Testing for Smooth Structural Changes in Time Series Models via Nonparametric Regression

Bin Chen
University of Rochester

Yongmiao Hong
Cornell University & Xiamen University

Abstract: Checking parameter stability of econometric models is a long standing problem. Almost all existing structural change tests in econometrics are designed to detect abrupt breaks. Little attention has been paid to smooth structural changes, which may be more realistic in economics. We propose a consistent test for smooth structural changes as well as abrupt structural breaks with known or unknown change points. The idea is to estimate smooth time-varying parameters by local smoothing and compare the fitted values of the restricted constant parameter model and the unrestricted time-varying parameter model. The test is asymptotically pivotal and does not require prior information about the alternatives. A simulation study highlights the merits of the proposed test relative to a variety of popular tests for structural changes. In an application, we strongly reject the stability of univariate and multivariate stock return prediction models in the post war and post oil-shocks periods.

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Key words: Kernel, Model stability, Nonparametric regression, Parameter constancy, Smooth structural change

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1. INTRODUCTION

Detection of structural changes in economic relationships is a long standing problem in econometrics. However, most existing tests are designed for abrupt structural breaks. As Hansen (2001) points out, "it may seem unlikely that a structural break could be immediate and might seem more reasonable to allow a structural change to take a period of time to take effect". Indeed, technological progress, preference change and policy switch are some leading driving forces of structural changes that usually exhibit evolutionary changes in the long term.

During the past two decades, time-varying time series models have appeared as a novel tool to capture the evolutionary behavior of economic time series. A leading example is the Smooth Transition Regression (STR) model developed by Lin and Teräsvirta (1994). By the use of a transition function, the STR model allows both the intercept and slope to change smoothly over time. Parametric models for time-varying parameters lead to more efficient estimation if the coefficient functions are correctly specified. However, economic theories usually do not suggest any concrete functional form for time-varying parameters; the choice of a functional form is somewhat arbitrary. A nonparametric time-varying parameter time series model is introduced by Robinson (1989, 1991) and further studied by Orbe, Ferreira and Rodriguez-Poo (2000, 2005) and Cai (2007). One advantage of this nonparametric model is that little restriction is imposed on the functional forms of the time-varying intercept and slope, except for the condition that they evolve over time smoothly. Motivated by its flexibility, we will use this model as the alternative to test smooth structural changes for a linear regression model.

To our knowledge, there are only two tests designed explicitly for smooth structural changes in the literature. Farley, Hinich and McGuire (1975) construct an F test by comparing a linear time series model with a parametric alternative whose slope is a linear function of time. Lin and Teräsvirta (1994) develop LM type tests against a STR alternative. An undesired feature of these tests is that they use a specific parametric time-varying parameter model. While these tests have best power against the assumed alternative, no prior information about the true alternative is usually available for practitioners. In such scenarios, it is highly desirable to develop consistent tests that have good power against all-round alternatives of structural changes.

This paper proposes a new consistent Wald-type test for smooth structural changes as well as abrupt structural breaks. The test complements the existing tests for abrupt structural breaks and avoids the difficulty associated with whether there are multiple breaks and/or whether breakpoints are unknown. We estimate smooth-changing parameters by local linear regression and compare them with the OLS parameter estimator. The proposed Wald-type test can be viewed as a generalization of Hausman's (1978) test from the parametric framework to the nonparametric framework. A generalized Chow's (1960) F-type test could also be constructed by comparing the sums of squared residuals (SSRs) between the restricted constant parameter model and the unrestricted time-varying parameter model (see Chen and Hong (2008) for details). Interestingly, unlike Chow's (1960) test, which is optimal in the context of the classical linear regression model with i.i.d. normal errors, the generalized Chow test is no longer optimal. We show that the generalized Hausman test is asymptotically more powerful than the generalized Chow test. For this reason, this paper focuses on the generalized Hausman test.

Compared with the existing tests for structural breaks in the literature, the proposed test has a number of appealing features. First, it is consistent against a large class of smooth time-varying parameter alternatives as well as multiple sudden structural breaks with unknown break points. Second, no prior information on a structural change alternative is needed. In particular, we do not need to know whether the structural changes are smooth or abrupt, and in the cases of abrupt structural breaks, we do not need to know the dates or the number of breaks. Third, different from many tests for structural breaks in the literature, the proposed test is asymptotically pivotal. The only inputs required are the OLS and local linear time-varying parameter estimators. The latter is in fact a locally weighted least squares estimator. Hence, any standard econometric software can be used to implement the test. Fourth, because only local information is employed in estimating parameters at each time point, the proposed test has symmetric power against structural breaks that occur either in the first or second half of the sample period. In contrast, some existing tests (e.g., Brown, Durbin and Evans' (1975) CUSUM test) have different powers against structural breaks that have same sizes but occur at different time points. Fifth, unlike such tests as Andrews' (1993) supremum test and Bai and Perron's (1998) double maximum test, no trimming of the boundary region near the end points of the sample period is needed for our test. Moreover, as a by-product, the nonparametric local linear estimators of the time-varying parameters can provide insight into the stability of the economic relationship.

In Section 2, we introduce the framework and state the hypotheses of interest. Section 3 describes our approach and the form of the test statistic. Section 4 derives the asymptotic null distribution and Section 5 investigates the asymptotic power. In Section 6, a simulation study is conducted to assess the reliability of the asymptotic theory in finite samples. Section 7 applies our test to stock return predictability models and documents strong evidence against model stability. Section 8 concludes. All mathematical proofs are collected in the Supplemental Appendix.

2. HYPOTHESES OF INTEREST

Consider the data generating process (DGP)

$$Y_t = \mathbf{X}_t'\alpha_t + \varepsilon_t, \qquad t = 1, .., T, \tag{2.1}$$

where Y_t is a dependent variable, $\mathbf{X}_t$ is a $d \times 1$ vector of explanatory variables, α_t is a $d \times 1$

possibly time-varying parameter vector, ε_t is an unobservable disturbance with $E(\varepsilon_t|\mathbf{X}_t) = 0$ almost surely (a.s.), d is a fixed positive integer, and T is the sample size. The regressor vector $\mathbf{X}_t$ can contain exogenous explanatory variables and lagged dependent variables. Thus, both static and dynamic regression models are covered.

Like the bulk of the literature on structure changes, we are interested in testing constancy of the regression parameter in (2.1). The null hypothesis of interest is

$$\mathbb{H}_0 : \alpha_t = \alpha \text{ for some constant vector } \alpha \in \mathbb{R}^d \text{ and for all } t.$$

The alternative hypothesis $\mathbb{H}_A$ is that $\mathbb{H}_0$ is false. Under $\mathbb{H}_0$, the unknown constant parameter vector α can be consistently estimated by (e.g.) the OLS estimator

$$\hat{\alpha} = \arg\min_{\alpha \in \mathbb{R}^d} \sum_{t=1}^{T} (Y_t - \mathbf{X}_t'\alpha)^2. \tag{2.2}$$

Under the alternative $\mathbb{H}_A$, α_t is a time-varying parameter vector. Examples include Chow's (1960) single break model, Hall and Hart's (1990) deterministic trend model, Lin and Teräsvirta's (1994) STR model. Tests for parametric structural change alternatives (e.g., Lin and Teräsvirta's (1994) LM tests) have best power against the assumed alternative. Unfortunately, usually no prior information about the structural change alternative is available in practice. To cover a wide range of alternatives, we consider the following smooth time-varying parameter model:

$$Y_t = \mathbf{X}_t'\alpha\,(t/T) + \varepsilon_t, \qquad t = 1, ..., T, \tag{2.3}$$

where $\alpha : [0,1] \to \mathbb{R}^d$ is an unknown smooth function except for a finite number of points over $[0,1]$. Discontinuities of $\alpha(\cdot)$ at a finite number of points in $[0,1]$ allow abrupt structural changes.

This model is introduced by Robinson (1989, 1991) and its nonparametric estimation has been considered in Robinson (1989, 1991), Orbe *et al.* (2000, 2005) and Cai (2007).[1] It avoids restrictive parameterization of $\alpha(\cdot)$. The specification that parameter $\alpha(\cdot)$ is a function of ratio t/T rather than time t only is a common scaling scheme in the literature (e.g., Phillips and Hansen 1990). The reason for this requirement is that a nonparametric estimator for α_t will not be consistent unless the amount of data on which it depends increases, and merely increasing the sample size will not necessarily improve estimation of α_t at some fixed point t, even if some smoothness condition is imposed on α_t. The amount of local information must increase suitably if the variance and bias of a nonparametric estimator of α_t are to decrease suitably. A convenient way to achieve this is to regard α_t as ordinates of smooth function $\alpha(\cdot)$ on an equally spaced

[1] These authors consider pointwise consistent nonparametric estimation of time-varying parameters $\alpha(t/T)$ and $\sigma^2(t/T)$, where $\text{var}(\varepsilon_t) = \sigma^2(t/T)$. They do not consider testing parameter constancy.

grid over $[0,1]$, which becomes finer as $T \to \infty$, and consider estimation of $\alpha(\tau)$ at fixed points $\tau \in [0,1]$. Consistent estimation of model (2.3) is considered in Robinson (1991) and Cai (2007) using local constant smoothing and local linear smoothing respectively.

