

Informational Content of Equivalence Scales based on Minimum Needs Income¹

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ABSTRACT

Uniform equivalence scales are routinely used for welfare comparisons and imply that utility function is IB/ESE (independent of base / equivalent scale exact). This condition imposes restrictions on the level of measurability and interpersonal comparability of preferences across households, so called informational basis, in that welfare ordering is Ordinal and Fully Comparable (OFC). We show that if one calculates equivalence scale at particular utility level, for example households living in poverty, the required informational basis is much weaker and requires full comparability only at a single point. Therefore, we introduce axiom of Ordinal Local Comparability (OLC) and show that equivalence scale based on Minimum Needs Income satisfies that axiom. We argue that subjective equivalence scale using the intersection method offers practical application of equivalence scale satisfying OLC.

KEY WORDS: Informational Basis, Poverty, Equivalence Scales, Subjective Poverty Line, Local-comparability, Intersection Method

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1. INTRODUCTION

The proper derivation and use of equivalence scales is a key element in conducting an appropriate analysis of inequality, poverty, or welfare in a particular society. Well-defined equivalence scales can adjust for economies of scale within the family and thus allow for cross-household and cross-person comparisons (Coutler et al., 1992). Because equivalence scale involves adjustment of income so that households of different types can achieve the same level of utility, each approach to deriving equivalence scale implicitly assumes certain properties of the utility function and social welfare functional. Of particular importance are conditions required for effective comparisons between households which are summarized by the informational basis of welfare ordering. The objective in this paper is to analyze the informational content underlying the equivalent scales calculated for a single group of households at a particular level of welfare and characterized by the same minimum needs income.

For practical reasons the most common approach is to use uniform equivalence scales where one assumes the same required adjustment of income across all households with different utility levels. Lewbel (1989) and Blackorby and Donaldson (1991, 1993) independently showed that in order to use uniform equivalence scale the household preferences must be consistent with IB/ESE (independent of base / equivalent scale exactness) condition. However, IB/ESE requires that the information structure supports interhousehold comparisons of utility levels and thus it implies welfare profiles to be Ordinal and Full Comparable (OFC). Such condition imposes restrictions on the preferences which may be difficult to satisfy or even verify.

Because OFC requires information about utility levels for members of the household, which is normally unobservable, Blackorby and Donaldson (1993) proposes a practical solution where one can derive equivalence scale for a single

reference group (ex. poverty utility level). We show that such approach implies Ordinal Local Comparability (OLC) and puts minimal restrictions on household preferences. Further, we demonstrate that subjective scale based on Minimum Income Needs Question (MINQ) proposed by Goedhart et al. (1977) satisfies OLC and can be regarded as theoretically sound and robust to most possible utility functions, and yet empirically viable and identifiable.

The paper is organized as follows. Section 2 introduces axiom of *ordinality and local comparability* (OLC), Section 3 presents assumptions underlying MINQ-based subjective equivalence scales derived using the intersection method and shows that they are consistent with axiom of OLC, and Section 4 concludes.

2. ORDINAL AND LOCAL COMPARABLE INFORMATIONAL BASIS

We begin by discussing Sen's underlying *informational basis* of household profiles (Sen, 1974), which is defined by a set of restrictions on the properties of the household utility profiles. They inform the social planner about available intra-household or inter-household utility comparisons. If intra-household cardinal comparisons of utility are disallowed then it is called *ordinal* setting. If inter-household cardinal comparisons are disallowed then we have an inter-household *non-comparable* setting. Therefore *informational basis* is characterized by a mixture of *measurability* assumptions concerning the degree cardinality of household utility, and *comparability* assumptions about the precise description of the degree of inter-household comparability of utilities.

Modern positive theory formalizes the restrictions imposed on the social welfare functionals, and eventually on the household utility profiles, by using *invariance axioms*. These conditions on the social preference orderings guarantee that the orderings

are insensitive to transforms of the utility profiles. In this section we introduce new axiom of *ordinality and local-comparability* (OLC), which can be viewed as an intermediate case between the axioms of *ordinality and non-comparability* (ONC) (Arrow, 1963) and *ordinality and full-comparability* (OFC) (D'Aspremont and Gevers, 1977, 2003) when a particular “data filter” for relevant information is applied (Fleurbaey, 2003, p. 350). In our case the only relevant information is contained in utility comparison for households who have incomes equal to poverty line.

