High-Frequency Measurement of the Non-Gaussian Macro-finance Dynamics

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Lunch Presentation
Greenville, North Carolina, January 29th, 2014

Contribution to Time Series Modeling

One Project - Two Papers:

- **Paper I:** ***Develops a Generic Framework*** to estimate high-frequency economic dynamics using data sampled at mixed frequency.
- **Paper II:** ***Make the First Attempt*** to make optimal portfolio choice using irregularly spaced high-frequency data.

Motivation

- **Not** all economic data are sampled **at the same frequency**.
- **Unavailability** of macroeconomic variables at high frequency.
- **Increasing Demand** for estimating **high-frequency** economic dynamics.

Literature

1. **Ghysels et al. (2006): MIDAS Method**
   - Temporal Aggregation.
   - Linear Regression.
2. **Aruoba et al. (2009): ADS Index**
   - **Gaussian State-Space Model.**
   - **Kalman Filter.**

Methodology

- **Non-Gaussian State Space Model: Particle Filter**
  - *To allow for fat-tailed shocks to the observable variables.*
  - *The setting of the model can be easily extended to arbitrary number of variables observed at any frequency.*
- **Data at Mixed Frequency:** April 1, 1960 to February 20, 2007.
  1. ***Daily****: Term Spread*
  2. ***Weekly****: Initial Claims for Unemployment Insurance*
  3. ***Monthly****: Employees on Non-farm Payrolls*
  4. ***Quarterly****: Real GDP Growth*

Model Specification

Measurement Equation:

$$\underbrace{\begin{bmatrix} \tilde{y}_t^1 \\ \tilde{y}_t^2 \\ \tilde{y}_t^3 \\ \tilde{y}_t^4 \end{bmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{bmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \\ \beta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_4 \end{bmatrix}}_{\mathbf{Z}_t} \underbrace{\begin{bmatrix} x_t \\ C_{W,t} \\ C_{M,t} \\ C_{Q,t} \end{bmatrix}}_{\alpha_\mathbf{t}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ \gamma_2 & 0 & 0 \\ 0 & \gamma_3 & 0 \\ 0 & 0 & \gamma_4 \end{bmatrix}}_{\Gamma_t} \underbrace{\begin{bmatrix} \tilde{y}_{t-W}^2 \\ \tilde{y}_{t-M}^3 \\ \tilde{y}_{t-Q}^2 \end{bmatrix}}_{\omega_\mathbf{t}} + \underbrace{\begin{bmatrix} u_t^1 \\ \tilde{u}_t^2 \\ \tilde{u}_t^3 \\ \tilde{u}_t^4 \end{bmatrix}}_{\epsilon_\mathbf{t}}, \quad (1)$$

Transition Equation:

$$\underbrace{\begin{bmatrix} x_t \\ C_{W,t} \\ C_{M,t} \\ C_{Q,t} \end{bmatrix}}_{\alpha_\mathbf{t}} = \underbrace{\begin{bmatrix} \rho & 0 & 0 & 0 \\ \rho & \zeta_{W,t} & 0 & 0 \\ \rho & 0 & \zeta_{M,t} & 0 \\ \rho & 0 & 0 & \zeta_{Q,t} \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} x_{t-1} \\ C_{W,t-1} \\ C_{M,t-1} \\ C_{Q,t-1} \end{bmatrix}}_{\alpha_{\mathbf{t-1}}} + \underbrace{\begin{bmatrix} e_t \\ e_t \\ e_t \\ e_t \end{bmatrix}}_{\eta_\mathbf{t}}. \quad (2)$$

Model Specification

To reduce the number of state variables:

$$\begin{aligned} C_{W,t} &= \zeta_t C_{W,t-1} + x_t \\ &= \zeta_t C_{W,t-1} + \rho_1 x_{t-1} + \rho_2 x_{t-2} + \ldots + +\rho_p x_{t-p} + \xi_t \end{aligned}$$

$$\zeta_t = \begin{cases} 0 & \text{if } t \text{ is the first day of the week} \\ 1 & \text{otherwise} \end{cases}$$

Model Estimation

Non-Gaussian State Space Model: Particle Filter

Step I : The likelihood function of the observable variables can be decomposed.