The specification of $\alpha_t = \alpha(t/T)$ does not regard the sampling of $(Y_t, \mathbf{X}_t')'$ as taking place on a grid on $[0,1]$, which would make the preservation of independence or weak dependence properties as T increases implausible. We note that the device of taking $(Y_t, \mathbf{X}_t' \,)'$ to be observations at intervals $1/T$ on a continuous process on $[0,1]$ that itself is independent of T would not work because it does not achieve the accumulation of new information as T increases, which is needed for consistency. Making parameter α_t depend on T is common in econometrics. A well-known example is local power analysis, where local alternatives are specified as a function of T.

Model (2.3) includes the class of locally stationary autoregressive models in Dahlhaus (1996):

$$Y_t = \alpha_0 \left(t/T\right) + \sum_{j=1}^{p} \alpha_j \left(t/T\right) Y_{t-j} + \varepsilon_t,$$

where $\varepsilon_t = \sigma(t/T)v_t$ and $v_t \sim i.i.d.N(0,1)$. A locally stationary process is a nonstationary time series whose behavior can be locally approximated by a stationary process. In time series analysis, it is often assumed that nonstationary economic time series can be transformed, by removing time trends and/or taking differences, into a stationary process. In fact, the transformed series may still not be stationary, even after trending components are removed. Locally stationary time series models nicely fill this gap and provide new insight into modelling economic time series.

We will assume that $\alpha(\cdot)$ is continuous except for a finite number of points on $[0,1]$. In other words, we permit $\alpha(\cdot)$ to have finitely many discontinuities. Hence, single structural break or multiple breaks with known or unknown break points, as often considered in this literature, are special cases of model (2.3). For example, suppose $\alpha(\cdot)$ is a jump function, namely,

$$\alpha\left(\tau\right) = \begin{cases} \alpha_0, & \text{if } \tau \leq \tau_0, \\ \alpha_1, & \text{otherwise.} \end{cases}$$

Then we obtain the single break model originally considered in Chow (1960).

3. NONPARAMETRIC TESTING

We now propose a consistent test for smooth structural changes. Recall that under $\mathbb{H}_0$, we have a constant parameter regression model $Y_t = \mathbf{X}_t'\alpha + \varepsilon_t$, where α can be consistently estimated by the OLS estimator $\hat{\alpha}$ in (2.2). Under the alternative $\mathbb{H}_A$, $\alpha_t = \alpha(t/T)$ is changing over time. The OLS estimator $\hat{\alpha}$ is no longer suitable because there exists no parameter α such that $E(Y_t|\mathbf{X}_t) = \mathbf{X}_t'\alpha$ a.s. under $\mathbb{H}_A$. However, a nonparametric estimator can consistently estimate the time-varying parameter α_t.

Various nonparametric methods could be used to estimate α_t. Robinson (1991) and Cai (2007) study the pointwise consistency and asymptotically normality of the kernel and local linear estimators respectively. Here, we use local linear smoothing, which includes the kernel method as a special case. Cai (2007) shows that although the kernel and local linear estimators share the same asymptotic properties at the interior points, the latter converges faster than the former in the boundary regions near the end points of the sample period. The use of local linear smoothing is quite suitable in the present context.[2] In particular, structural changes near the boundary regions are notoriously difficult to detect, as shown by many previous works in the literature (e.g., Chu, Hornik and Kuan 1995). Unlike many existing tests, no trimming is needed for the local linear smoother, which can estimate structural changes near boundary regions for sufficiently large samples. Thus, it is expected to give better power in such cases.

Put $\mathbf{Z}_{st} = \left(1, \frac{s-t}{T}\right)'$, and $k_{st} = k\left(\frac{s-t}{Th}\right)$, where the kernel $k(\cdot) : [-1, 1] \to \mathbb{R}^+$ is a prespecified symmetric probability density, and $h \equiv h(T)$ is a bandwidth such that $h \to 0$ and $Th \to \infty$ as $T \to \infty$. For notational simplicity, we have suppressed the dependence of $\mathbf{Z}_{st}$ and k_{st} on T and h. Examples of $k(\cdot)$ include the uniform, Epanechniov and quartic kernels.

We note that although local linear smoothing can enhance the convergence rate of the asymptotic bias in the boundary regions $[1, Th] \cup [T - Th, T]$ from h to h^2, the scale is different from that of an interior point. As shown in Cai (2007), the asymptotic bias at an interior point is proportional to $h^2 \int_{-1}^{1} u^2 k(u)du$ while that at a boundary point is proportional to $h^2 b(c)$, where $b(c) = (\mu_{2c}^2 - \mu_{1c}\mu_{3c})/(\mu_{0c}\mu_{2c} - \mu_{1c}^2)$, $\mu_{ic} = \int_{-c}^{1} u^i k(u)du$, $i = 0, 1, 2, 3$ and $0 < c < 1$. Although the convergence rate is the same as in the interior region, the asymptotic variance at a boundary point tends to be larger, because fewer observations contribute to the estimator in the boundary regions. These differences would complicate the form of our test statistics. Moreover, the observations contained in the boundary regions $[1, Th] \cup [T - Th, T]$ are rather substantial. For example, if $h = (1/\sqrt{12})T^{-1/5}$ is used, then about 23%, 17% and 10% of the observations fall into the boundary regions when the sample size $T = 100, 500$ and $5,000$ respectively.

To make the behavior of the local linear estimator at boundary points similar to that at interior points, we follow Hall and Wehrly (1991) to reflect the data in the boundary regions, obtaining pseudodata $\mathbf{X}_t = \mathbf{X}_{-t}$, $Y_t = Y_{-t}$ for $-\lfloor Th \rfloor \le t \le -2$, where $\lfloor Th \rfloor$ denotes the integer part of Th, and $\mathbf{X}_t = \mathbf{X}_{2T-t}$, $Y_t = Y_{2T-t}$ for $T + 1 \le t \le T + \lfloor Th \rfloor$. We use the synthesized data (i.e., the union of the original data and the pseudodata) to estimate α_t. By construction, symmetric data points are available in the original boundary regions $[1, Th] \cup [T - Th, T]$. Besides

[2] Both local linear smoothing and the conventional kernel method are local smoothing. Global smoothing (e.g., series approximation) is another class of nonparametric method. The coefficient function $\alpha(\cdot)$ may not have a nice shape and many terms are needed when using a serial approximation, which complicates the estimation. On the other hand, structural change is the local behavior of parameters and hence local smoothing is expected to have better approximation in many cases.

in nonparametric regression estimation, this method has also been described as "reflection about the boundaries" by Cline and Hart (1991) in nonparametric density estimation. It has not been used to estimate time-varying coefficients in the previous literature.

The local linear parameter estimator is obtained by minimizing the local SSR:

$$\min_{\beta \in \mathbb{R}^{2d}} \sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} \left[Y_s - \alpha_0' \mathbf{X}_s - \alpha_1' \left(\frac{s-t}{T} \right) \mathbf{X}_s \right]^2 = \sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} (Y_s - \boldsymbol{\beta}' \mathbf{Q}_{st})^2, \tag{3.1}$$

where $\boldsymbol{\beta} = (\alpha_0', \alpha_1')'$ is a $2d \times 1$ vector, α_j is a $d \times 1$ coefficient vector for $(\frac{s-t}{T})^j \mathbf{X}_s$, $j = 0, 1$, $\mathbf{Q}_{st} = \mathbf{Z}_{st} \otimes \mathbf{X}_s$ is a $2d \times 1$ vector, and $\otimes$ is the Kronecker product. Note that the device of using pseudodata does not affect the estimation at interior points $[Th, T - Th]$. By solving the optimization problem in (3.1), we obtain the solution:

$$\hat{\beta}_t = \left(\sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} \mathbf{Q}_{st} \mathbf{Q}_{st}' \right)^{-1} \sum_{s=t-\lfloor Th \rfloor}^{t+\lfloor Th \rfloor} k_{st} \mathbf{Q}_{st} Y_s, \ t = 1, ..., T. \tag{3.2}$$

This is a locally weighted least square estimator. As pointed out by Cai (2007), the local linear estimator could be regarded as the OLS estimator of the transformed model

$$k_{st}^{1/2} Y_s = k_{st}^{1/2} \mathbf{X}_s' \alpha_0 + k_{st}^{1/2} \left(\frac{s-t}{T} \right) \mathbf{X}_s' \alpha_1 + k_{st}^{1/2} \varepsilon_s, \qquad s = 1, ..., T.$$

Hence the estimation can be implemented by standard econometric software.

