) + \beta^-_{jw}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jm}\phi(RJ^-_{q,t-22,t}) + \epsilon_{t+h}.\end{aligned} \tag{23}$$

This specification fully decomposes realized measure of the qth order jump power variation into upside and downside components. The predictors therefore contain both upside and downside jump power variations, i.e. $RJ^+_{q,t}, RJ^+_{q,t-5,t}, RJ^+_{q,t-22,t}$ and $RJ^-_{q,t}, RJ^-_{q,t-5,t}, RJ^-_{q,t-22,t}$. We set the range of q to vary from from 2.1 to 6 for this specification.

Specification 6: HAR-RV-C-APJ(q) Model,

$$\begin{aligned}\phi(RV_{t+h}) &= \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta_{jd}\phi(RJA_{q,t}) + \beta_{jw}\phi(RJA_{q,t-5,t}) + \beta_{jm}\phi(RJA_{q,t-22,t}) + \epsilon_{t+h}.\end{aligned} \tag{24}$$

This class of models uses RMs of signed jump power variations, measures of jump asymmetry, as explanatory variables for RV forecast. Specifically, predictors are $RJA_{q,t}$, $RJA_{q,t-5,t}$ and $RJA_{q,t-22,t}$, calculated using (17).

The estimation of the above models is simply done by OLS regression. We report the parameters and measures of fit. Across all the specifications, there is a potential issue of the serial correlation due to the long forecast horizons ($h = 5, 22$). Though serial correlation does not affect the consistency of the estimated parameters, robust estimates of covariance matrix need be addressed. In our empirical experiments, we apply both standard and robust heteroskedasticity-and-autocorrelation-consistent (HAC) estimators of covariance matrix [12] .

In a different forecast experiment, we construct realized measures of the truncated jump power variations following the similar approach as in DS (2010). Specifically, we define the RMs of jump power variations truncated at a fixed level γ to be:

$$RJ_q(\gamma) = \sum_{i=1}^{n} |r_{i,n}|^q I_{|r_{i,n}|<\gamma}.$$

Similarly for the RMs of downside jump power variations truncated at fixed level γ,

$$RJ_q^-(\gamma) = \sum_{i=1}^{n} |r_{i,n}|^q I_{-\gamma<r_{i,n}<0},$$

and for realized measure of upside jump power variations truncated at fixed level γ,

$$RJ_q^+(\gamma) = \sum_{i=1}^{n} |r_{i,n}|^q I_{0<r_{i,n}<\gamma}.$$

Then, if one is interested in jumps with magnitude less than γ in the forecast of future RV, the time series of $RJ_{q,t}, RJ_{q,t}^-, RJ_{q,t}^+$ on the right hand side of forecasting equation(20) (21) (22) (23) (24) could be replaced by $RJ_{q,t}(\gamma), RJ_{q,t}^-(\gamma), RJ_{q,t}^+(\gamma)$. In this context, we assume that the modeler has a predetermined knowledge of γ. In our empirical implementation, we choose γ on the basis of the sample of maximum of monthly increments which represents monthly abnormal events. A question we would want to see is whether the choice of larger γ has an impact on the volatility prediction.

Note that we could obtain the optimal value of q for prediction of volatility under a certain measure of fit criteria such as the minimum mean square error. However, this is not the aim of our paper. By using a wide rage of values of q, we are more interested in capturing the pattern of the predictive powers of the RMs of jump power variations. The pattern also helps in approximating the optimal value of q in the prediction.

Regarding the predictive regression of the above models, note that for specification 2, we need to estimate 60 linear regression equations, depending on q from 0.1 to 6. For each specification

[12] For Hac estimator, we use Newey-West estimator.

3,4,5 and 6, we need to estimate 40 linear regression equations, depending on q from 2.1 to 6. A straightforward way to assess the usefulness of the RMs is to compare the predictive accuracy, measured by mean square errors or R^2 across all values of q. In the next section, we discuss the forecast evaluation methods that are being used in empirical implementation.

4.2 In-Sample and Out-of-Sample Forecast Evaluation

For each specification, we fit the above forecasting equations by ordinary least square. The forecast horizons that we examine in this paper are set to be $h = 1, 5, 22$ which are the one day ahead, one week ahead and one month ahead horizon, respectively. Our model specifications extend the standard HAR-RV models, as presented in previous section. For each specification, we have 40 sub-models, corresponding to 40 different values of q. Once a measure of fit is obtained, we present it as a function of q and the relationship between q and the measure of fit could be plotted. Regarding the measure of fit, a straightforward way is using in-sample adjusted R^2. The other favored measure of fit is the out-of-sample R^2, calculated from projection of the predictive RV sample on the sample of forecasted RV implied by the model. For the pairwise model comparisons, we use the Diebold-Mariano (DM:1995) test and quadratic loss function.

Specifically, the entire sample of T observations is divided into two samples, the estimation sample containing R observations and the prediction sample containing $P = T - R$ observations. The traditional in-sample adjusted R^2 is calculated using entire sample T.

For the out of sample forecast, we calculate the R^2 using recursive, rolling or fixed estimation schemes. If the forecast horizon $h = 1$ and the recursive estimation are used, the model is to be fitted by P regressions using data chunks from 1 to R, 1 to $R + 1$, ...1 to $T - 1$. Alternatively, we could use the rolling scheme where the P regressions are implemented using data chunks 1 to R, 2 to $R+1, ..., T-P$ to $T-1$. The fixed scheme requires the estimation using the entire sample. After this step, we could calculate P predicted values implied by the models. Next, the out-of-sample R^2s are obtained by simply regressing the prediction sample on the forecasts implied by the models. Note that the above procedure is presented with forecasting horizon $h = 1$. For the general forecast horizon h and the recursive scheme, the models are fitted P times using data chunk from 1 to $R - h + 1$, 1 to $R + 1$, ...1 to $T - h$. For the rolling scheme, P model-implied forecasts are obtained by the estimation using data chunk from 1 to $R - h + 1$, 2 to $R + 1, ..., T - P - h + 1$ to $T - h$.

Now turn to predictive equality accuracy test of Diebold and Mariano (DM: 1995), we could formally make a pairwise comparisons of any two models by applying this test. Suppose we are interested in the comparison of two models $i = 1, 2$ using the times series y_t, $t = 1, 2, ..., T$. The mean square forecast error (MSFE) is defined as

$$MSFE = \sum_{\tau=R-h+2}^{T-h+1} \left(y_{t+h} - \widehat{y}_{i,t+h}\right)^2,$$

where $\widehat{y}_{i,t+h}$ is the forecast for horizon h for model i. Denote $\varepsilon_{i,t+h}$ to be model's prediction error of model i. The hypothesis could be set up as. The null $H_0 : E(\varepsilon^2_{1,t+h}) - E\left(\varepsilon^2_{2,t+h}\right) = 0$ and alternative H_1 : not H_0. The actual statistics is constructed as: $DM = P^{-1}\sum_{k=1}^{P}(d_t/\widehat{\sigma})$ where $d_t = \widehat{\varepsilon}^2_{1,t+h} - \widehat{\varepsilon}^2_{2,t+h}$, a the comparative measure of fit between the two models. $\widehat{\sigma}$ is the the estimator of standard deviation of $(\sum_{k=1}^{P} d_t)/P$. The choice of this estimator could be set as a heteroskedasticity and auto-correlation robust estimator (HAC). In addition, to the acceptance and rejection outcome of the test on the basis of the test statistics, we could also infer that the negative statistics implies that model 2 is preferred to model 1 as it's statistics measure of fit over the out of sample forecast is superior.

4.3 Alternative Models to HAR-RV and Other Issues

Given the main focus of our paper is to assess the predictability of the new group of variables that capture information generated by jumps, we use the simple predictive regression models, i.e. the extended HAR-RV in this paper. For an alternative to the extended HAR-RV class of models, the GARCH-based model as in BKS (2010) could be considered. The other approach is using stochastic volatility models. Both approaches require us to treat true volatility as an unobserved variable. In this context, RVs are additional variables that capture rich information generated by high frequency data sets. Stochastic Volatility (SV) model in discrete time is discussed in depth by Shephard (2005). We could use models that inputs RV variable into volatility equation. One way is to estimate the bivariate return - stochastic volatility system building on Liesfield and Richard (2003) filtering framework. In the context of mixed data sampling, one could also implement non-linear regression MIDAS as used in Ghysels and Sohn (2009) as an alternative to HAR-RV model.

In addition, with the choice of volatility estimator, variables that capture information generated by jumps (RMs of Jump Power Variations), jump test statistics, and predictive models as discussed in previous sections, before moving to the discussion on empirical findings, it is useful make an comparative overview on empirical strategies implemented in our paper and other related papers. The following table summarizes the selected papers that examine jumps and higher order power transformation of absolute returns to predict future volatility.

Summary of Related Work using RMs of Power Variations for Volatility Prediction[13]

Paper	HFD	Jumps	Dow/Upside	Power q	Jump Test	Truncation
DGE (1993)	No	No	No	0-5	No	No
LM (2005)	Yes	Yes	No	0-2	No	No
ABD (2007)	Yes	Yes	No	2	Yes	No
GS (2009)	Yes	No	No	0-2	No	No
PS (2011)	Yes	Yes	Yes	2	No	No
Duong (2012)	Yes	Yes	Yes	0-6	Yes	Yes

In the above table, note that our work makes a thorough examination of jumps variations by using a wide range of values of q compared to other papers and we also consider jump test adjusted RMs in predictions. In the next section, we present empirical findings of our paper.