2.1. Notation

Lets assume population of N households, characterized by two attributes: the total actual income of the household, $y \in \mathbb{R}_+$, and the level of needs captured by a set of welfare-relevant “non-income” characteristics, α (i.e., household size and composition). Suppose we partition the population into n disjoint groups, according to their level of needs and that the needs can be ranked, $\alpha = 1, 2, \dots, n$ ($1 \leq n \leq N$). The set of all possible characteristics is denoted by Φ . This allows us to decompose the whole population into disjoint and exhaustive homogeneous subgroups, ordered in ascending order according to their needs (from less needed to more needed groups). That is, if $\alpha > \bar{\alpha}$ then the level of needs in the group α is greater that in that of $\bar{\alpha}$. The only difference within any subgroup is actual income, y .

Further, let us assume that differences in needs are incorporated in the indirect utility functions $V: \mathbb{R}_+^{m+1} \cup \Phi \rightarrow \mathbb{R}$, whose typical image $V(p, y, \alpha)$, indicates the indirect utility associated with a household with income y in the group α , facing an m -vector of prices. We assume that any two households with the same needs have identical preferences. We also assume V is continuous and increasing function in income, $\Delta V(p, y, \alpha) / \Delta y > 0$, with the property that $V(p, y, \alpha) < V(p, y, \bar{\alpha})$, $\alpha \neq \bar{\alpha}$, and if $\alpha >$

$\bar{\alpha}$, for all $p \in \mathbb{R}_{++}^m$, $y \in \mathbb{R}_+$ and $\alpha, \bar{\alpha} \in \Phi$. It means that the utility associated with households in a group decreases with the levels of needs, that is, $\Delta V(p, y, \alpha)/\Delta \alpha < 0$.³

2.2. Equivalence Scales

From inverting $V(p, y, \alpha)$, we obtain the expenditure functions $e(p, u, \alpha)$, giving the minimum cost of utility $u = V(p, y, \alpha)$. From consumption duality we obtain $y = e(p, u, \alpha)$ where e is the expenditure required to achieve the level of utility u , for a household of needs α , facing prices p . The equivalence scale $ES(p, u, \alpha)$ is the relative cost of being in a household with needs α relative to the benchmark household with needs α^r (say single adult), while maintaining the same level of utility u . Assuming all households face the same prices, the equivalence scale satisfies:

$$ES(p, u, \alpha) = \frac{e(p, u, \alpha)}{e(p, u, \alpha^r)} \quad (1)$$

The function $ES(p, u, \alpha)$ is then the equivalence scale of household with characteristic α with respect to the reference household with characteristic α^r , having income y and utility u .

The function $ES(p, u, \alpha)$ can be implicitly defined in terms of the indirect utility function as the value d such that:

$$u = V(p, y, \alpha) = V(p, \frac{y}{d}, \alpha^r) \quad (2)$$

³We also assume this technical assumption as in Blackorby and Donaldson (1991). for all $p \in \mathbb{R}_{++}^m$, $y \in \mathbb{R}_+$ and $\alpha, \bar{\alpha} \in \Phi$, there exist $\bar{y} \in \mathbb{R}_+$, such that: $V(p, y, \alpha) = V(p, \bar{y}, \bar{\alpha})$. Note that it is unique because V is increasing in y .

2.3. Informational basis

Note that the value of $d = ES(p, u, \alpha)$ in equation (2), is only meaningful up to those monotone transformations of utility for which d does not change. The set of monotone transformations that keep d unchanged characterizes the informational basis, which describes the degree of measurability and comparability of the level of utilities among households.

For the least restrictive case that still provides informative equivalence scales we require at least ordinal utility framework (utility is equivalent up to any monotone transformation within the households) and allow for some comparability among households (some restrictions on inter-households comparability must be done). These conditions provide enough information to specify informational basis.

Ordinal local-comparability axiom (OLC). The value of $d = ES(p, \bar{u}, \alpha)$ does not change, if for all α, p and y , V is replaced by \tilde{V} :

$$\tilde{V}(p, y, \alpha) = \varphi(V(p, y, \alpha)), \quad V = \bar{u}$$

and

$$\tilde{V}(p, y, \alpha) = \varphi_\alpha(V(p, y, \alpha)), \quad V \neq \bar{u}$$

where φ and φ_α are increasing functions.