Put $\mathbf{e}_1 = (1, 0)'$. Then the local linear estimator for α_t is given by

$$\hat{\alpha}_t = (\mathbf{e}_1' \otimes \mathbf{I}_d) \hat{\beta}_t, \ t = 1, ..., T. \tag{3.3}$$

We note that with the reflection method for the boundary regions, one can also use the kernel method, which is equivalent to a local linear estimation with the restriction $\alpha_1 = \mathbf{0}$. The test statistic has the same asymptotic distribution for both local constant and linear estimators.

With $\hat{\alpha}_t$, we can construct a Wald-type test by comparing the OLS and nonparametric regression estimators. This can be interpreted as a generalized Hausman's (1978) test. Hausman's (1978) test is a convenient specification test that compares two parameter estimators, where one is efficient but inconsistent under the alternative, and the other is inefficient but consistent under the alternative. Here we extend Hausman's (1978) idea from a parametric regression to a nonparametric time-varying parameter regression, where the OLS regression estimator $\mathbf{X}_t' \hat{\alpha}$ can be viewed as an efficient estimator for $E(Y_t | \mathbf{X}_t)$, and the nonparametric time-varying parameter regression estimator $\mathbf{X}_t' \hat{\alpha}_t$ can be viewed as an inefficient but consistent estimator for $E(Y_t | \mathbf{X}_t)$

under $\mathbb{H}_A$. We compare these parametric and nonparametric fitted values via a sample quadratic form:

$$\hat{Q} = \frac{1}{T}\sum_{t=1}^{T}(\mathbf{X}_t'\hat{\alpha}_t - \mathbf{X}_t'\hat{\alpha})^2.$$

The statistic $\hat{Q}$ converges to 0 under $\mathbb{H}_0$, but to a strictly positive constant under $\mathbb{H}_A$, giving the test asymptotic unit power. Any significant departure of $\hat{Q}$ from 0 is evidence of structural changes.[3] Formally, our generalized Hausman test is a standardized version of $\hat{Q}$:

$$\hat{H} = \left(T\sqrt{h}\hat{Q} - \hat{A}_H\right)/\sqrt{\hat{B}_H}, \tag{3.4}$$

where $\hat{A}_H = h^{-1/2}C_A\text{trace}(\hat{\Omega}\hat{M}^{-1})$, $\hat{B}_H = 4C_B\text{trace}(\hat{M}^{-1}\hat{\Omega}\hat{M}^{-1}\hat{\Omega})$, $C_A = T^{-1}h^{-1}\Sigma_{j=-\lfloor Th\rfloor}^{\lfloor Th\rfloor}$ $(1-\frac{|j|}{T})k(\frac{j}{Th})[k(\frac{j}{Th}) + h\int_{-1}^{1}k\left(\frac{j}{Th}+2u\right)du] = \int_{-1}^{1}k^2(u)\,du + o(1)$, $C_B = T^{-1}h^{-1}\Sigma_{j=1}^{T-1}(1-\frac{j}{T})$ $[\int_{-1}^{1}k(u)k(u+\frac{j}{Th})du]^2 = \int_0^1[\int_{-1}^{1}k(u)k(u+v)du]^2dv + o(1)$, $\hat{\mathbf{M}} = T^{-1}\Sigma_{t=1}^{T}\mathbf{X}_t\mathbf{X}_t'$, and $\hat{\Omega} = T^{-1}\Sigma_{t=1}^{T}\hat{\varepsilon}_t^2\mathbf{X}_t\mathbf{X}_t'$. Note that C_A and C_B are independent of the random sample $\{\mathbf{X}_t, Y_t\}_{t=1}^{T}$. The factors $\hat{A}_H$ and $\hat{B}_H$ are approximately the mean and variance of $T\sqrt{h}\hat{Q}$. They have taken into account the impact of conditional heteroscedasticity and higher order serial dependence in the residual $\{\varepsilon_t\}$. As a result, the $\hat{H}$ test is robust to conditional heteroscedasticity and time-varying higher order conditional moments of unknown form in $\{\varepsilon_t\}$. If ε_t is conditional homoscedastic, $\hat{A}_H$ and $\hat{B}_H$ can be simplified to $h^{-1/2}dC_A\hat{\sigma}^2$ and $4dC_B\hat{\sigma}^4$ respectively, where $\hat{\sigma}^2 = T^{-1}\Sigma_{t=1}^{T}(Y_t - \mathbf{X}_t'\hat{\alpha}_t)^2$ is the residual variance estimator.

4. ASYMPTOTIC DISTRIBUTION

To derive the asymptotic distribution of $\hat{H}$, we impose the following regularity conditions.

Assumption A.1: *$\{\mathbf{X}_t', \varepsilon_t\}'$ is a $(d+1)\times 1$ stationary β-mixing process with mixing coefficients $\{\beta(j)\}$ satisfying $\Sigma_{j=1}^{\infty}j^2\beta(j)^{\frac{\delta}{1+\delta}} < C$ for some $0<\delta<1$.*

Assumption A.2: *$\{\varepsilon_t\}$ is a martingale difference sequence (m.d.s.) such that $E(\varepsilon_t|\mathcal{F}_{t-1}) = 0$ and $E(\varepsilon_t^2) = \sigma^2$, where $\mathcal{F}_{t-1} = \{\mathbf{X}_t, \mathbf{X}_{t-1}, ..., \varepsilon_{t-1}, \varepsilon_{t-2}, ...\}$.*

Assumption A.3: *(i) The $d\times d$ matrix $\mathbf{M} = E(\mathbf{X}_t\mathbf{X}_t')$ is finite and positive definite; (ii) $E(\mathbf{X}_{ti}^8) < \infty$ for $i = 1, ..., d$; (iii) $E(Y_t^8) < \infty$.*

Assumption A.4: *$\hat{\alpha}$ is a parameter estimator such that $\sqrt{T}(\hat{\alpha} - \alpha^*) = O_P(1)$, where $\alpha^* = p\lim_{T\to\infty}\hat{\alpha}$ and $\alpha^* = \alpha$ under $\mathbb{H}_0$, where α is given in $\mathbb{H}_0$.*

Assumption A.5: *$k : [-1,1] \to \mathbb{R}^+$ is a symmetric bounded probability density function.*

Assumption A.6: *The bandwidth $h = cT^{-\lambda}$ for $0<\lambda<1$ and $0<c<\infty$.*

[3] Alternatively, we could compare $\hat{\alpha}_t$ and $\hat{\alpha}$ directly and the asymptotic derivation is similar. However, multiplying the coefficients by $\mathbf{X}_t$ gives comparison between fitted values of the restricted and unrestriced models and the asymptotic pivatality of our test.

The β-mixing condition in Assumption A.1 imposes restriction on temporal dependence in $\{\mathbf{X}_t', \varepsilon_t\}'$. Assumption A.2 allows dynamic regression models where $\mathbf{X}_t$ contains both exogenous and lagged dependent variables, and conditional heteroscedasticity of unknown form.[4] We note that our m.d.s. assumption is weaker than Lin and Terësvirta's (1994), who assume that $\{\varepsilon_t\}$ is a m.d.s. with $\lim_{t\to\infty} E(\varepsilon_t^2|\mathcal{F}_{t-1}) = \sigma^2$. Assumption A.2 requires that the linear regression model is correctly specified under $\mathbb{H}_0$ and the violation of correct model specification may lead to spurious rejection of model stability. Assumption A.3 are moment conditions on $\mathbf{X}_t$ and Y_t, commonly assumed in the regression literature. It could be relaxed to allow time-varying moments (i.e., $\mathbf{M}(t/T) = E(\mathbf{X}_t\mathbf{X}_t')$ is a function of standardized time t/T) at the cost of more tedious proof and test statistic. Assumption A.4 holds for any $\sqrt{T}$-consistent estimator for α under $\mathbb{H}_0$. We allow but are not restricted to the OLS estimator $\hat{\alpha}$ in (2.2).