5 Empirical Findings

5.1 Data Description

For empirical implementation in this paper, we implement the forecasting experiments on S&P 500 futures for the period 1993-2009. We also look at Dow 30 components in the period 1993-2008 as in DS (2010). The data source for stocks is the TAQ database. In the data processing, we follow the common practice in the literature by eliminating from the sample those days with infrequent trades (less than 60 transactions at our 5 minute frequency).

One problem in data handling involves determining the method to filter out an evenly-spaced sample. In the literature, two methods are often applied - *previous tick* filtering and *interpolation* (Dacorogna, Gencay, Müller, Olsen, and Pictet (2001)). As shown in Hansen and Lund (2006), in applications using quadratic variation, the *interpolation* method should not be used, as it leads to realized volatility with value 0 (see *Lemma 3* in their paper). Therefore, we use the *previous tick* method (i.e. choosing the last price observed during any interval). We restrict our data-set to regular time (i.e. 9:30am to 4:00pm) and ignore ad hoc transactions outside of this time interval. To reduce microstructure effects, the suggested sampling frequency in the literature is from 5 minutes to 30 minutes[14]. As mentioned above, we choose the 5 minute frequency, yielding a maximum of 78 observations per day.

[13] The table summarizes the selected papers that examine jumps and higher order power transformation of absolute returns to predict future volatility. The first column is the list of papers under consideration. The second, third and fourth column provide information whether the paper in the list utilizes high frequency data (HFD), jumps, downside/upside jumps, respectively. The fifth column provides the range of order q used in each paper. The sixth and seventh column provide information whether the paper implements jump test adjustment technique as in ABD (2007) and whether the paper looks at truncated jumps (truncation) in the construction of jump variables for volatility prediction, respectively.

[14] See Aït-Sahalia, Y., Mykland, P. A., and Zhang, L. (2005)

5.2 Prediction without Jump Test

First, we calculate all daily RMs as discussed in section 3.4 for S&P 500 futures. For each realized measure, we end up with a time-series of size $T = 4123$. In the out-of-sample forecasting experiments, we choose prediction sample size, $P = 410$, and estimation sample size, $R = 3713$, respectively.[15]

Models considered in our empirical application are discussed in Section 3. We present all the specifications in Table 2. For a quick summary, the specification 1 (benchmark model), HAR-RV-C incorporates only RMs of continuous component variations as predictors. The specification 2, the HAR-RV-C-PV ($q > 0$) uses RMs of continuous component variations and RMs of qth (jump) power variation components as predictors[16]. The specification 3, the HAR-RV-C-UJ ($q > 2$) uses RMs of continuous component variations and RMs of the qth order "upside" jump power variation components as predictors. The specification 4, HAR-RV-C-DJ ($q > 2$) utilize continuous component variations and the qth order "downside" jump power variation components as predictors. The specification 5, HAR-RV-C-UDJ ($q > 2$) builds directly on specification 3 and 4, and uses both RMs of the qth order upside and downside jump power variation components in predictions. The specification 6, HAR-RV-C-APJ examines variables that capture jump asymmetry by incorporating RMs of the signed qth order jump power variations in the prediction. The formulation of the time series, $RPV_{q,t}, RJ^{+}_{q,t}$, $RJ^{-}_{q,t}$ and $RJA_{q,t}$ is shown in details Section 3.4.

The empirical analyses of exchange rates, equity index returns, and bond yields in ABD (2007) suggest that the volatility jump component is both highly important and distinctly less persistent than the continuous component, and that separating the "rough" jump movements from the smooth continuous movements results in significant in-sample volatility forecast improvements (i.e. the linear and nonlinear HAR-RV-CJ models perform better than the other two classes of models).

We first provide a brief discussion on the performance of the models for S&P futures. In our paper, the predictive performance of a model is measured by both its in-sample and out-of sample R^2, which is similar to approach taken in ABD (2007). We also carry out the Diebold-Mariano (1995) predictive equivalence tests to determine whether the choice of order q matters for the qth order jump power variations in forecasting future RV.

Turning to our regression results, Table 1 reports the regression estimates, in-sample and out-of-sample R^2 values for the linear, square root and log HAR-RV-C models at daily ($h = 1$), weekly ($h = 5$) and monthly ($h = 22$) prediction horizons. The entries in bracket are t-statistics calculated using the Newey-West estimator with auto-correlation up to 44 lags[17]. Regarding in-sample and out-of-sample R^2s, as shown in the table, the square root models and log models perform much

[15] We also implement other choices for P =210, 310, 510, 610,710 and results show the same pattern, which are available upon request.

[16] For q>2,the realized power variation converges to the jump power variation. For q<2, the standardized realized power variation converges to power variation of continuous component as discussed in Section 3.

[17] HAC estimator is known to be robust to both heteroscedasticity and auto-correlation.

better than their linear counterpart regardless of the prediction horizon. For instance, at the forecasting horizon $h = 1$, the in-sample and out-of-sample R^2 of square root models are 0.45 and 0.34 while those of the linear counterpart are 0.35 and 0.24, respectively. In addition, the estimates of β_{cd}, β_{cw}, β_{cm} and t-statistics confirm the long memory persistent feature of volatility. For the linear model with $h = 1$, the t-statistics of the monthly forecast parameter is 7.81, implying that the continuous component of the previous month could help the one-day-ahead prediction of volatility. This statistical pattern holds for square root and log models across all forecast horizons. In addition, at prediction horizon $h = 22$, while the in-sample R^2s are large, the out-of sample results show an opposite direction.

In the formulation of RMs of jump power variations, $RPV_{q,t}$, $RJ^+_{q,t}$, $RJ^-_{q,t}$, and $RJA_{q,t}$, order q is gridded by 0.1 from 2.1 and 6, i.e $q = \{2.1, 2.2, ..., 5.8, 5.9, 6\}$. The choice of maximal $q = 6$ is sufficient to determine the effect of large jumps and their predictive power[18]. With $q > 2$, the realized (jump) power variation, $RPV_{q,t}$, converges asymptotically to jump power variations of log-price process. In addition, larger q effectively eliminates the effect of continuous component and smaller jumps and magnify the impacts of large jumps. In the presentation of results, we choose $q = 2.5$ and $q = 5$, the two representative cases for small and large jump power variations. Table 3A, 3B, 3C, 3D report predictive regression estimates of the two cases. Each table involves linear, square root or log model. All the numbers in the brackets are t-statistics. For the in-sample forecast results, jump coefficients are not statistically significant for $q = 5$ (large jumps). The results hold across all model specifications. For $q = 2.5$, the t-statistics are significant for β_{jm} in HAR-RV-C-PV linear and square root models. Similarly, the t-statistics are 2.366 and 2.1 for forecasting horizon $h = 1$ in HAR-RV-C-PDJ linear and square root models. Regarding the full "decomposition" HAR-RV-C-PDUJ model, we find that the downside jumps have an impact on future RV at one-day-ahead forecast horizon ($h = 1$). In particular, for linear model, Table 3C shows that the t-statistics for β^-_{jd} is 2.138. All the upward jump variations have small impacts on the prediction. More interestingly, correlation between the past $RAJ(q)$ and and future RV is strong across all forecast horizons (daily, weekly and monthly) for all models under consideration (linear, square root, log). Table 4 depicts the findings in the group of log models, showing the t-statistics for β_{jd} of 10.05 (daily), 9.15 (weekly), 10.01(monthly) for case $q = 2.5$ and 10.76 (daily), 9.91 (weekly), 11.08 (monthly) for case $q = 5$. The finding strongly suggests that jump asymmetry matters for modeling future RV, at least at the shorter horizon. In addition, the asymmetry holds for both large and small jumps.

Turn to the analysis of the predictive comparison, our prediction experiments show improvements for both in sample and out of sample once RMs of jump power variations are used as additional predictors in volatility forecasting. For example, at the forecast horizons $h = 1$ and $h = 5$, the out-of sample R^2s of the HAR-RV-C square root models are 0.341 for $h = 1$ and 0.244 for

[18] In our implementation, for $q > 6$, the prediction results are almost the same as the case $q = 6$ and therefore are not presented.

$h = 5$ compared to that numbers of 0.368 and 0.262 of HAR-RV-C-PV models. This is equivalent to 8% and 7.5% increases in R^2 if we switch from HAR-RV-C to HAR-RV-C-PV models. However, as shown in the table, the continuous component, RVC, dominates in all prediction experiments, which is consistent with the previous findings in the literature on volatility forecasting using high frequency data. There are little improvements in R^2 for HAR-RV-C-PDUJ in the prediction. Interestingly, the table suggests in-sample and out-of sample R^2 be smaller for the larger q when we examine prediction experiments for the case $q = 2.5$ and $q = 5$. Once we consider a wider range of values of q, this pattern is clear as shown next.

Table 4 shows the Diebold and Mariano (DM) test statistics for fixed, recursive and rolling schemes. In the construction of the statistics, denoted in the table as DM Stat, we make a restriction for q to be larger or equal to 2.5, i.e. ($q = \{2.5 + k * 0.1\}_{k=0}^{k=35}$) and then search for the values of q that yield the maximal and minimal mean square errors. More specifically, q_b denotes the the value of q that yields biggest R^2 and q_s denotes the value of q that yields the smallest R^2. The table shows that q_b is smaller than q_s. In addition, for most of the models, the value of q_b is 2.5 and the value of q_s is 6. We test whether the predictions of future volatility using RMs of (jump) power variations as predictors differ if q_b and q_s are used. The results of DM tests show that most of the t-statistics are significant, regardless of which forecast scheme is used. In particular, the results are stronger for downside jump measures.