The underlying informational basis is very general and supports ordinal and local inter-household comparability of utilities at a particular utility level.

In this case, intra-household ordinality is satisfied because for the same household and for all $(p, \bar{p}, y, \bar{y}, \alpha)$:

$$V(p, y, \alpha) = V(\bar{p}, \bar{y}, \alpha) \Leftrightarrow \varphi(V(p, y, \alpha)) = \varphi(V(\bar{p}, \bar{y}, \alpha)), \quad V = \bar{u}$$

$$V(p, y, \alpha) \geq V(\bar{p}, \bar{y}, \alpha) \Leftrightarrow \varphi_\alpha(V(p, y, \alpha)) \geq \varphi_\alpha(V(\bar{p}, \bar{y}, \alpha)), \quad V \neq \bar{u}$$

Moreover, when you compare different households, inter-household level comparability is satisfied at $V = \bar{u}$ because for all $(p, \bar{p}, y, \bar{y}, \alpha, \bar{\alpha})$:⁴

$$V(p, y, \alpha) = V(\bar{p}, \bar{y}, \bar{\alpha}) \Leftrightarrow \varphi(V(p, y, \alpha)) = \varphi(V(\bar{p}, \bar{y}, \bar{\alpha})), \quad V = \bar{u}$$

but it is not satisfied at $V \neq \bar{u}$ because for all $(p, \bar{p}, y, \bar{y}, \alpha, \bar{\alpha})$:

$$V(p, y, \alpha) \geq V(\bar{p}, \bar{y}, \bar{\alpha}) \Leftrightarrow \varphi_\alpha(V(p, y, \alpha)) \not\geq \varphi_{\bar{\alpha}}(V(\bar{p}, \bar{y}, \bar{\alpha})), \quad V \neq \bar{u}.$$

The class of monotone transformations under this condition is wider and it requires much less information than under the standard conditions assumed in the literature. Standard uniform equivalence scale $ES(p, u, \alpha)$ in equation (1) is constant for all levels of u . Blackorby and Donaldson (1991, 1993) showed that it implies *ordinal full-comparability* (OFC). Therefore, the class of monotone transformations required under OFC is the following (Blackorby and Donaldson, 1991, 1993, d'Aspremont and Gevers, 2003 and Bossert and Weymark, 2004):⁵

$$\bar{V}(p, y, \alpha) = \varphi(V(p, y, \alpha)), \quad \text{for all } V$$

⁴ The reader will note that under this specification, full inter-household level comparability is also satisfied for household belonging to the same group. Less demanding informational basis is still possible by admitting individual specific monotone transformations.

⁵This condition is implicit in Sen (1974). Hammond (1976) and d'Aspremont and Gevers (1977) call it *cardinality*. Roberts (1980) and Fleurbaey and Hammond (2004) call it *ordinality and level comparability* because it allows for inter-household comparison of utility levels but rules out comparability of utility differences.

which clearly describe a more restrictive set of functions. In this case, intra-household ordinality is satisfied because for all $(p, \bar{p}, y, \bar{y}, \alpha)$:

$$V(p, y, \alpha) \geq V(\bar{p}, \bar{y}, \alpha) \Leftrightarrow \varphi(V(p, y, \alpha)) \geq \varphi(V(\bar{p}, \bar{y}, \alpha)),$$

and inter-household level comparability is satisfied because for all $(p, \bar{p}, y, \bar{y}, \alpha, \bar{\alpha})$:⁶

$$V(p, y, \alpha) \geq V(\bar{p}, \bar{y}, \bar{\alpha}) \Leftrightarrow \varphi(V(p, y, \alpha)) \geq \varphi(V(\bar{p}, \bar{y}, \bar{\alpha})),$$

Furthermore, if we have to keep $ES(p, u, \alpha)$ unchanged for all utility levels,⁷ as standard equivalence scales do, we need to restrict the class of ordinal utility even further to the class whose expenditure functions can be multiplicatively written as, for all (p, u, α) , (Lewbel, 1989):

$$e(p, u, \alpha) = \phi(p, u)\hat{e}(p, \alpha)$$

Although this is a restrictive condition, it is still wider than homotheticity, which in turn is equivalent to:

$$e(p, u, \alpha) = \phi(u)\hat{e}(p, \alpha)$$

Note that in this case, $ES(p, u, \alpha)$ does not depend on u :

⁶Note that to incorporate inter-household difference comparability, so that for all $(A, B, \alpha, \bar{\alpha})$:

$$V(A, \alpha) - V(B, \alpha) \geq V(A, \bar{\alpha}) - V(B, \bar{\alpha}) \Leftrightarrow \bar{V}(A, \alpha) - \bar{V}(B, \alpha) \geq \bar{V}(A, \bar{\alpha}) - \bar{V}(B, \bar{\alpha})$$

a stronger informational framework is required which guarantees invariance under common affine transformations.