Assumption A.5 implies $\int_{-1}^{1} k(u)du = 1$, $\int_{-1}^{1} uk(u)du = 0$ and $\int_{-1}^{1} u^2k(u)du < \infty$. All examples noted in Section 3 satisfy this assumption. It is possible to use kernels with infinite support, such as the Gaussian kernel $k(u) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}u^2)$, $-\infty < u < \infty$. However, we only use kernels with bounded support to simplify our analysis. Assumption A.6 implies $h \to 0$ and $Th \to \infty$. This is the standard condition for bandwidth and covers the optimal rate $h \propto T^{-\frac{1}{5}}$ of the nonparametric estimation for α_t. In practice, h can be chosen via a simple rule-of-thumb approach, namely $h = (1/\sqrt{12})T^{-\frac{1}{5}}$, where $1/\sqrt{12}$ is the standard deviation of $U(0,1)$, which could be viewed as the limiting distribution of the grid points $\frac{t}{T}$, $t = 1, ..., T$, as $T \to \infty$. More sophistically, an automatic method such as cross-validation (CV) may be used. Define a "leave-one-out" estimator $\hat{\alpha}_{-t} = (\mathbf{e}_1' \otimes \mathbf{I}_d)\hat{\boldsymbol{\beta}}_{-t}$, where $\hat{\boldsymbol{\beta}}_{-t} = (\Sigma_{s=t-\lfloor Th\rfloor, s\neq t}^{t+\lfloor Th\rfloor} k_{st}\mathbf{Q}_{st}\mathbf{Q}_{st}')^{-1}\Sigma_{s=t-\lfloor Th\rfloor, s\neq t}^{t+\lfloor Th\rfloor} k_{st}\mathbf{Q}_{st}Y_s$. Then a data-driven choice of h is $\hat{h}_{CV} = \arg\min_{c_1\leq h\leq c_2} CV(h)$, where $CV(h) = \Sigma_{t=1}^{T}(Y_t - \mathbf{X}_t'\hat{\alpha}_{-t})^2$. We investigate this method in the simulation study.

We now state the asymptotic distribution of $\hat{H}$ under $\mathbb{H}_0$.

Theorem 1: *Suppose Assumptions A.1−A.6 and $\mathbb{H}_0$ hold. Then (i) $\hat{H} \xrightarrow{d} N(0,1)$ as $T \to \infty$. (ii) Suppose in addition* $\mathrm{var}(\varepsilon_t|\mathbf{X}_t) = \sigma^2$ *a.s., then $\hat{A}_H = h^{-1/2}dC_A\hat{\sigma}^2$ and $\hat{B}_H = 4dC_B\hat{\sigma}^4$.*

As an important feature of $\hat{H}$, the use of the restricted parametric estimator $\hat{\alpha}$ in place of the regression parameter α under $\mathbb{H}_0$ has no impact on the limit distribution of $\hat{H}$. Intuitively, $\hat{\alpha}$ converges to α faster than the nonparametric estimator $\hat{\alpha}_t$. Consequently, the asymptotic distribution of $\hat{H}$ is solely determined by the nonparametric estimator $\hat{\alpha}_t$. In small samples, the distribution of $\hat{H}$ may not be well approximated by the asymptotic N(0,1) distribution. Accurate finite sample critical values can be obtained via bootstrap; see Section 6 for more discussion.

5. ASYMPTOTIC POWER

[4] Assumption A.2 rules out linear regression models with endogeneity. For such cases, we could compare a two-stage least square (2SLS) estimator and a local 2SLS estimator, and construct a test statistic accordingly. This is left for future research.

To study the asymptotic power of $\hat{H}$ under $\mathbb{H}_A$, we impose the following assumption:

Assumption A.7: *The coefficient function* $\alpha : [0,1] \rightarrow R^d$ *is continuous except for a finite number of discontinuity points on* $[0,1]$ *and* $sup_{u_0 \in (0,1)} \left\| \lim_{u \to u_0^+} \alpha(u) - \lim_{u \to u_0^-} \alpha(u) \right\| \leq C.$

This allows both smooth structural changes and abrupt structural breaks with known or unknown break points. For abrupt structural breaks, the break size is bounded.

Theorem 2: *Suppose Assumptions A.1−A.7 hold. Then for any sequence of nonstochastic constants* $\{M_T = o(T\sqrt{h})\}$, $P(\hat{H} > M_T) \rightarrow 1$ *under* $\mathbb{H}_A$ *as* $T \rightarrow \infty$.

Theorem 2 suggests that $\hat{H}$ is consistent against all alternatives to $\mathbb{H}_0$ at any given significance level, subject to Assumption A.7. Thus, for T sufficiently large, $\hat{H}$ can detect any structural changes, including those occur close to the starting and ending points of the sample period because no trimming is used. This is rather appealing because no prior information about the alternative is available in practice. It avoids the blindness of searching for possible alternatives of structural changes. We note that for (and merely for) simplicity, stationarity for $\mathbf{X}_t$ is assumed under $\mathbb{H}_A$. One could allow $\mathbf{X}_t$ to be a locally stationary process.

To gain more insight into the power property of $\hat{H}$, we consider two classes of local alternatives. Case 1 [*Local Smooth Structural Change*]:

$$\mathbb{H}_{1A}(j_T) : \alpha(u) = \alpha + j_T g(u), \ u \in [0,1],$$

where $g : [0,1] \rightarrow \mathbb{R}^d$ is a twice continuously differentiable vector function with $\sup_{u\in[0,1]} \|g(u)\| \leq C$ and $\sup_{u\in[0,1]} \|d^2 g(u)/du^2\| \leq C$. The term $j_T g(u)$ characterizes the departure of the smooth-changing $\alpha(u)$ from α at each point $u \in [0,1]$ and j_T is the speed at which the departure vanishes to 0 as $T \rightarrow \infty$. For notational simplicity, we have suppressed the dependence of $\alpha(u)$ on T.
Case 2 [*Local Sharp Structural Change at Some Point* u_0]:

$$\mathbb{H}_{2A}(b_T, r_T) : \alpha(u) = \alpha + b_T f[(u - u_0)/r_T], \ u \in [0,1],$$

where u_0 is a given point in $[0,1]$, $f : \mathbb{R} \rightarrow \mathbb{R}^d$ is a twice continuously differentiable vector function with $\sup_{z\in\mathbb{R}} \|f(z)\| \leq C$ and $\sup_{z\in\mathbb{R}} \|d^2 f(z)/dz^2\| \leq C$, $b_T = b(T) \rightarrow 0$, and $r_T = r(T) \rightarrow 0$ as $T \rightarrow \infty$. This alternative is studied by Rosenblatt (1975) and Horowitz and Spokoiny (2001) in different contexts. Under $\mathbb{H}_{2A}(b_T, r_T)$, the coefficient function $\alpha(u)$ eventually becomes a nonsmooth spike at location u_0 (i.e., a sudden break) as $T \rightarrow \infty$, due to the existence of the shrinking width parameter r_T. Here, r_T controls the sharpness of the structural change around u_0, and b_T is the speed at which the departure of $\alpha(u)$ from α at each point $u \in [0,1]$ vanishes to 0 as $T \rightarrow \infty$. For concreteness, we use OLS estimation under $\mathbb{H}_0$ in Theorem 3 below.

Theorem 3: *Suppose Assumptions A.1–A.6 hold and let $\hat{\alpha}$ be the OLS estimator. (i) Under $\mathbb{H}_{1A}(j_T)$ with $j_T = T^{-1/2}h^{-1/4}$, $\hat{H} \xrightarrow{d} N(\delta_1, 1)$ as $T \to \infty$, where $\delta_1 = [\int_0^1 g(u)' \mathbf{M} g(u)\, du - \int_0^1 g(u)' du \mathbf{M} \int_0^1 g(u)\, du]/\sqrt{B_H}$,* where $B_H = 4trace(\mathbf{M}^{-1}\Omega\mathbf{M}^{-1}\Omega) \int_0^1 [\int_{-1}^1 k(u)k(u+v)du]^2 dv$, $\mathbf{M} = E(\mathbf{X}_t\mathbf{X}_t')$ and $\Omega = E(\varepsilon_t^2\mathbf{X}_t\mathbf{X}_t')$. *(ii) Under $\mathbb{H}_{2A}(b_T, r_T)$ with $b_T \to 0$, $r_T \to 0$, $b_T^2 r_T = T^{-1}h^{-1/2}$ and $h = o(r_T)$, $\hat{H} \xrightarrow{d} N(\delta_2, 1)$ as $T \to \infty$, where $\delta_2 = [\int_{-\infty}^{\infty} f(z)' \mathbf{M} f(z)\, dz]/\sqrt{B_H}$.*[5]

Our test has nontrivial power against the class of smooth alternatives $\mathbb{H}_{1A}(j_T)$ with rate $j_T = T^{-1/2}h^{-1/4}$, which is slightly slower than the parametric rate $T^{-1/2}$ as $h \to 0$.[6] In contrast, parametric tests such as Andrews' (1993) supremum and Bai and Perron's (1998) double maximum tests can have nontrivial power against $\mathbb{H}_{1A}(j_T)$ with $j_T = T^{-1/2}$. Thus, these parametric tests could be more powerful than ours against the smooth alternatives $\mathbb{H}_{1A}(j_T)$. This is the cost we have to pay to construct a consistent smoothed test $\hat{H}$.