Finally, the pattern involving R^2s suggested in the above discussion is confirmed by our figures shown in the appendix. In Figure 1, we plot the in-sample adjusted R^2s of all linear and nonlinear models across horizon $h = 1, 5$ and 22. The vertical axis ranges from 0 to 1 for the value of the R^2. The horizontal axis ranges from 0.1 to 6, representing the 60 grid points of value of q, i.e. $q = \{0 + 0.1 * k\}_{k=1}^{60}$. In those plots, except for HAR-RV-PV models, we focus on the part of the curves on the right side of 2 as convergence to jump power variation occurs only when $q > 2$ (see e.g. Todorov and Tauchen (2009) and BKS (2010)). The purple curve represents the R^2s of HAR-RV-C-PDUJ model (full decomposition). The light blue curve represents the R^2s of HAR-RV-C-AJP model. The dark purple curve represents HAR-RV-C-PUJ model. The light green curve represents HAR-RV-C-PDJ. The dark blue curve and orange curve represent the R^2s of HAR-RV-C-PV and the benchmark model HAR-RV-C model, respectively. Notably, for $q > 2$ the R^2 is monotonically decreasing in q. These results are consistent with what we found in Table 3. With the monotonic pattern, all the curves look very close to one another, except for HAR-RV-C-PDUJ and HAR-RV-C-AJP model. HAR-RV-C-PDUJ model is slightly better for daily and weekly horizon while HAR-RV-C-AJP model is slightly superior for monthly horizon. It is also clear that the R^2s of all models are higher than those of the benchmark model. This suggests that higher order jumps be helpful in prediction of future RV. The predictive power of large jumps power variations dies out as q becomes bigger. As the higher order jump power variations are dominated by large jumps, the observation suggests that large jumps play less important role in the prediction and dies out when the value of q is larger than 6.

Finally, we also plot the results for the mean square errors across power q, as shown in Figure 4. The shapes of plots are in opposite direction to R^2, supporting our earlier findings. In addition to S&P futures, we also implement the volatility prediction applied to individual stocks in the Down 30. We get the similar pattern. However, the optimal values of q in the prediction for those stocks are smaller than 2 and mostly stick around 1, which is consistent with the findings in GS (2009). We show the results for several individual stocks of the Dow 30 components in Figure 5[19]. The patterns are obviously similar to S&P futures.

5.3 Prediction with Jump Test and Truncated Jump

In the previous section, we present a set of results which are purely based on the RMs of jump power variations which are not adjusted for the jump tests. Theoretically, the realized measures should converge to jump power variations. In finite sample with the sampling choice of 5 minutes ($n = 78$ per day), ABD (2007) develop a straightforward procedure to separate the variation of log-price process due to jumps. We follow this approach to adjust the realized measures of jumps for any day that jump does not occur. In particular day, we first test for jumps using the simple jump test procedure and set the realized measures of jumps to be 0 once the jump statistics is significant. With the new time series of $RPV_{q,t}, RJ^{+}_{q,t}$, $RJ^{-}_{q,t}$ and $RJA_{q,t}$, we then carry out the similar forecast experiments as in previous section.

The results show similar pattern as earlier findings. Figure 2 plots the in-sample R^2s for all specification and horizons while Figure 3 plot the out of sample R^2s for linear, square root at the forecast horizon $h = 1$ and $h = 5$. Across all plots, HAR-RV-C-PDUJ is slightly better for and daily and weekly horizon while HAR-RV-C-AJP is slightly better for monthly horizon. Regarding all specifications from 2 to 6 of linear square root and log models, the R^2s are higher than those of the benchmark model. This result is consistent with the earlier findings that higher order jumps help in prediction of future. In addition, similar to the above discussion, large jumps play less important role in the prediction and dies out. In comparison of out-of sample R^2s between jump test and no jump test cases, we see a marginal improvement in the jump test case. Though the increase is very small, this would suggest jump test might be helpful in the forecast experiments using jump variations.

As an additional remark, as shown in Figure 3A and 3B for S&P futures, the out-of-sample results for the no-jump-test case point out the scenarios where the optimal predictive values of q are larger than 2, as opposed to the the results found in earlier in the literature where q is around 1. Interestingly, the curves change when jump tests are implemented. For linear models at horizon $h = 1$ and $h = 5$, the optimal q is larger than 2 in Figure 3A (no jump tests) and is less than 2 in Figure 3B (with jump tests). Conversely, for the linear model, the optimal q is larger than 2 when

[19] We present the result for 4 stocks. Results for other stocks in Dow 30 are avaialbe upon request.

jump tests are implemented. This illustration therefore also suggests that the implementation of jump tests could affect the results of the prediction.

Now turn to the truncated jumps variables, as discussed in section 4.1, we truncate large jumps on the basis of percentiles of the time series of monthly largest increments, as implemented in DS (2010). For the experiments, we pick $\gamma = 5\text{th}, 10\text{th}$ and 25th percentile of the sample spanning period 1993-2009, i.e. we discard all the jumps larger than this threshold and construct the new time series $RPV_{q,t}(\gamma)$, $RJ^{+}_{q,t}(\gamma)$, $RJ^{-}_{q,t}(\gamma)$ and $RJA_{q,t}(\gamma)$. We then implement forecast using specifications as in 4.1. Interestingly, the results are almost the same as in the above discussion, implying that the larger jumps matter little in the prediction of future volatility.

In summary, our analysis demonstrates that: (i) Continuous component dominates in the predictions. (ii) There is a strong correlation between our jump power variation based jump asymmetry variable and future realized volatility and downside jumps matters more than upward jumps in the prediction. (iii) Incorporation of downside and upside jump power variations might help in prediction but to a limited extent in term of both in-sample and out-of-sample prediction. (iv) There is a strong pattern that higher order jump power variations help less in the prediction of realized volatility, regardless of model specifications that we consider.(v) We find the evidence that the optimal value of q could be larger than 2, depending on the set-up and jump implementation. (vi) Finally, the implementation of jump tests might change the results in the predictions.

6 Concluding Remarks

In this paper, we build on the recent theoretical results of Jacod (2008) and Barndorff-Nielsen and Shephard (2004, 2006) and BKS (2010) to assess large jump power variations, downside (upside) jump power variations, and asymmetry jump power variations. In particular, we look at the role of those variables in the prediction of future realized volatility. We do so by extending the class of approximate long memory model, HAR-RV. Our results are consistent with the earlier findings in the literature, such as ABD (2007) that continuous component dominates in the prediction of future realized volatility. The separation of continuous and jump components could help in increasing in-sample and out-of- sample R^2. In addition, we find a pattern of predictability in which past "large" jump power variations help less in the prediction of future realized volatility, than past "small" jump power variations. This suggests the "larger" jumps might help less in the prediction of future realized volatility than "smaller" jumps. Regarding jump asymmetry, there is an evidence that the signed jump power variation has a strong correlation with future RV. Our results also show that downside jump power variation might matter for modeling future RV. Moreover, in various experimental setups, the (forecast) best values of q are larger than 2 for S&P futures. Finally, incorporation of downside and upside jump power variations does improve predictability, albeit to a limited extent.

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Table 1: Daily, Weekly and Monthly HAR-RV-C Predictive Regression for S&P 500 futures (Benchmark)*

Linear Model				Square Root Model				Log Model			
β_0	β_d	β_w	β_m	β_0	β_d	β_w	β_m	β_0	β_d	β_w	β_m
h=1 (Daily Forecast)											
0.000	0.089	0.060	1.654	-0.002	0.067	0.117	1.001	-0.200	0.167	0.099	0.716
(-0.67)	(1.93)	(0.38)	(7.81)	(-1.569)	(2.70)	(1.70)	(11.91)	(-1.15)	(7.04)	(1.66)	(11.71)
$R^2_{in}(R^2_{out})$ = 0.35(0.244)				$R^2_{in}(R^2_{out})$ = 0.45(0.341)				$R^2_{in}(R^2_{out})$ = 0.45(0.388)			
h=5 (Weekly Forecast)											
0.000	0.058	-0.075	1.832	-0.001	0.055	0.037	1.077	-0.346	0.134	0.142	0.688
(-0.71)	(0.51)	(-0.43)	(10.31)	(-1.00)	(0.94)	(0.40)	(12.62)	(-1.84)	(5.80)	(2.40)	(10.85)
$R^2_{in}(R^2_{out})$ = 0.35(0.169)				$R^2_{in}(R^2_{out})$ = 0.44(0.244)				$R^2_{in}(R^2_{out})$ = 0.43(0.296)			
h=22 (Monthly Foreast)											
0.000	-0.034	0.387	1.375	0.000	0.011	0.137	0.978	-0.772	0.076	-0.014	0.847
(0.32)	(-0.89)	(3.47)	(11.86)	(0.18)	(0.41)	(1.92)	(15.04)	(-3.08)	(3.24)	(-0.19)	(12.39)
$R^2_{in}(R^2_{out})$ = 0.33(0.026)				$R^2_{in}(R^2_{out})$ = 0.41 (0.0357)				$R^2_{in}(R^2_{out})$ = 0.38 (0.033)			

Table 2: Summary of Model Specifications for RV Forecasting

Specification 1

$$\phi(RV_{t+h}) = \beta_0+\beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) + \epsilon_{t+h}$$

Specification 2 (HAR-RV-C-PV(q))

$$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t,t-5}) + \beta_{cm}\phi(RVC_{t,t-22}) \\ +\beta_{jd}\phi(RPV_{q,t}) + \beta_{jw}\phi(RPV_{q,t,t-5}) + \beta_{jm}\phi(RPV_{q,t,t-22}) + \epsilon_{t+h}$$

Specification 3 (HAR-RV-C-UJ(q))

$$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t,t-5}) + \beta_{cm}\phi(RVC_{t,t-22})+ \\ +\beta^+_{jd}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jw}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jm}\phi(RJ^+_{q,t-22,t}) + \epsilon_{t+h}$$

Specification 4 (HAR-RV-C-DJ(q))

$$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t,t-5}) + \beta_{cm}\phi(RVC_{t,t-22})+ \\ +\beta^-_{jd}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jw}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jm}\phi(RJ^-_{q,t-22,t}) + \epsilon_{t+h}$$

Specification 5 (HAR-RV-C-UDJ(q))

$$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ +\beta^+_{jd}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jw}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jm}\phi(RJ^+_{q,t-22,t}) \\ +\beta^-_{jd}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jw}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jm}\phi(RJ^-_{q,t-22,t}) + \epsilon_{t+h}$$

Specification 6 (HAR-RV-C-APJ(q))

$$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ +\beta_{jd}\phi(RJA_{q,t}) + \beta_{jw}\phi(RJA_{q,t-5,t}) + \beta_{jm}\phi(RJA_{q,t-22,t}) + \epsilon_{t+h}$$

* The table 1 summarizes the estimation of HAR-RV-C model at daily (h=1), weekly (h=5) and montly (h=22) horizon. For each horizon, the first row entries are the parameter estimates, the second row entries in bracket are t-statistics. The third row reports R_I and R_O, the in and out-of- sample R-square of the predictive regressions, respectively.