$$f(\mathbf{Y_t}|\psi) = \prod_{s=1}^{t} f(\mathbf{Y_s}|\mathbf{Y}_{s-1}, \psi) = \prod_{s=1}^{t} \int f(\mathbf{Y_s}|\alpha_\mathbf{s}, \psi) f(\alpha_\mathbf{s}|\mathbf{Y}_{s-1}, \psi) d\alpha_\mathbf{s}.$$

where

$$f(\mathbf{Y_s}|\alpha_\mathbf{s}, \psi) = St(\mathbf{Y_s}|\mathbf{Z_s}\alpha_\mathbf{s} + \Gamma_s \omega_\mathbf{s}, \mathbf{H_s}, \lambda)$$

is the Student's-t density function with mean $\mathbf{Z_t}\alpha_\mathbf{t} + \Gamma_t \omega_\mathbf{t}$, variance $\mathbf{H_t}$ and λ degrees of freedom.

Model Estimation

Non-Gaussian State Space Model: Particle Filter

Step II : For each t, the particle filter delivers a sample of draws on $\alpha_{\mathbf{t}}$ from the filtered distribution $f(\alpha_{\mathbf{t}}|\mathbf{Y_{t-1}}, \psi)$. These draws allow to estimate the one-step ahead density of $\mathbf{y_t}$:

$$f(\mathbf{y_t}|\mathbf{Y_{t-1}}, \psi) = \int St(\mathbf{Y_t}|\mathbf{Z_t}\alpha_{\mathbf{t}}+\Gamma_t\omega_{\mathbf{t}}, \mathbf{H_t}, \lambda)f(\alpha_{\mathbf{t}}|\mathbf{Y_{t-1}}, \psi)d\alpha_{\mathbf{t}}.$$

by simple Monte-Carlo averaging of $St(\mathbf{Y_t}|\mathbf{Z_t}\alpha_{\mathbf{t}}+\Gamma_t\omega_{\mathbf{t}}, \mathbf{H_t}, \lambda)$ over the draws of $\alpha_{\mathbf{t}}$ from $f(\alpha_{\mathbf{t}}|\mathbf{Y_{t-1}}, \psi)$.

Model Estimation

Non-Gaussian State Space Model: **Particle Filter**

Step III : In particular, I consider the auxiliary particle filter introduced in Chib et al. (2002).

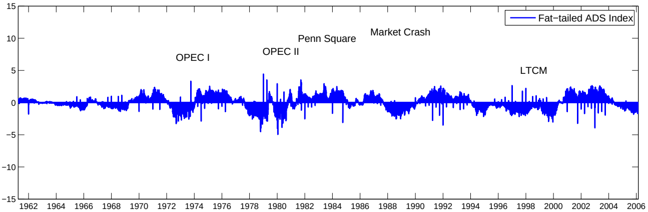
- This filter first creates a group of proposal values $\alpha_{\mathbf{t}}^{\mathbf{1}}, \ldots, \alpha_{\mathbf{t}}^{\mathbf{R}}$.
- These values are then reweighted to produce draws $\{\alpha_{\mathbf{t}}^{\mathbf{1}}, \ldots, \alpha_{\mathbf{t}}^{\mathbf{M}}\}$ that correspond to draws from the target distribution.
- Typically, we take **R** to be five or ten times larger than **M**.

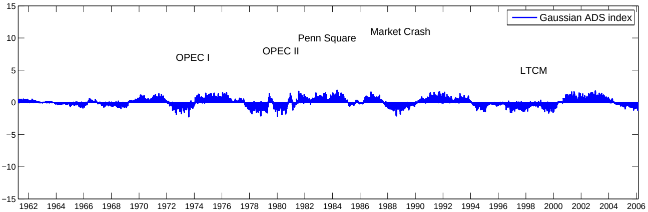
Computation Challenge & Solution

- **Challenge I: Data Intensive at High Frequency**
  Data intensity increases as the data frequency becomes higher.
  The time consumed in iteration increases the computation complexity.
- **Challenge II: Large Number of Loops**
  Particle Filter requires thousands of sweeps in each iteration.
  A simple parallel computation cannot improve the computation efficiency.
- **Solution: Introducing MEX-Files in Matlab**
  Replace Loops with High-Dimensional Array Operation.