However, our test can have better power than the aforementioned parametric tests against the class of nonsmooth sharp alternatives $\mathbb{H}_{2A}(b_T, r_T)$ for suitable sequences of b_T and r_T. For example, suppose we choose $h = T^{-1/5}$, $r_T = T^{-1/5}(\ln\ln T)^{\varepsilon}$ and $b_T = T^{-7/20}(\ln\ln T)^{-\varepsilon/2}$ for small $\varepsilon > 0$. Since $b_T r_T = o\left(T^{-1/2}\right)$, it is not difficult to show that the noncentrality parameters of the aforementioned parametric tests converge to 0 as $T \to \infty$. In this case, our test is asymptotically more powerful than those parametric tests under $\mathbb{H}_{2A}(b_T, r_T)$.

In the classical normal linear regression model, Chow's (1960) F test enjoys the optimal power property against a single structural break. We can also construct a generalized Chow's F-type test by comparing the SSRs between the restricted constant parameter model and the unrestricted time-varying parameter model, namely

$$\hat{C} = \left[\sqrt{h}(SSR_0 - SSR_1) - \hat{A}_C\right] / \sqrt{\hat{B}_C},$$

where $SSR_0 = \Sigma_{t=1}^T (Y_t - \mathbf{X}_t'\hat{\alpha})^2$, $SSR_1 = \Sigma_{t=1}^T (Y_t - \mathbf{X}_t'\hat{\alpha}_t)^2$, $\hat{A}_C$ and $\hat{B}_C$ are some suitable centering and scaling factors (see Chen and Hong (2008)). Interestingly, this optimality property disappears for $\hat{C}$ in the present nonparametric setup, because it is asymptotically less powerful than $\hat{H}$ under the same local or global alternative. This is established in Theorem 4 below.

Theorem 4: *(i) Suppose the conditions of Theorem 3 hold and the same kernel $k(\cdot)$ and bandwidth h are used for both the $\hat{H}$ and $\hat{C}$ tests. Then $\hat{H}$ is asymptotically more efficient than $\hat{C}$ under both $\mathbb{H}_{1A}(j_T)$ and $\mathbb{H}_{2A}(b_T, r_T)$ respectively. (ii) Suppose Assumptions A.1–A.7 hold.*

[5] It may first seem odd that the noncentrality parameter δ_2 does not depend on the given location point $u_0 \in [0, 1]$. This is due to the fact that $\mathbf{M}$ is not time-varying. If we allow for the time-varying second moment of $\mathbf{X}_t$, δ_2 would depend on u_0.

[6] We note that no "curse of dimensionality" problem exists here as the nonparametric regression is implemented with respect to the scalar t/T.

Then $\hat{H}$ is asymptotically more efficient than $\hat{C}$ in terms of the Bahadur asymptotic efficiency criterion.

Theorem 4 (i) suggests that $\hat{H}$ is more efficient than $\hat{C}$ in terms of the Pitman asymptotic efficiency criterion, which is suitable for local power analysis. Theorem 4(ii) shows that the relative efficiency of $\hat{H}$ over $\hat{C}$ carries over to the global alternative. The Pitman (Pitman 1979) and Bahadur (Bahadur 1960) asymptotic relative efficiency criteria are the limit ratios of the sample sizes required by the two tests to achieve the same asymptotic p-value under the same *local* or *global* alternative respectively. Theorem 4 implies that under the same set of conditions, including the same local or global alternative, the same bandwidth and kernel, the generalized Hausman test is asymptotically more efficient than the generalized Chow test.

Intuitively, the relative efficiency of $\hat{H}$ over $\hat{C}$ follows because the direct comparison of fitted values between the restricted and unrestricted models has a smaller variation than the comparison of the SSRs between two models. To see this, we decompose

$$SSR_0 - SSR_1 = 2\sum_{t=1}^{T}(\hat{\alpha}_t - \hat{\alpha})'\mathbf{X}_t\varepsilon_t + \sum_{t=1}^{T}(\mathbf{X}_t'\hat{\alpha}_t - \mathbf{X}_t'\hat{\alpha})^2 + \text{ Remainder term.} \qquad (5.1)$$

The asymptotic distribution of $\hat{C}$ is jointly determined by the first two terms in (5.1), whose total variance is larger than the variance of the second term, thus causing lower power than $\hat{H}$. The asymptotic distribution of $\hat{H}$ is determined by the second term of (5.1) only.

The relative efficiency of $\hat{H}$ over $\hat{C}$ is sizable. It can be shown that with the choice of bandwidth $h = cT^{-\lambda}$, both Pitman and Bahadur relative efficiencies of $\hat{H}$ to $\hat{C}$ are

$$\text{RE}(\hat{H}:\hat{C}) = \left\{\int_{-1}^{1}\left[2k\left(v\right) - \int_{-1}^{1}k\left(u\right)k\left(u+v\right)du\right]^2 dv \Big/ \int_{-1}^{1}\left[\int_{-1}^{1}k\left(u\right)k\left(u+v\right)du\right]^2 dv\right\}^{\frac{1}{2-\lambda}}.$$

Suppose the bandwidth rate parameter $\lambda = 1/5$, which gives the optimal bandwidth rate for estimating $\alpha(t/T)$. Then for most commonly used kernels, such as the uniform, Epanechniov and quartic kernels, we have $\text{RE}(\hat{H}:\hat{C}) = 2.80, 2.04, 1.99$ respectively.

Caution may be taken when the generalized Hausman and Chow tests reject $\mathbb{H}_0$. It is possible that the rejection is due to a nonlinear relationship or other model misspecifications rather than structural changes. For example, the choice of an inappropriate functional form and omitted variables can result in spurious structural changes. Of course, this is not particular to the proposed tests, but relevant to all existing tests for structural breaks.

6. FINITE SAMPLE PERFORMANCE

We now compare the finite sample performances of the proposed tests and those of Lin and Terȁsvirta (1994), Andrews (1993), Bai and Perron (1998) and Elliott and Müller (2006).

To examine the size of all tests under $\mathbb{H}_0$, we consider the following DGP:

DGP S.1 [*No Structural Change*]:

$$\begin{cases} Y_t = 1 + 0.5X_t + \varepsilon_t, \\ X_t = 0.5X_{t-1} + \nu_t,\ \nu_t \sim i.i.d.N\,(0,1)\,. \end{cases}$$

To examine robustness of tests, we consider three cases for the regression error ε_t : (i) $\varepsilon_t \sim i.i.d.N\,(0,1)$; (ii) $\varepsilon_t = \sqrt{h_t}u_t,\ h_t = 0.2 + 0.5\varepsilon_{t-1}^2,\ u_t \sim i.i.d.N\,(0,1)$; (iii) $\varepsilon_t = \sqrt{h_t}u_t,\ h_t = 0.2 + 0.5X_t^2,\ u_t \sim i.i.d.N\,(0,1)$. Note that $var\,(\varepsilon_t|X_t) \neq \sigma^2$ under Case (iii). We generate $5,000$ data sets of the random sample $\{X_t, Y_t\}_{t=1}^T$ for $T = 100, 250$ and 500 respectively.

We use the uniform kernel for both $\hat{H}$ and $\hat{C}$ tests. Our simulation experience suggests that the choice of $k(\cdot)$ has little impact on the performance of the tests. For space, we report results based on the simple rule-of-thumb bandwidth $h = (1/\sqrt{12})T^{-\frac{1}{5}}$, which attains the optimal rate for local linear fitting.[7] We compare $\hat{H}$ and $\hat{C}$ with a variety of popular tests, namely, Lin and Terȁsvirta's (1994) LM test based on the first order Taylor expansion, Andrews' (1993) supremum LM test, Bai and Perron's (1998) $UD\max$ test, and Elliott and Müller's (2006) qLL test.[8] Following Andrews (1993), we choose the trimming region $\Pi = [0.15, 0.85]$ for the tests of Andrews (1993) and Bai and Perron (1998). For Bai and Perron's (2003) test, we set the upper bound of the number of breaks at 5. We consider both heteroscedasticity robust and homoscedasticity-specific versions of all tests (the latter are all denoted as -het), following Elliott and Müller (2006).

Table 1 reports the rejection rates of all tests under DGP S.1 at the 5% significance level, using asymptotic theory. Under *iid* and ARCH errors, both $\hat{H}$ and $\hat{C}$ overreject $\mathbb{H}_0$ when $T = 100$, but not excessively and improve as T increases; the $\hat{H}$ and $\hat{C}$ tests derived under conditional homoscedasticity and i.i.d. have better sizes than $\hat{H}$-het and $\hat{C}$-het respectively. Under conditional heteroscedastic errors, the $\hat{H}$ and $\hat{C}$ tests derived under conditional homoscedasticity display strong overrejection, as is expected. For other tests, Andrews' supremum LM is quite conservative, especially its heteroscedasticity version. In contrast, Bai and Perron's $UD\max$ test shows quite big overrejection. Overall, Lin and Terȁsvirta's (1994) LM and Elliott and Müller's (2006) qLL tests have best sizes for small samples, but our tests also have reasonable sizes.