Table 3A: HAR-RV-C-PV(q) Predictive Regression (q=2.5 and 5) for S&P 500 Futures *

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
β_0	q=2.5	0.001	0	0.001	0.006	0.005	0.008	-0.519	-0.702	-1.228
		-3.035	-1.737	-2.107	-3.444	-2.885	-3.286	(-1.902)	(-2.369)	(-3.290)
	q=5	0.001	0	0.001	0.003	0.004	0.006	-0.38	-0.55	-1.024
		-3.171	-2.345	-2.468	-2.486	-2.455	-3.136	(-1.620)	(-2.173)	(-3.203)
β_{cd}	q=2.5	0.066	0.004	-0.072	0.02	0.024	0.031	0.174	0.136	0.075
		-1.448	-0.031	(-1.090)	-0.599	(0.347	-0.674	-6.808	-5.691	-2.978
	q=5	0.073	0.019	-0.078	0.056	0.056	0.029	0.169	0.133	0.071
		-1.771	-0.17	(-1.319)	-1.786	(0.961	-0.71	-6.893	-5.73	-2.944
β_{cw}	q=2.5	-0.122	-0.251	0.421	0.015	-0.116	0.111	0.087	0.128	-0.017
		(-0.812)	(-1.419)	-2.79	-0.177	(-1.058)	-1.247	-1.346	-2.102	(-0.217)
	q=5	-0.067	-0.197	0.429	0.061	-0.065	0.088	0.098	0.137	-0.012
		(-0.443)	(-1.151)	-2.944	-0.777	(-0.616)	-1.015	-1.594	-2.289	(-0.164)
β_{cm}	q=2.5	0.701	1.23	0.686	0.52	0.695	0.37	0.681	0.653	0.793
		-2.472	-4.925	-2.215	-3.628	-4.685	-1.933	-10.055	-9.149	-10.088
	q=5	1.25	1.569	1.029	0.854	0.973	0.784	0.691	0.666	0.818
		-6.84	-10.869	-5.276	-9.239	-10.025	-7.699	-10.764	-9.918	-11.084
β_{jd}	q=2.5	0.072	0.214	0.165	0.108	0.067	-0.042	-16.351	-9.511	-2.187
		-0.434	1.317)	-0.7	-1.942	-0.879	(-0.426)	(-1.441)	(-0.787)	(-0.143)
	q=5	18.272	56.886	71.9	0.625	-0.05	-0.84	-2597	-1047	3180
		-0.351	1.171)	-1.002	-0.563	(-0.035)	(-0.451)	(-0.877)	(-0.302)	-0.889
β_{jw}	q=2.5	0.788	0.793	-0.186	0.321	0.471	0.08	27.949	30.637	10.902
		-1.628	1.775)	(-0.543)	-1.916	-2.573	-0.544	-0.872	-0.97	-0.362
	q=5	194.637	195.392	-76.421	3.984	7.134	3.415	2032	6163	127
		-1.158	-1.469	(-0.808)	-1.064	-1.884	-1.223	-0.215	-0.68	-0.016
β_{jm}	q=2.5	1.18	0.46	1.236	0.387	0.195	0.746	32.416	26.803	51.586
		-2.238	-0.893	-1.976	-1.969	-0.793	-2.742	-0.984	-0.726	-1.389
	q=5	114.426	2.21	211.491	0.879	-1.852	2.98	10776	6132	10073
		-0.714	-0.017	-1.346	-0.245	(-0.454)	-0.987	-1.066	-0.59	-1.056
R^2_{in}	q=2.5	0.376	0.372	0.333	0.463	0.452	0.418	0.452	0.434	0.383
	q=5	0.368	0.368	0.333	0.455	0.446	0.414	0.451	0.434	0.383
R^2_{out}	q=2.5	0.315	0.199	0.033	0.368	0.262	0.04	0.39	0.297	0.032
	q=5	0.244	0.167	0.027	0.344	0.24	0.037	0.389	0.296	0.032

* The table 3A summarizes the regression parameter estimates for HAR-RV-C model at daily (h=1), weekly (h=5) and montly (h=22) horizon. For each parameters corresponding to q=2.5 or q=5, the first row entries are the parameter estimates. The entries in bracket in the second row are t-statistics. The four rows at the bottom report R_{in} and R_{out}, the in-sample and out-of- sample R-squares of the predictive regressions for the case q=2.5 and q=5, respectively.

Table 3B: HAR-RV-C-PDJ(q) Predictive Regression (q=2.5 and 5) for S&P 500 Futures *

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
β_0	q=2.5	0.001	-0.702	-1.228	0.006	0.005	0.008	-0.525	-0.710	-1.236
		(3.024)	(-2.369)	(-3.290)	(3.462)	(2.886)	(3.281)	(-1.917)	(-2.385)	(-3.300)
	q=5	0.001	-0.550	-1.024	0.004	2.489	0.006	-0.384	-0.554	-1.027
		(3.175)	(-2.173)	(-3.203)	(2.533)	(0.004)	(3.145)	(-1.632)	(-2.184)	(-3.203)
β_{cd}	q=2.5	0.069	0.136	0.075	0.023	0.025	0.030	0.174	0.136	0.074
		(1.465)	(5.691)	(2.978)	(0.675)	(0.359)	(0.653)	(6.812)	(5.653)	(2.955)
	q=5	0.078	0.133	0.071	0.058	0.968	0.027	0.169	0.133	0.071
		(1.779)	(5.730)	(2.944)	(1.813)	(0.058)	(0.673)	(6.894)	(5.715)	(2.927)
β_{cw}	q=2.5	-0.122	0.128	-0.017	0.018	-0.113	0.109	0.087	0.130	-0.016
		(-0.800)	(2.102)	(-0.217)	(0.222)	(-1.037)	(1.226)	(1.345)	(2.125)	(-0.203)
	q=5	-0.070	0.137	-0.012	0.063	-0.633	0.085	0.099	0.136	-0.012
		(-0.456)	(2.289)	(-0.164)	(0.804)	(-0.061)	(0.976)	(1.601)	(2.286)	(-0.156)
β_{cm}	q=2.5	0.656	0.653	0.793	0.503	0.682	0.371	0.679	0.651	0.792
		(2.211)	(9.149)	(10.088)	(3.442)	(4.541)	(1.966)	(10.050)	(9.121)	(10.043)
	q=5	1.228	0.666	0.818	0.845	10.021	0.788	0.690	0.666	0.817
		(6.486)	(9.918)	(11.084)	(8.960)	(0.982)	(7.668)	(10.738)	(9.897)	(11.055)
β_{jd}	q=2.5	0.115	-9.511	-2.187	0.143	0.091	-0.057	-33.777	-16.717	-2.734
		(0.347)	(-0.787)	(-0.143)	(1.798)	(0.859)	(-0.411)	(-1.493)	(-0.731)	(-0.091)
	q=5	21.144	-1047	3180	0.720	-0.083	-1.051	-5414	-2223	7005
		(0.204)	(-0.302)	(0.889)	(0.449)	(-0.424)	(-0.411)	(-0.915)	(-0.341)	(0.994)
β_{jw}	q=2.5	1.548	30.637	10.902	0.441	0.654	0.126	56.900	57.260	17.944
		(1.638)	(0.970)	(0.362)	(1.917)	(2.516)	(0.623)	(0.893)	(0.901)	(0.304)
	q=5	390	6163	127	5.415	1.885	5.192	3453	12940	-688
		(1.184)	(0.680)	(0.016)	(1.058)	(17.888)	(1.385)	(0.182)	(0.710)	(-0.043)
β_{jm}	q=2.5	2.528	26.803	51.586	0.582	0.300	1.038	65.742	56.053	106.019
		(2.366)	(0.726)	(1.389)	(2.104)	(0.863)	(2.748)	(0.999)	(0.755)	(1.416)
	q=5	263	6132	10073	1.719	-0.446	3.727	22752	12109	20505
		(0.817)	(0.590)	(1.056)	(0.348)	(-5.091)	(0.858)	(1.104)	(0.576)	(1.051)
R^2_{in}	q=2.5	0.376	0.372	0.333	0.463	0.452	0.418	0.452	0.434	0.383
	q=5	0.365	0.368	0.333	0.455	0.446	0.414	0.451	0.434	0.383
R^2_{out}	q=2.5	0.318	0.201	0.033	0.364	0.260	0.039	0.390	0.297	0.032
	q=5	0.244	0.167	0.027	0.345	0.241	0.037	0.390	0.296	0.032

* See notes in Table 3A.