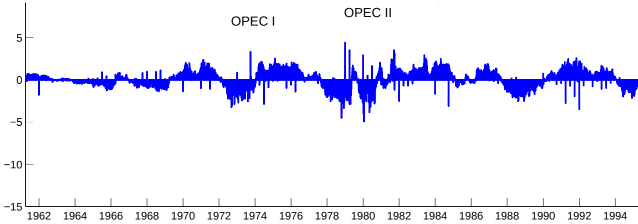
Extracted Daily Index

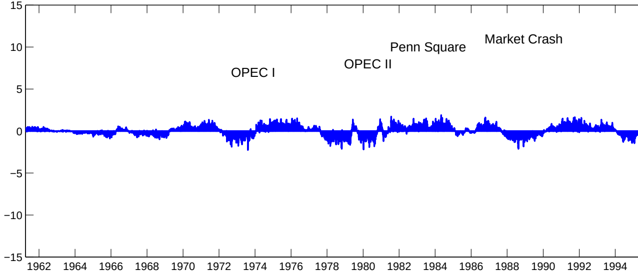
Figure 3.1 Daily Business Conditions Index: Fat-tail vs. Gaussian



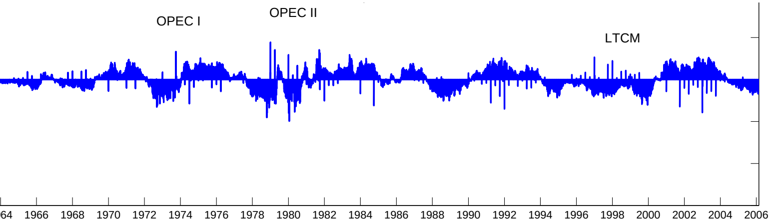


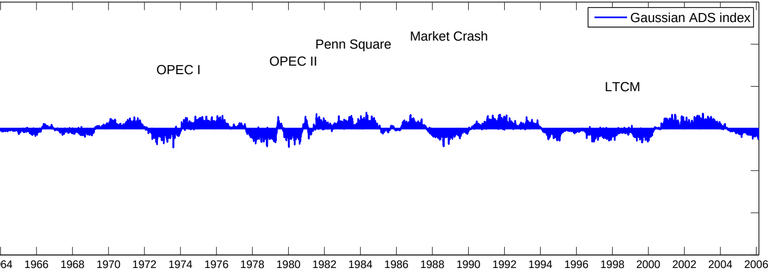
Extracted Daily Index





Extracted Daily Index





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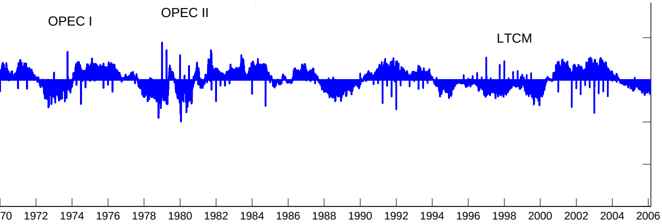

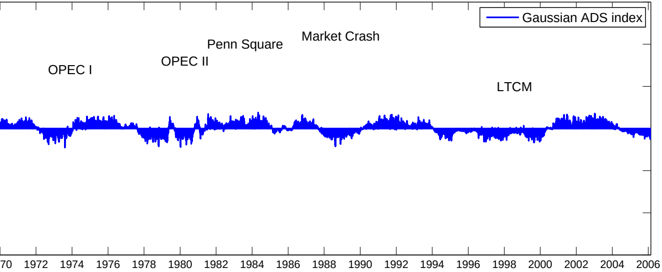

Reference

Aruoba, S Borağan, Francis X Diebold, and Chiara Scotti, "Real-time measurement of business conditions," *Journal of Business & Economic Statistics*, 2009, *27* (4), 417–427.

Bai, Jennie, Eric Ghysels, and Jonathan Wright, "State space models and MIDAS regressions," *Econometric Reviews*, 2009, *45*.

Chib, Siddhartha, Federico Nardari, and Neil Shephard, "Markov chain Monte Carlo methods for stochastic volatility models," *Journal of Econometrics*, 2002, *108* (2), 281–316.

Ghysels, Eric, Pedro Santa-Clara, and Rossen Valkanov, "Predicting volatility: getting the most out of return data sampled at different frequencies," *Journal of Econometrics*, 2006, *131* (1), 59–95.