[7] We also try the rule-of-thumb bandwidth with different scaling parameters and the CV bandwidth described in Section 4. Simulation results, reported in the Supplemental Appendix, show that empirical sizes and powers are a bit sensitive to the bandwidth selection without bootstrap. However, the nonparametric bootstrap described below alleviates the sensitivity to the choice of the bandwidth.

[8] For space, simulation results for Brown *et al.*'s (1975) CUSUM test, Hackl's (1980) MOSUM test, Lin and Terȁsvirta's (1994) $LM2, LM3$, Andrews and Ploberger's (1994) exponential and average LM tests, and Bai and Perron's (1998) $WD\max$ test can be found in the Supplemental Appendix.

Because the sizes of our tests using asymptotic theory differ from the nominal level in small samples and are a bit sensitive to bandwidth selection, we consider a nonparametric bootstrap:

Step (i): Estimate the model via OLS and nonparametric regression respectively and compute the $\hat{H}$ statistic and the nonparametric residual $\hat{\varepsilon}_t = Y_t - \mathbf{X}_t'\hat{\alpha}_t$; Step (ii): Obtain a wild bootstrap residual $\hat{\varepsilon}_t^*$ from the centered nonparametric residual $\bar{\varepsilon}_t = \hat{\varepsilon}_t - T^{-1}\Sigma_{t=1}^{T}\hat{\varepsilon}_t$ and construct a bootstrap sample $\{\mathbf{X}_t, Y_t^*\}_{t=1}^{T}$, where $Y_t^* = \mathbf{X}_t'\hat{\alpha} + \hat{\varepsilon}_t^*$;[9] Step (iii): Compute the bootstrap statistic $\hat{H}^*$, in the same way as $\hat{H}$, with $\{\mathbf{X}_t, Y_t^*\}_{t=1}^{T}$ replacing the original sample $\{\mathbf{X}_t, Y_t\}_{t=1}^{T}$; Step (iv): Repeat steps (ii) and (iii) B times to obtain B bootstrap test statistics $\{\hat{H}_l^*\}_{l=1}^{B}$, where B is sufficiently large; Step (v): Compute the bootstrap p-value $p^* \equiv B^{-1}\Sigma_{l=1}^{B}\mathbf{1}(\hat{H}_l^* > \hat{H})$.

We generate 500 data sets of random sample $\{\mathbf{X}_t, Y_t\}_{t=1}^{T}$ and use $B = 99$ bootstrap iterations for each simulated data set. Table 1 shows that the bootstrap indeed approximates the finite sample distribution of test statistics more accurately. Table 7A in the Supplemental Appendix shows that the bootstrap p-values are not sensitive to the choice of bandwidth h.

To investigate the power of all tests in detecting structural changes, we consider five alternatives: (i) a single break, (ii) multiple breaks, (iii) non-persistent temporal breaks, (iv) smooth structural changes, and (v) unit root in parameters, respectively:

DGP P.1 [*Single Structural Break*]:[10]

$$Y_t = \begin{cases} 1 + 0.5X_t + \varepsilon_t, & \text{if } t \leq 0.3T, \\ 1.2 + X_t + \varepsilon_t, & \text{otherwise.} \end{cases}$$

DGP P.2 [*Multiple Structural Breaks*]:

$$Y_t = \begin{cases} 0.6 + 0.3X_t + \varepsilon_t, & \text{if } 0.1T \leq t \leq 0.2T \text{ or } 0.7T \leq t \leq 0.8T, \\ 1.5 + X_t + \varepsilon_t, & \text{if } 0.4T \leq t \leq 0.5T, \\ 1 + 0.5X_t + \varepsilon_t, & \text{otherwise.} \end{cases}$$

DGP P.3 [*Non-persistent Temporal Structural Breaks*]:

$$Y_t = \begin{cases} 1 + 0.5X_t + \varepsilon_t, & \text{if } t \leq 0.4T \text{ or } t \geq 0.6T, \\ 1.5 + X_t + \varepsilon_t, & \text{otherwise.} \end{cases}$$

DGP P.4 [*Smooth Structural Changes*]:

$$Y_t = F(\tau)(1 + 0.5X_t) + \varepsilon_t,$$

[9] We generate a wild bootstrap residual according to the formula that $\hat{\varepsilon}_t^* = a\bar{\varepsilon}_t$ with probability $1 - a/\sqrt{5}$ and $\hat{\varepsilon}_t^* = (1-a)\bar{\varepsilon}_t$ with probability $a/\sqrt{5}$, where $a = (1+\sqrt{5})/2$.

[10] For robustness, we consider different locations of structural changes. For space, results are reported in the Supplemental Appendix.

where $\tau = \frac{t}{T}$ and $F(\tau) = 1.5 - 1.5 \exp\left[-3(\tau - 0.5)^2\right]$.

DGP P.5 [*Unit Root in Parameters*]:

$$Y_t = \rho_{1t} + \rho_{2t} X_t + \varepsilon_t,$$

where $\rho_{jt} = \rho_{jt-1} + u_{jt}$, $u_{jt} \sim i.i.d.N(0,1)$, and $j = 1, 2$.

For each of DGPs P.1-P.5, we generate 1,000 data sets of the random sample $\{X_t, Y_t\}_{t=1}^T$ for $T = 100$, 250 and 500 respectively. Table 2 reports the rejection rates of all tests with empirical critical values (ECVs) under DGPs P.1-P.5 at the 5% level. We first consider the deterministic single break (DGP P.1), namely, a single break with a given break point and size. For the interior break point, all tests have power against DGP P.1, although Andrews' (1993) sup-LM-het is most powerful among all heteroscedasticity-robust tests. The $\hat{H}$-het test performs slightly better than LM-het, $UD\max$-het and qLL-het tests when T is small. Results for the single break at $t = 0.1T$ are report in the Supplemental Appendix. Here, $\hat{H}$-het outperforms LM-het, $UD\max$-het, sup-LM-het and qLL-het tests. We note that all homoscedasticity-specific tests are more powerful than their heteroscedasticity-robust counterparts, and $\hat{H}$ is more powerful than $\hat{C}$ under DGP P.1, confirming our theory.

Next, we consider multiple breaks. Under DGP P.2, the $\hat{H}$ and $\hat{C}$ tests dominate all other tests. Lin and Teräsvirta's LM test has no power even when $T = 500$. Bai and Perron's $UD\max$ test improve a lot upon Andrews' (1993) single break test, which confirms Perron's (2006) observation that "while the test for one break is consistent against alternatives involving multiple changes, its power in finite samples can be rather poor".

Under DGP P.3, the break lasts only for some period of time. The $\hat{H}$ and $\hat{C}$ tests outperform other tests for all sample sizes. Lin and Teräsvirta's LM test has low or little power. $UD\max$ and qLL tests perform slightly worse than ours but better than Andrews' (1993) sup-LM test.

DGP P.4 is an alternative with non-monotonic smooth structural changes. This is a STR model considered in Lin and Teräsvirta (1994), where the transition function is a second-order logistic function. Not surprisingly, Lin and Teräsvirta's LM test, which is based a first-order Taylor expansion, has no power. Our tests and qLL test outperform other tests.

Finally, we consider the alternative with unit root in parameters (DGP P.5). Again, the $\hat{H}$ test outperforms all other tests. The qLL test is slightly less powerful than $\hat{H}$, but more powerful than LM and sup-LM tests. In most cases, tests using bootstrap critical values have similar power to tests using ECVs.

To sum up, (i) the empirical sizes of the $\hat{H}$ and $\hat{C}$ tests are larger than the nominal levels, but they improve as the sample size increases. Under conditional homoscedastic errors, the homoscedasticity-specific tests, $\hat{H}$ and $\hat{C}$, have better sizes than heteroscedasticity-robust tests

$\hat{H}$-het and $\hat{C}$-het respectively. Under conditional heteroscedasticity errors, $\hat{H}$-het and $\hat{C}$-het continue to have reasonable levels, but homoscedasticity-specific tests strongly overreject the correct model. Other homoscedasticity-specific and heteroscedasticity-robust tests have similar patterns. (ii) Our tests have reasonable all-around power against both smooth and abrupt structural changes. They outperform all other tests in detecting smooth structural changes; they have good power against various multiple structural breaks, including the alternatives where the break occurs near the boundary of the sample period. (iii) Our tests are not always the most powerful in detecting each of the alternatives considered. However, they have relatively omnibus power against all five DGPs, provided the sample size is sufficiently large. Tests for parametric smooth structural changes are powerful against the specified alternatives but they may have low power if the polynomial order is not high enough; tests with the trimmed range also have a danger of omitting breaks occurring near the boundary of the sample period; and tests for single break may have rather poor power against alternatives involving multiple breaks. (iv) The generalized Hausman test $\hat{H}$ is more powerful than the generalized Chow test $\hat{C}$ in most cases, confirming our asymptotic theory. (v) The heteroscedasticity-robust generalized Hausman and Chow tests have similar power to their homoscedasticity-specific counterparts respectively in most cases. This feature is not shared by other tests.