Table 3C: HAR-RV-C-PDUJ(q) Predictive Regression (q=2.5 and 5) for S&P 500 Futures*

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
β_0	q=2.5	0.001	0.000	0.001	0.006	0.006	0.008	-0.569	-0.748	-1.279
		(2.950)	(1.648)	(2.055)	(3.550)	(2.901)	(3.284)	(-2.038)	(-2.468)	(-3.357)
	q=5	0.001	0.000	0.001	0.004	0.004	0.006	-0.415	-0.582	-1.026
		(3.221)	(2.326)	(2.607)	(2.727)	(2.561)	(3.128)	(-1.746)	(-2.262)	(-3.156)
β_{cd}	q=2.5	0.067	0.004	-0.072	0.019	0.023	0.031	0.173	0.136	0.074
		(1.477)	(0.030)	(-1.135)	(0.583)	(0.335)	(0.671)	(6.758)	(5.646)	(2.952)
	q=5	0.072	0.020	-0.074	0.056	0.054	0.028	0.168	0.133	0.071
		(1.845)	(0.182)	(-1.300)	(1.778)	(0.940)	(0.712)	(6.859)	(5.733)	(2.958)
β_{cw}	q=2.5	-0.102	-0.239	0.407	0.018	-0.110	0.113	0.094	0.133	-0.011
		(-0.633)	(-1.439)	(2.755)	(0.217)	(-1.034)	(1.291)	(1.462)	(2.172)	(-0.141)
	q=5	-0.040	-0.185	0.407	0.067	-0.056	0.084	0.104	0.140	-0.013
		(-0.235)	(-1.132)	(2.757)	(0.834)	(-0.557)	(0.980)	(1.673)	(2.333)	(-0.174)
β_{cm}	q=2.5	0.578	1.153	0.713	0.509	0.680	0.364	0.668	0.643	0.782
		(1.713)	(3.750)	(2.468)	(3.549)	(4.638)	(1.943)	(9.919)	(8.889)	(9.762)
	q=5	1.160	1.504	1.062	0.831	0.951	0.793	0.682	0.659	0.818
		(5.209)	(8.904)	(6.022)	(8.645)	(10.120)	(8.485)	(10.528)	(9.598)	(10.980)
β_{jd}^-	q=2.5	-1.116	0.419	1.836	-0.303	-0.114	0.082	-144	89	75
		(-0.659)	(0.395)	(2.138)	(-1.002)	(-0.317)	(0.263)	(-1.037)	(0.784)	(0.638)
	q=5	-526	8	532	-5.807	-4.136	3.523	-16818	-7675	39133
		(-0.785)	(0.026)	(1.961)	(-0.831)	(-0.700)	(0.667)	(-0.426)	(-0.312)	(1.644)
β_{jw}^-	q=2.5	0.523	-0.579	0.404	0.002	0.588	0.608	214	-130	-173
		(0.111)	(-0.241)	(0.082)	(0.002)	(0.756)	(0.699)	(0.449)	(-0.340)	(-0.412)
	q=5	400	245	328	-2.498	16.943	11.885	-22222	58161	-39700
		(0.292)	(0.347)	(0.215)	(-0.144)	(1.479)	(0.954)	(-0.199)	(0.747)	(-0.433)
β_{jm}^-	q=2.5	14.004	9.028	-3.694	2.772	1.644	0.097	946	989	1155
		(1.682)	(0.798)	(-0.495)	(1.435)	(0.742)	(0.048)	(1.217)	(1.063)	(1.262)
	q=5	3597	2303	-1813	47.313	18.591	-22.755	277860	154197	28836
		(1.692)	(0.825)	(-0.916)	(1.689)	(0.572)	(-0.717)	(1.530)	(0.735)	(0.122)

Table 3C: HAR-RV-C-PDUJ(q) Predictive Regression (q=2.5 and 5) for S&P 500 Futures (Continued)*

β^+_{jd}	q=2.5	1.272	0.001	-1.536	0.460	0.212	-0.141	113.345	-109.981	-81.229
		(0.755)	(0.001)	(-1.547)	(1.595)	(0.574)	(-0.481)	(0.817)	(-0.884)	(-0.690)
	q=5	567.860	103.841	-397.293	6.768	4.195	-4.735	11699	5426	-33271
		(0.833)	(0.331)	(-1.310)	(1.018)	(0.658)	(-0.897)	(0.293)	(0.190)	(-1.373)
β^+_{jw}	q=2.5	0.893	2.094	-0.688	0.445	0.062	-0.498	-172.658	185.572	187.449
		(0.184)	(0.792)	(-0.131)	(0.373)	(0.089)	(-0.551)	(-0.362)	(0.493)	(0.434)
	q=5	-79.637	108.397	-441.164	7.847	-7.332	-6.750	22772	-48575	40677
		(-0.056)	(0.143)	(-0.274)	(0.440)	(-0.785)	(-0.507)	(0.208)	(-0.656)	(0.428)
β^+_{jm}	q=2.5	-11.313	-7.911	6.100	-2.226	-1.355	0.968	-875	-934	-1049
		(-1.438)	(-0.726)	(0.784)	(-1.167)	(-0.623)	(0.462)	(-1.131)	(-1.013)	(-1.163)
	q=5	-3255.071	-2218.650	2196.218	-45.618	-20.656	26.622	-252245	-138328	-8874
		(-1.634)	(-0.795)	(1.066)	(-1.648)	(-0.628)	(0.813)	(-1.469)	(-0.678)	(-0.038)
R^2_{in}	q=2.5	0.378	0.373	0.335	0.464	0.452	0.418	0.480	0.434	0.384
	q=5	0.372	0.369	0.335	(0.456	0.447	.0415	0.452	0.434	0.383
R^2_{out}	q=2.5	0.342	0.212	0.03	0.373	0.263	0.038	0.391	0.298	0.033
	q=5	0.249	0.169	0.027	(0.352	0.246	0.035	0.390	0.297	0.032

*See notes in Table 3A.

Table 3D: HAR-RV-C-APJ(q) Predictive Regression (q=2.5 and 5) for S&P 500 Futures*

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
β_{cd}	q=2.5	0.001	0.000	0.001	0.006	0.005	0.008	-0.519	-0.702	-1.228
		(3.035)	(1.737)	(2.107)	(3.449)	(2.885)	(3.285)	(-1.902)	(-2.369)	(-3.290)
	q=5	0.001	0.000	0.001	0.003	0.004	0.006	-0.380	-0.550	-1.024
		(3.171)	(2.345)	(2.468)	(2.484)	(2.451)	(3.135)	(-1.620)	(-2.173)	(-3.203)
β_{cw}	q=2.5	0.066	0.004	-0.072	0.019	0.023	0.032	0.174	0.136	0.075
		(1.448)	(0.031)	(-1.090)	(0.575)	(0.338)	(0.680)	(6.807)	(5.688)	(2.977)
	q=5	0.073	0.019	-0.078	0.056	0.055	0.028	0.169	0.133	0.071
		(1.771)	(0.170)	(-1.319)	(1.780)	(0.947)	(0.710)	(6.893)	(5.730)	(2.944)
β_{cm}	q=2.5	-0.122	-0.251	0.421	0.013	-0.115	0.111	0.087	0.129	-0.017
		(-0.812)	(-1.419)	(2.790)	(0.163)	(-1.053)	(1.242)	(1.348)	(2.104)	(-0.216)
	q=5	-0.067	-0.197	0.429	0.058	-0.063	0.088	0.098	0.137	-0.012
		(-0.443)	(-1.151)	(2.944)	(0.741)	(-0.602)	(1.005)	(1.594)	(2.289)	(-0.164)
β_{jd}	q=2.5	0.701	1.230	0.686	0.522	0.695	0.370	0.681	0.653	0.793
		(2.472)	(4.925)	(2.215)	(3.649)	(4.681)	(1.934)	(10.058)	(9.152)	(10.092)
	q=5	1.250	1.569	1.029	0.858	0.973	0.784	0.691	0.666	0.818
		(6.840)	(10.869)	(5.276)	(9.293)	(9.994)	(7.693)	(10.764)	(9.918)	(11.084)
β_{jw}	q=2.5	0.072	0.214	0.165	0.078	0.048	-0.030	-16.138	-9.318	-2.037
		(0.434)	(1.317)	(0.700)	(1.970)	(0.898)	(-0.431)	(-1.431)	(-0.777)	(-0.134)
	q=5	18.273	56.886	71.902	0.450	-0.002	-0.591	-2596.867	-1046.806	3180.150
		(0.352)	(1.171)	(1.002)	(0.571)	(-0.002)	(-0.449)	(-0.876)	(-0.302)	(0.889)
	q=2.5	0.788	0.793	-0.186	0.229	0.330	0.057	27.695	30.419	10.641
		(1.628)	(1.775)	(-0.543)	(1.930)	(2.588)	(0.549)	(0.867)	(0.967)	(0.355)
	q=5	194.636	195.391	-76.421	2.935	4.928	2.420	2031.944	6162.489	127.111
		(1.158)	(1.469)	(-0.808)	(1.108)	(1.897)	(1.230)	(0.215)	(0.680)	(0.016)
β_{jm}	q=2.5	1.180	0.460	1.236	0.271	0.139	0.528	32.365	26.705	51.504
		(2.238)	(0.893)	(1.976)	(1.947)	(0.805)	(2.740)	(0.985)	(0.726)	(1.391)
	q=5	114.426	2.211	211.490	0.499	-1.222	2.100	10776.175	6131.779	10073.197
		(0.714)	(0.017)	(1.346)	(0.197)	(-0.435)	(0.991)	(1.066)	(0.590)	(1.056)
R^2_{in}	q=2.5	0.376	0.372	0.335	0.463	0.451	0.418	0.452	0.434	0.384
	q=5	0.368	0.368	0.335	0.455	0.445	0.415	0.451	0.434	0.383
R^2_{out}	q=2.5	0.315	0.199	0.033	0.368	0.262	0.040	0.39	0.297	0.032
	q=5	0.244	0.167	0.027	0.343	0.241	0.037	0.389	0.296	0.032

*See notes in Table 3A.

Table 4: Diebold - Mariano Predictive Tests for Jump Variations for S&P 500 futures*

Panel A: Recursive Scheme

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
	DM Stat	5.30	2.75	-3.04	3.42	2.60	3.25	2.08	2.84	2.29
HAR-C-PV	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.20	2.50	2.50
	$.q_s$	4.40	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
	DM Stat	4.05	1.61	-2.40	3.20	2.90	3.41	2.51	3.20	2.64
HAR-C-PDJ	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.20	2.50	2.50
	$.q_s$	4.30	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
	DM Stat	5.80	2.36	-1.69	3.20	2.05	3.30	2.20	1.49	1.31
HAR-C-PUJ	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.10	2.50	2.50
	$.q_s$	4.40	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
	DM Stat	6.16	2.89	-1.75	3.51	2.31	3.16	2.19	2.28	1.63
HAR-C-PDUJ	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.10	2.50	2.50
	$.q_s$	4.40	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00

Panel B: Rolling Scheme

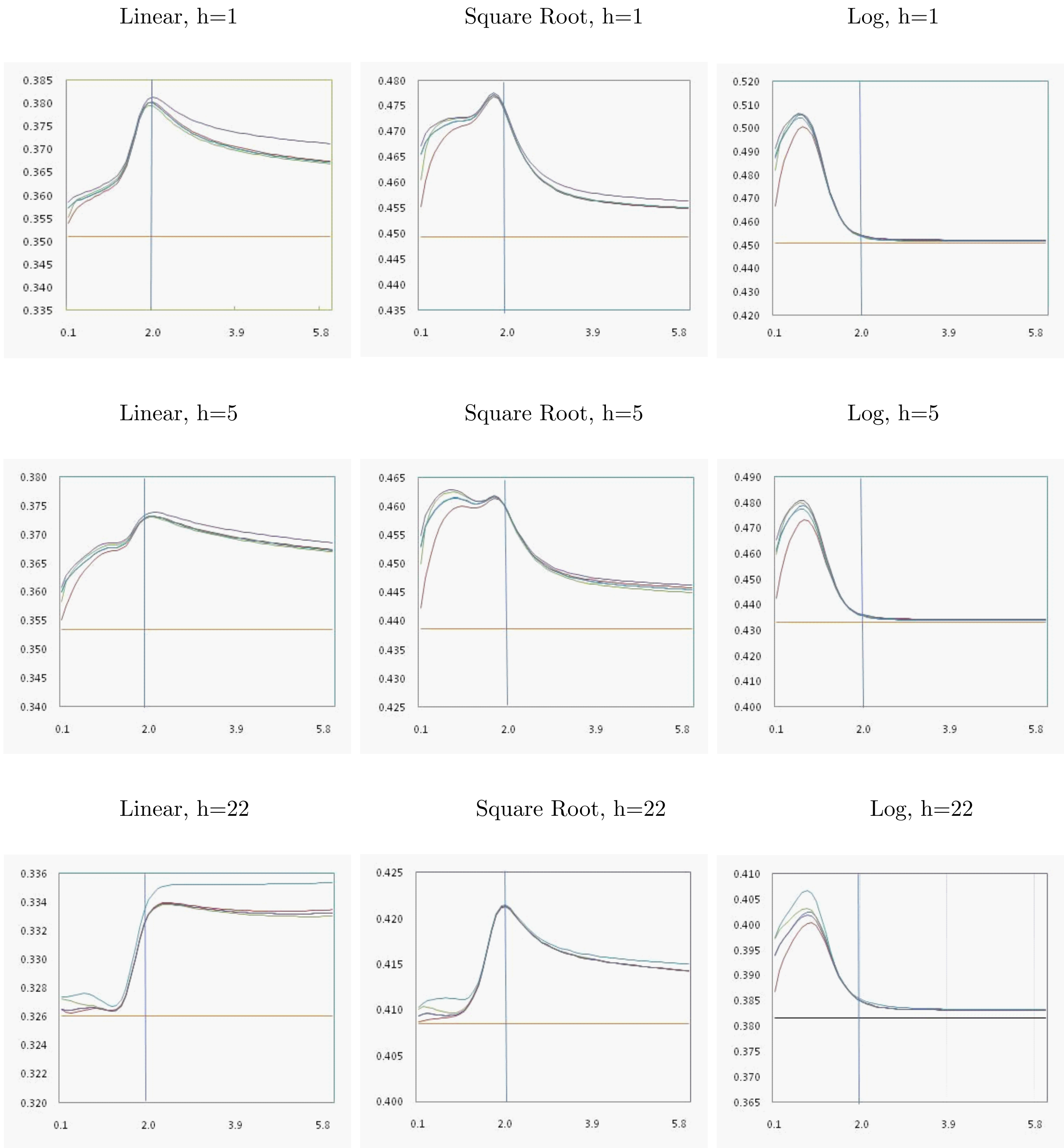
	DM Stat	6.17	3.19	-3.41	3.29	2.55	3.19	0.99	2.87	2.49
HAR-C-PV	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.40	2.50	2.50
	$.q_s$	4.20	4.70	5.40	6.00	6.00	6.00	6.00	6.00	6.00
	DM Stat	4.28	2.15	-2.67	3.09	2.84	3.32	-3.18	3.30	2.83
HAR-C-PDJ	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.40	2.50	2.50
	$.q_s$	4.20	4.70	5.40	6.00	6.00	6.00	2.50	6.00	6.00
	DM Stat	6.56	2.86	-1.89	3.04	1.99	3.20	0.92	1.46	1.53
HAR-C-PUJ	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.30	2.50	2.50
	$.q_s$	4.20	4.70	5.40	6.00	6.00	6.00	6.00	6.00	6.00
	DM Stat	7.13	3.40	-1.75	3.35	2.26	3.11	2.11	2.24	1.88
HAR-C-PDUJ	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.10	2.50	2.50
	$.q_s$	4.20	4.70	5.30	6.00	6.00	6.00	6.00	6.00	6.00

Panel C: Fixed Scheme

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
HAR-C-PV	DM Stat	6.17	3.11	-3.23	4.27	3.15	3.41	-4.82	3.32	2.35
	q_b	2.5	2.5	2.5	2.5	2.5	2.5	3.5	2.5	2.5
	.q_s	4.3	4.8	5.8	6.0	6.0	6.0	2.5	6.0	6.0
HAR-C-PDJ	DM Stat	4.23	1.98	-2.46	3.67	3.25	3.52	-3.75	3.46	2.66
	q_b	2.5	2.5	2.5	2.5	2.5	2.5	3.5	3.5	3.5
	.q_s	4.3	4.3	4.3	6.0	6.0	6.0	6.0	6.0	6.0
HAR-C-PUJ	DM Stat	6.82	2.76	-1.84	3.86	2.43	3.37	0.21	1.80	1.23
	q_b	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	.q_s	4.3	5.8	5.8	6.0	6.0	6.0	6.0	6.0	6.0
HAR-C-PDUJ	DM Stat	7.13	3.05	-1.80	4.37	2.87	3.34	1.73	3.19	1.85
	q_b	2.5	2.5	2.5	2.5	2.5	2.5	3.2	2.5	2.5
	.q_s	4.3	4.9	3.4	6.0	6.0	6.0	6.0	6.0	6.0