7. STABILITY OF RETURN PREDICTION MODELS

Stock return predictability is an important yet controversial issue in empirical finance. Numerous studies document the predictability of stock returns using various lagged financial and macroeconomic variables, such as the dividend price ratio, earning price ratio, book to market ratio, term spread, default premium, interest rates, inflation rate as well as corporate payout and financing activity. Most existing works focus on in-sample tests.

A recent critique that challenges the conventional wisdom of return predictability emphasizes that predictive regressions have poor out-of-sample performance. Welch and Goyal (2008) show that all aforementioned financial and macroeconomic variables fail to yield better out-of-sample forecasts of the US equity premium than the simple historical mean equity returns. This striking finding triggers vigorous debates in the profession. One possible reason that significant in-sample evidence of predictability is often accompanied by weak or insignificant out-of-sample evidence of predictability is the existence of structural changes. Indeed, Clark and McCracken (2005) present analytical evidence on the effects of structural breaks on the tests for equal forecast accuracy and encompassing, as used in Welch and Goyal (2008), and show that out-of-sample predictive evidence can be harder to detect because the results of out-of-sample tests are highly dependent on the timing of the predictability.

We now use our tests to check whether the predictive regression of stock returns is stable over time. Some existing studies have considered structural breaks in the equity premium but results are mixed. For example, Kim, Morley and Nelson (2005) find a one-time structural break in the equity premium in the 1940s but no additional breaks in the postwar period. Paye and Timmermann (2006) examine the stability of return prediction models for 10 OECD countries. They find strong evidence against stability in a multivariate regression with the dividend yield, short rate, term spread and default spread, but in the univariate regressions, they find fairly weak evidence on instability in the dividend yield regression or default premium regression. Using Elliott and Müller's (2006) test, Rapach and Wohar (2006) cannot reject structural stability in 3 of the 8 predictive regressions (the price earning ratio, term spread and short rate) for S&P 500 returns. As emphasized by Paye and Timmermann (2006), all existing tests focus on occasional, large shifts in coefficients rather than a gradual evolution. We will avoid this restriction by using our tests, which have power against both smooth structural changes and sudden breaks.

We consider a standard predictive regression $Y_{t+1} = \alpha + \beta' X_t + \varepsilon_{t+1}$, where $Y_{t+1} = \log[(P_{t+1} + D_{t+1})/P_t] - r_t$, P_t is the S&P 500 index, D_t is the dividend paid on the S&P 500 index, r_t is the 3-month Treasury bill rate and X_t is a predetermined predictor. Following Welch and Goyal (2008) and Rapach, Strauss and Zhou (2009), we consider fourteen financial and economic variables: (i) *Log dividend price ratio (D/P):* the log difference between dividends and the S&P 500 index, where dividends are computed via a one year moving sum; (ii) *Log dividend yield (D/Y):* the log difference between dividends and the lagged S&P 500 index; (iii) *Log earnings price ratio (E/P):* the log difference between earnings and the lagged S&P 500 index, where earnings are computed via a one year moving sum; (iv) *Log dividend payout ratio (D/E):* the log difference between dividends and earnings; (v) *Stock variance (SVAR):* the sum of squared daily returns on the S&P 500 index; (vi) *Book-to-market ratio (B/M):* the ratio of book value to market value for the Dow Jones Industrial Average; (vii) *Net equity expansion (NTIS):* the ratio of twelve-month moving sums of net issues by NYSE-listed stocks to the total end-of-year market capitalization of NYSE stocks; (viii) *Treasury bill rate (TBL):* the 3 month treasury bill rate; (viiii) *Long-term yield (LTY):* the long-term government bond yield; (x) *Long-term return (LTR):* the return on long-term government bonds; (xi) *Term spread (TMS):* the difference between the long-term yield and the Treasury bill rate; (xii) *Default yield spread (DFY):* the difference between BAA- and AAA-rated corporate bond yields; (xiii) *Default return spread (DFR):* the difference between long-term corporate bond and long-term government bond returns; (xiiii) *Inflation (INFL):* the CPI-based inflation rate in the previous period. All data are from Welch and Goyal (2008).

We apply our tests to monthly and quarterly stock returns and compare them with the $UD\max$ and qLL tests, which have overall good finite sample performance in our simulation study and have been used in Paye and Timmermann (2006) and Rapach and Wohar (2006). We consider the post war sample: January 1947 to December 2005 and the post oil-shocks subsample:

January 1976 to December 2005.

Table 3 reports the bootstrap p-values of heteroscedasticity-robust and homoscedasticity-specific versions of our tests, $UD\max$ and qLL for univariate predictor regressions with each of the above fourteen predictors using monthly data. The bootstrap $p-$values, based on 499 bootstrap iterations, are computed as described in Section 6. For robustness, we mainly focus on heteroscedasticity-robust tests although homoscedasticity-specific tests generally yield smaller p-values. For the whole sample, we find strong evidence against the model stability for all predictors considered: all bootstrap $p-$values of our tests and qLL tests are smaller than 1%. It is also evident from the figures in the Supplemental Appendix that the nonparametric estimators of the slope coefficient β for all univariate predictor regressions do change over time. For the subsample of 1976 to 2005, our $\hat{H}$ test is able to reject the model stability of all predictors except E/P at the 10% level and all except D/P, D/Y, E/P and ITY at the 5% level. Bai and Perron's (1998) $UD\max$ and Elliott and Müller's (2006) qLL tests also yield strong rejection in the whole sample. However, for the post oil-shocks subsample, $UD\max$ cannot reject E/P and B/M, and qLL cannot reject D/P, D/Y and E/P at the 10% level.

Next, we test the stability of popular multivariate predictor models, including two bivariate predictor regressions with D/P and TBL, D/P and E/P, one trivariate predictor regression with D/P, E/P and TBL, one quadrivariate predictor regression with D/P, TBL, TMS and DFR. The bivariate and trivariate models have been studied by Ang and Bekaert (2007), and the quadrivariate model has been studied by Paye and Timmermann (2006). The strong evidence of model instability in the whole sample carries over to the multivariate predictor regressions. Our tests reject the stability of all multivariate models at all conventional significance levels; $UD\max$ is able to reject all multivariate models at the 5% level but qLL does not find structural break in the trivariate predictor regression. For the post oil-shocks subsample, our tests and $UD\max$ have similar p-values: they reject the bivariate predictor regression with D/P and TBL, the trivariate and quadrivariate predictor regressions at the 5% level, and the bivariate predictor regression with D/P and E/P at the 10% level. On the other hand, qLL test finds no evidence against model stability for bivariate predictor regressions.

Tables 9A in the Supplemental Appendix summarizes the results for quarterly data, where the evidence against stability is a bit weaker for univariate predictor regressions. For the whole sample, our heteroscedasticity-robust $\hat{H}$ test rejects all univariate predictor regressions except E/P, D/E, LTR and INFL at the 5% level; $UD\max$ cannot reject E/P and B/M and qLL cannot reject E/P, D/E and INFL at the 5% level. For the subsample, all tests can barely find evidence against model stability for univariate predictor regressions: our tests can only reject the null with D/P, $UD\max$ can only marginally reject the stability of DFR and qLL rejects none. We conjecture that the weak evidence against model stability is mainly due to the small sample size.

However, interestingly, our tests firmly reject the stability hypothesis for all multivariate models in both periods considered. In particular, the p-values of our tests are essentially 0 for the whole sample. In comparison, $UD\max$ is able to reject all except the quadrivariate model and qLL is only able to reject the bivariate predictor regression with D/P and E/P at the 10% level for the whole sample; for the subsample, qLL is able to reject the trivariate and quadrivariate models but $UD\max$ does not find any evidence against model stability.

To sum up, our tests strongly reject the stability of univariate and multivariate return prediction models in the post war and the post oil-shocks sample periods. Our findings support the argument of Rapach *et al.* (2009) that "model uncertainty and instability seriously impair the forecasting ability of individual predictive regression models". The rejection may be due to model misspecification and how to reconstruct the models needs further investigation.

8. CONCLUSION

Detection and identification of structural breaks have attracted a great amount of attention in econometrics over the past several decades. We have contributed to this literature by proposing a nonparametric Wald-type test for smooth structural changes as well as abrupt structural breaks. Our test has intuitive appeal because it can be regarded as the generalization of Hausman's (1978) test from a parametric context to a nonparametric context. It is asymptotically pivotal, does not require trimming data, does not require prior information on the alternative, and is consistent against all smooth structural changes as well as multiple abrupt structural breaks. Simulation studies show that the proposed test performs reasonably in finite samples. We apply the proposed test to stock return prediction models and find strong evidence against model stability.