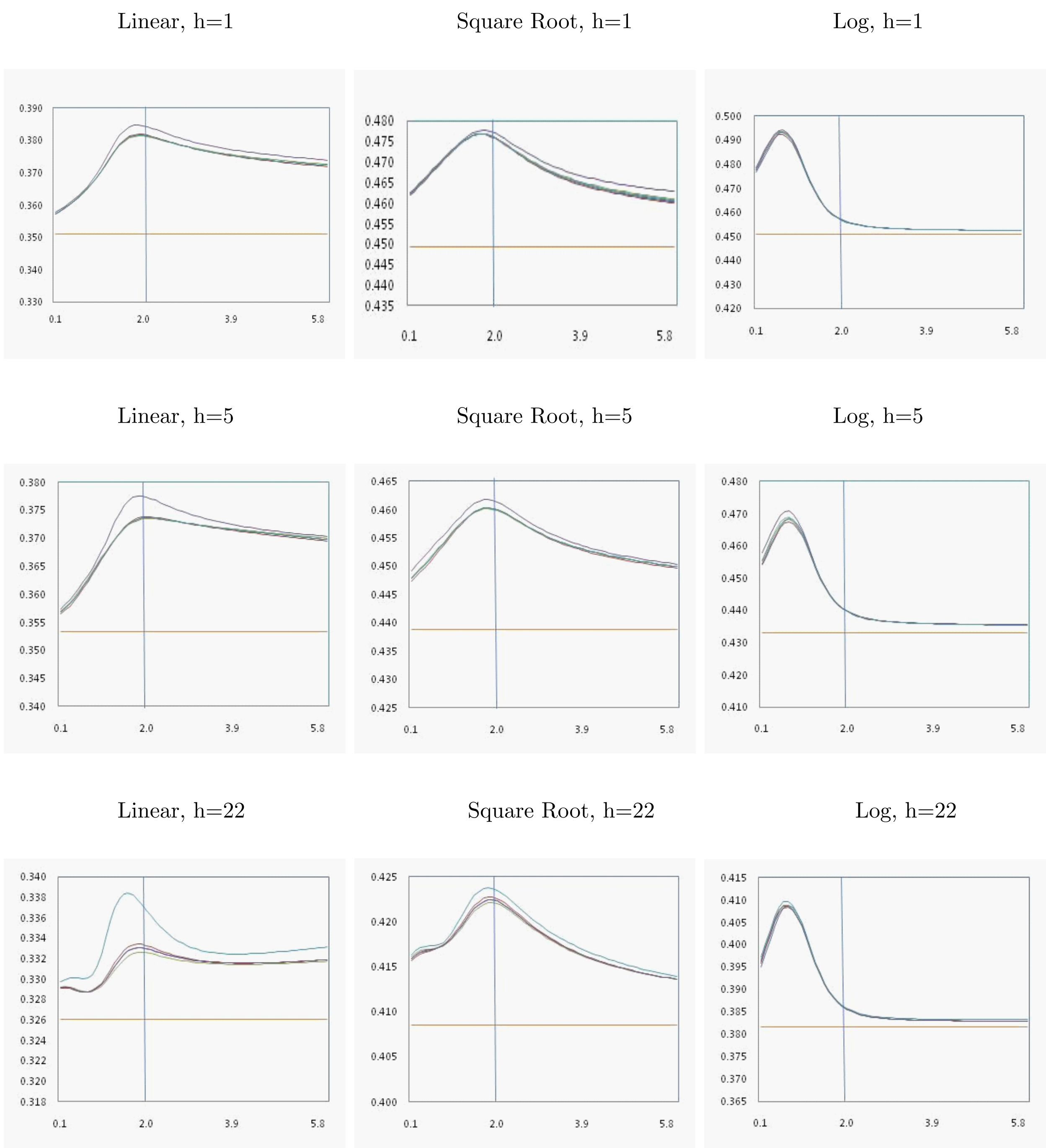
* The table reports Diebold-Mariano (1995) test statistics, calculated using Hac estimators with auto-correlated lags up to 44 as discussed in section 4.2, for linear, square root and log specifications HAR- C-PV, HAR-C-PDJ, HAR-C-DUJ, HAR-C-AJP at forecast horizon h=1,5,22 respectively. For each specification, the entries in the first rorws, DM Stat are statistics. The entries in the second row, $q_b(2.5 <= q_b <= 6)$ is the value of q that yields the highest R-square and $q_s(2.5 <= q_b <= 6)$ is the value of q that yields the smallest R-square.

Figure 1: In-sample R^2 for S&P 500 futures, No Jump Test*



* The figure depicts 9 plots in-sample R-square of 6 specifications summarized in Table 2 for all linear, square root and log models across forecast horizon h=1, 5 and 22. For each plot, the vertical axis represents R-square, with range from 0 to 1. The horizontal represents the order q with range from 0 to 6 , i.e, q=0.1,0.2,...,5.9,6. The orange line plots R-square of HAR-RV-C. The purple curve plots R-square of HAR-RV-C-PDUJ model. The dark purple represents HAR-RV-C-PUJ and the light green represents HAR-RV-C-PDJ and dark blue is for HAR-RV-C-PV. The light blue plots R-square of HAR-RV-C-AJP.

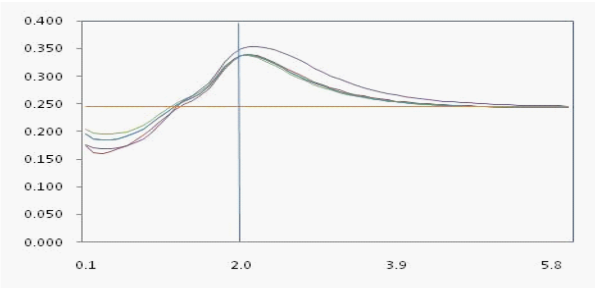
Figure 2: In-sample R^2 for S&P 500 futures, with Jump Test*



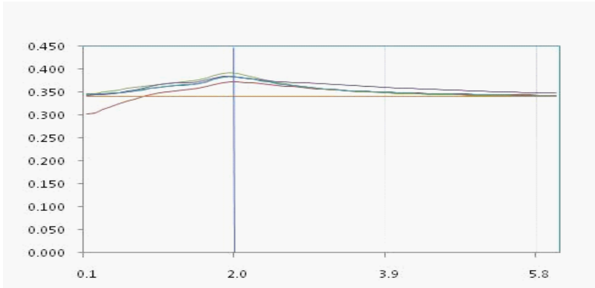
* The Figure depicts 9 plots in-sample Rš where jump tests are taken into account. See footnote in Figure 1.

Figure 3A: Out of sample R^2 for S&P 500 futures, No Jump Test*

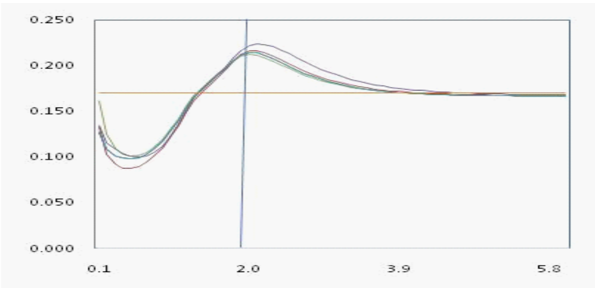
Linear, h=1



Square Root, h=1



Linear, h=5



Square Root, h=5

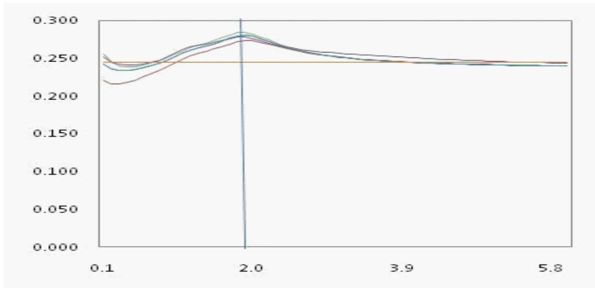
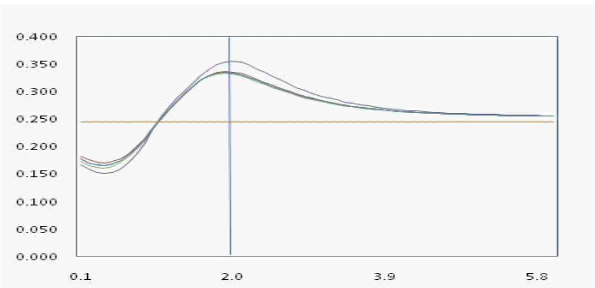
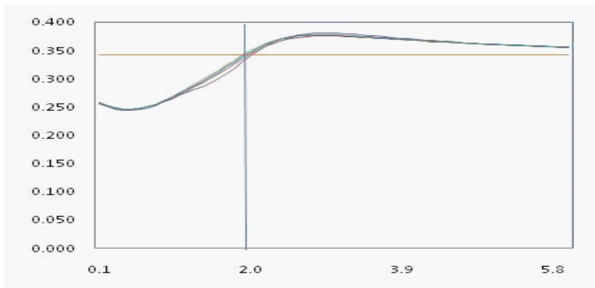


Figure 3B: Out of sample R^2 for S&P 500 Futures, with Jump Test*

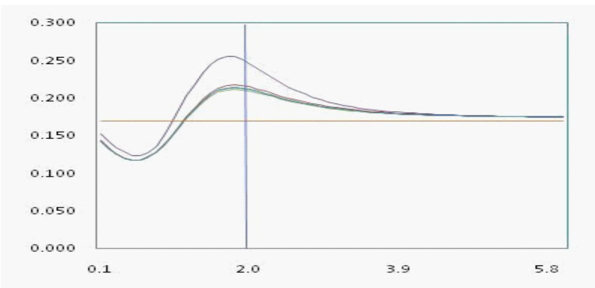
Linear, h=1



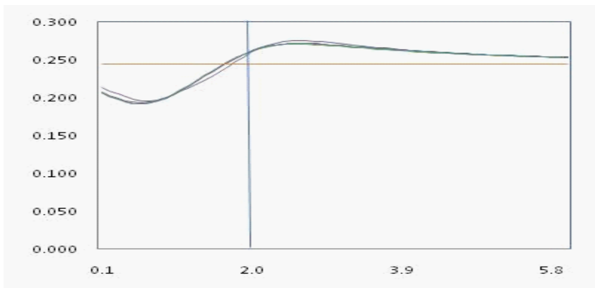
Square Root, h=1



Linear, h=5



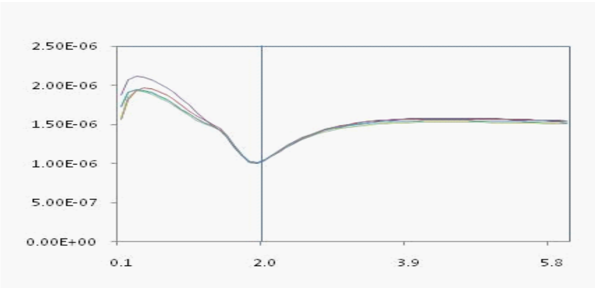
Square Root, h=5



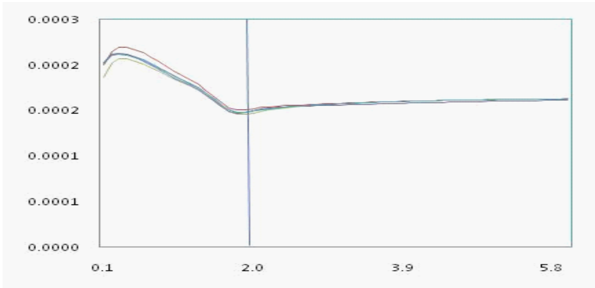
* The Figure 3A depicts 4 plots in-sample Rš of 6 specifications summarized in Table 2 for linear, square root across forecast horizon h=1, 5 See footnote in Figure 1 for further details. Figure 3B takes jump tests into account.

Figure 4A: Mean Square Errors for S&P 500 futures, No Jump Test*

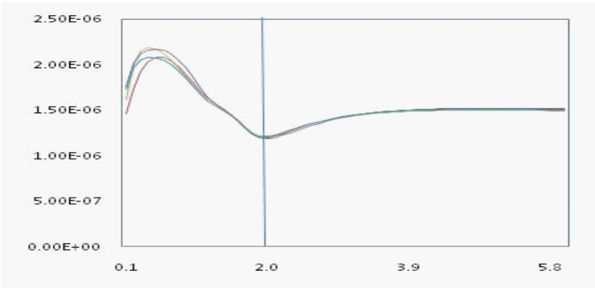
Linear, h=1



Square Root, h=1



Linear, h=5



Square Root, h=5

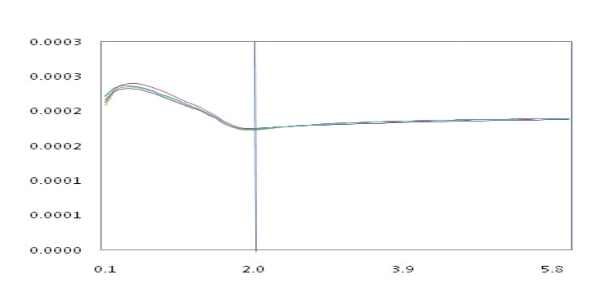
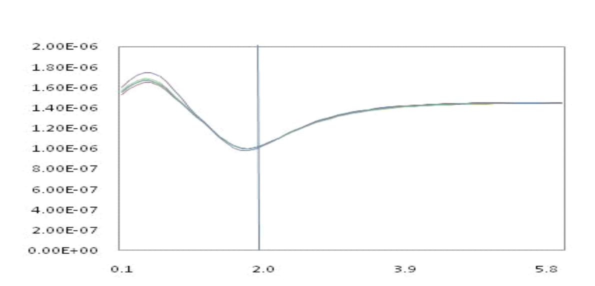
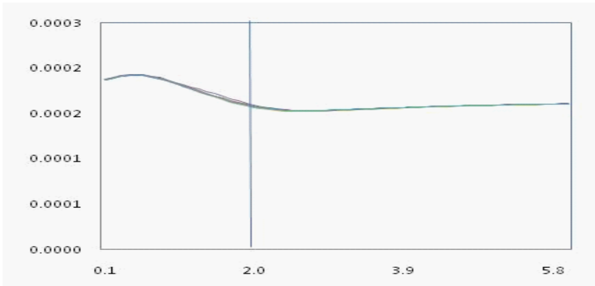


Figure 4B:Mean Square Errors for S&P 500 futures, Jump Test*

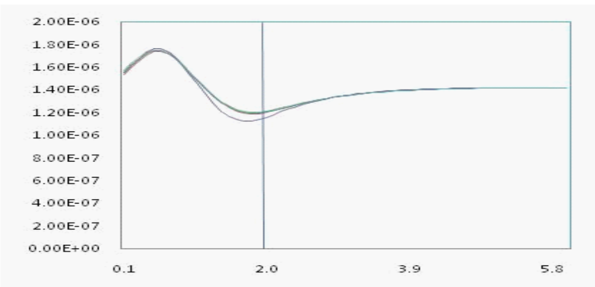
Linear, h=1



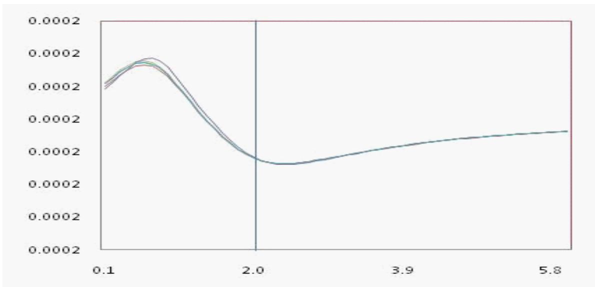
Square Root, h=1



Linear, h=5



Square Root, h=5

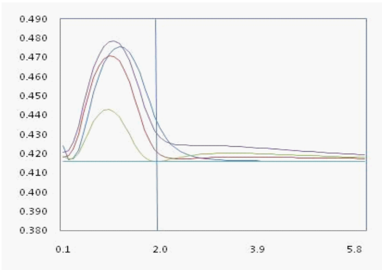


* The figure 4A depicts 4 plots mean square errors of specifications from 2 to 6 summarized in Table 2 for linear, square root across forecast horizon h=1, 5. Figure 4B takes jump tests into account. See footnote in Figure 1 for further details.

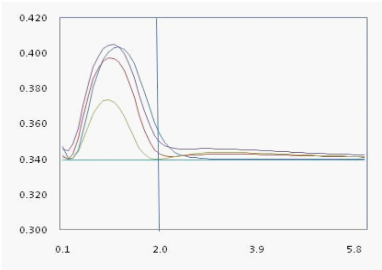
Figure 5: R^2 for Dow 30 components for Square Root Models, No Jump Test*

Panel A: Intel

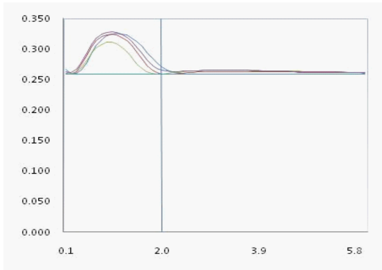
Square Root, h=1



Square Root, h=5

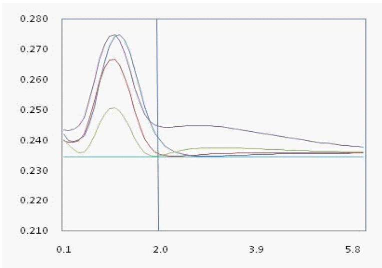


Square Root, h=22

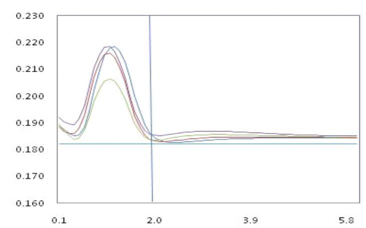


Panel B: City

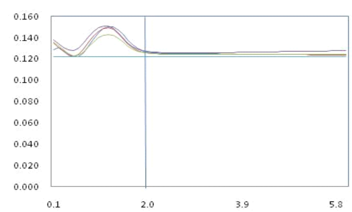
Square Root, h=1



Square Root, h=5

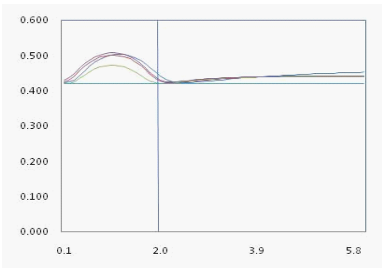


Square Root, h=22

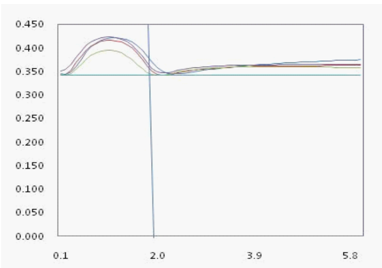


Panel C: Microsoft

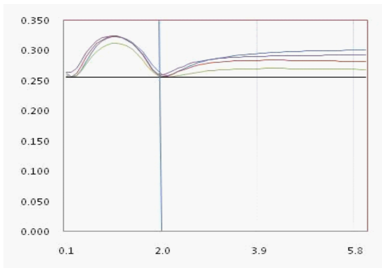
Square Root, h=1



Square Root, h=5

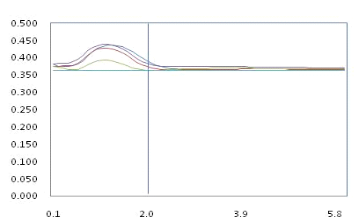


Square Root, h=22

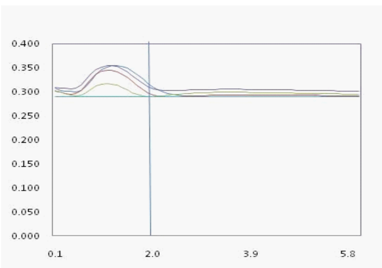


Panel D: Home Depot

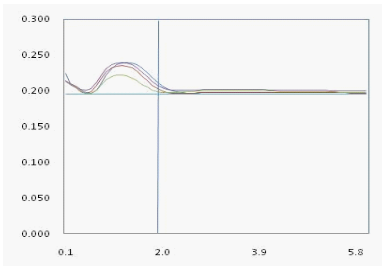
Square Root, h=1



Square Root, h=5



Square Root, h=22



* The Panel A,B,C,D in Figure 5 depict the in-sample Rš for the 4 representative stocks in Down 30 components. The four stocks are Intel, Citi, MSFT and Home Depot, respectively and the models are square root at daily, weekly and monthly forecast horizon. See footnotes in Figure 1A for further details about the plot.