Extensions of the proposed method to linear regression models with nonstationary regressors or serially correlated and endogenous errors with time-varying variances are possible. Moreover, our approach can be adopted to test whether an ARMA model or a GARCH model has smooth structural changes, using the log-likelihood criterion. Also, it can be used to test whether a time trend follows a polynomial of time, with the stochastic component being a weakly stationary but not necessarily m.d.s.. All these are left for future research.

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Table 1. Empirical Levels of Tests

	ε_t **~i.i.d.N(0,1)**			ε_t **~ARCH(1)**			$\varepsilon_t \mid X_t \sim N(0, f(X_t))$		
	100	250	500	100	250	500	100	250	500
BCV									
$\hat{H}$ **-het**	.058	.072	.038	.064	.050	.048	.058	.050	.062
$\hat{H}$	.066	.076	.040	.076	.048	.050	.086	.082	.084
$\hat{C}$ **-het**	.064	.060	.046	.058	.044	.056	.066	.052	.062
$\hat{C}$	.066	.066	.046	.070	.044	.060	.094	.074	.078
ACV									
$\hat{H}$ **-het**	.095	.078	.053	.116	.087	.066	.120	.092	.071
$\hat{H}$	.079	.071	.047	.097	.080	.062	.350	.428	.471
$\hat{C}$ **-het**	.066	.053	.042	.077	.060	.051	.081	.061	.048
$\hat{C}$	.052	.047	.035	.067	.050	.048	.243	.288	.328
LM-het	.043	.054	.048	.044	.047	.053	.045	.045	.050
LM	.046	.054	.049	.052	.048	.052	.150	.168	.177
Sup-LM-het	.018	.035	.043	.013	.029	.037	.010	.022	.034
Sup-LM	.029	.043	.045	.048	.050	.055	.205	.282	.330
UDMax-het	.138	.085	.067	.153	.082	.066	.260	.132	.096
UDMax	.051	.052	.050	.095	.072	.068	.338	.393	.431
qLL-het	.060	.050	.051	.081	.067	.055	.060	.058	.054
qLL	.065	.055	.052	.088	.074	.061	.454	.503	.514

Notes: (1) $\hat{H}$ and $\hat{C}$ are the generalized Hausman and Chow tests, LM is Lin and Teräsvirta's (1994) LM test based on the first-order Taylor expansion; Sup-LM is Andrews' (1993) supremum LM test; UDMax is Bai and Perron's (1998) double maximum test; qLL is Elliott and Müller's (2006) efficient test based on a "quasi local level" model. *-het denotes the heteroscedasticity-robust version of the corresponding * test. (2) BCV: bootstrap critical values; ACV: asymptotic critical values. (3) 5% significance level.

Table 2. Empirical Powers of Tests

	DGP P.1			DGP P.2			DGP P.3			DGP P.4			DGP P.5		
	Single Break			Multiple Breaks			Non-persistent Temporal Breaks			Smooth Changes			Unit-root-in-parameters		
	100	250	500	100	250	500	100	250	500	100	250	500	100	250	500
BCV															
$\hat{H}$ **-het**	.416	.848	.998	.288	.682	.948	.328	.762	.980	.414	.910	.998	.610	.988	.998
$\hat{H}$	.436	.856	.998	.304	.690	.956	.342	.788	.982	.454	.912	.998	.636	.988	.998
$\hat{C}$ **-het**	.330	.704	.962	.268	.664	.966	.324	.726	.946	.380	.804	.986	.550	.990	.998
$\hat{C}$	.340	.726	.968	.268	.682	.970	.334	.728	.958	.388	.828	.986	.568	.990	.998
ECV															
$\hat{H}$ **-het**	.416	.857	.990	.308	.646	.954	.319	.748	.990	.443	.884	.998	.617	.983	1.00
$\hat{H}$	.416	.857	.990	.306	.665	.957	.325	.765	.987	.452	.886	.998	.626	.987	1.00
$\hat{C}$ **-het**	.314	.728	.972	.276	.615	.942	.306	.693	.971	.378	.786	.992	.555	.982	1.00
$\hat{C}$	.352	.731	.972	.296	.621	.949	.330	.700	.977	.401	.793	.992	.564	.985	1.00
LM-het	.404	.814	.993	.052	.054	.061	.059	.058	.059	.065	.055	.059	.557	.889	.971
LM	.458	.853	.993	.053	.045	.061	.046	.050	.046	.074	.072	.063	.585	.896	.972
Sup-LM-het	.427	.888	.999	.115	.231	.478	.129	.330	.665	.215	.598	.942	.563	.945	.999
Sup-LM	.501	.923	1.00	.145	.286	.554	.143	.348	.708	.281	.675	.964	.623	.964	.999
UDMax-het	.393	.871	.996	.183	.400	.773	.236	.634	.973	.258	.683	.978	.524	.785	.966
UDMax	.494	.922	1.00	.240	.556	.914	.262	.736	.993	.345	.781	.989	.631	.815	.978
qLL-het	.376	.865	.996	.217	.637	.928	.222	.709	.968	.418	.892	.997	.613	.979	1.00
qLL	.428	.873	.996	.297	.657	.942	.324	.734	.976	.440	.897	.997	.645	.983	1.00

Notes: (1) $\hat{H}$ and $\hat{C}$ are the generalized Hausman and Chow tests, LM is Lin and Teräsvirta's (1994) LM test based on the first-order Taylor expansion; Sup-LM is Andrews' (1993) supremum LM test; UDMax is Bai and Perron's (1998) double maximum test; qLL is Elliott and Müller's (2006) efficient test based on a "quasi local level" model. *-het denotes the heteroscedasticity-robust version of the corresponding * test. (2) BCV: bootstrap critical values; ECV: empirical critical values. (3) 5% significance level.

Table 3. Stability test for monthly excess return

	$\hat{H}$ -het	$\hat{H}$	$\hat{C}$ -het	$\hat{C}$	UDmax-het	UDmax	qLL-het	qLL
Univariate Predictor Regressions								
	1947 January – 2005 December							
D/P	.000	.000	.000	.000	.000	.000	.000	.000
D/Y	.002	.002	.000	.000	.000	.000	.000	.000
E/P	.006	.006	.006	.004	.016	.000	.000	.000
D/E	.000	.000	.000	.000	.000	.000	.000	.000
SVAR	.000	.000	.000	.000	.000	.000	.000	.000
B/M	.000	.000	.000	.000	.000	.000	.000	.000
NTIS	.000	.000	.000	.000	.000	.000	.000	.000
TBL	.000	.000	.000	.000	.000	.000	.000	.000
LTY	.000	.000	.000	.000	.000	.000	.000	.000
LTR	.000	.000	.000	.000	.000	.000	.000	.000
TMS	.000	.000	.000	.000	.000	.000	.000	.000
DFY	.000	.000	.000	.000	.000	.000	.000	.000
DFR	.000	.000	.000	.000	.000	.000	.000	.000
INFL	.000	.000	.000	.000	.000	.000	.000	.000
	1976 January – 2005 December							
D/P	.058	.046	.016	.018	.052	.006	.459	.417
D/Y	.092	.088	.068	.056	.094	.012	.505	.425
E/P	.301	.293	.557	.559	.156	.060	.158	.150
D/E	.000	.000	.004	.002	.002	.000	.000	.000
SVAR	.000	.000	.000	.000	.010	.010	.000	.000
B/M	.036	.024	.014	.014	.110	.030	.048	.046
NTIS	.000	.000	.000	.000	.006	.000	.000	.000
TBL	.012	.010	.000	.000	.028	.004	.012	.014
LTY	.072	.066	.030	.030	.014	.002	.058	.066
LTR	.000	.000	.002	.000	.000	.000	.000	.000
TMS	.000	.000	.000	.000	.002	.000	.000	.000
DFY	.006	.004	.030	.030	.012	.000	.010	.008
DFR	.000	.000	.000	.000	.008	.002	.000	.000
INFL	.000	.000	.016	.006	.000	.000	.000	.000
Multivariate Predictor Regressions								
	1947 January – 2005 December							
Bi 1	.000	.000	.000	.000	.008	.000	.004	.004
Bi 2	.002	.000	.004	.002	.006	.000	.004	.004
Tri	.000	.000	.000	.000	.004	.000	.166	.130
Quadri	.000	.000	.000	.000	.030	.000	.014	.008
	1976 January – 2005 December							
Bi 1	.006	.006	.004	.002	.034	.010	.176	.116
Bi 2	.076	.074	.052	.044	.074	.036	.503	.471
Tri	.020	.024	.016	.016	.018	.006	.022	.026
Quadri	.042	.046	.022	.022	.012	.002	.022	.016

Notes: (1) $\hat{H}$ and $\hat{C}$ are the generalized Hausman and Chow tests, UDMax is Bai and Perron's (1998) double maximum test; qLL is Elliott and Müller's (2006) efficient test based on a "quasi local level" model. *-het denotes the heteroscedasticity-robust version of the corresponding * test. (2) Bootstrap p values with B=499.