⁷ Such condition is also called Independence of Base (IB) introduced by Lewbel (1989) or Equivalent-Scale Exactness (ESE) proposed by Blackorby and Donaldson (1991,1993).

$$ES(p, u, \alpha) = \frac{\hat{e}(p, \alpha)}{\hat{e}(p, \alpha^r)}$$

Alternatively, we can express this condition in terms of the indirect utility functions (the income-ratio comparability condition by Blackorby and Donaldson, 1991), for all $(p, \bar{y}, \tilde{y}, \bar{\alpha}, \tilde{\alpha})$:

$$V(p, \bar{y}, \bar{\alpha}) = V(p, \tilde{y}, \tilde{\alpha}) \Leftrightarrow V(p, \lambda \bar{y}, \bar{\alpha}) = V(p, \lambda \tilde{y}, \tilde{\alpha}) \quad \text{for } \lambda > 0$$

Note that the informational framework described by OLC is more demanding than in the Arrovian *ordinal non-comparability* case (ONC) where the more general transformations are allowed:

$$\hat{V}(p, y, \alpha) = \varphi_\alpha(V(p, y, \alpha)), \quad \text{for all } V$$

Note that even though it is desirable to construct equivalence scales satisfying only ONC, it is not possible because by definition some degree of comparability is required.

2.4. Practical solution

Under our framework for single equivalence scale we opt for a practical solution suggested by Blackorby and Donaldson (1993) which can be very relevant for policy analysis and for identification of the demand system. We compare utility levels at a particular level where it is required to “make ends meet,” that is, when $\bar{u} = u_{min}$. Then we obtain:

$$ES(p, u_{min}, \alpha) = \frac{e(p, u_{min}, \alpha)}{e(p, u_{min}, \alpha^r)} \quad (3)$$

We assume that at the poverty line all households have the same comparable utility.

Assumption 1 *Poverty equivalent utility*

At the poverty line $y_{min}^*(\alpha)$, all households attain same utility. For all p and $\alpha \in \Phi$:

$$u_{min} = V(p, y_{min}^*(\alpha), \alpha)$$

Note that the benefit of adopting assumption 1 in the context of OLC is that we can write equivalent scales in equation (3) as:

$$ES(p, u_{min}, \alpha) = \frac{e(p, u_{min}, \alpha)}{e(p, u_{min}, \alpha^r)} = \frac{y_{min}^*(p, \alpha)}{y_{min}^*(p, \alpha^r)} \quad (4)$$

Specification in (4) turns out very useful because it is sufficient to deal with the identification problem common in estimation of the demand system. Blundell and Lewbel (1991) showed that with the use of the demand data we can at the most estimate preferences over goods conditioned on households characteristics, but we cannot infer anything about preferences over characteristics themselves. In other words, demand system estimation does not capture an important aspect of the demographic characteristic costs. However, they showed that we can decompose:

$$ES(p, u_{min}, \alpha) = \frac{e(p, u_{min}, \alpha)}{e(p, u_{min}, \alpha^r)} = \frac{e(p, u_{min}, \alpha)/e(p^r, u_{min}, \alpha)}{e(p, u_{min}, \alpha^r)/e(p^r, u_{min}, \alpha^r)} \frac{e(p^r, u_{min}, \alpha)}{e(p^r, u_{min}, \alpha^r)}$$

It follows from this equation that the equivalence scale in price regime p equals the product of a ratio of household-specific cost of living indices, which is identified from demand data alone, and the equivalence scale in the price reference regime p^r , which cannot be estimated. Notice that the second component is equivalent to equation (4) and therefore its existence resolves the identification problem. In fact, Blundell and Lewbel (1991) suggested using external information using subjective data to provide information in (4) and our paper provides such alternative approach.

We devote the rest of the paper to identify $y_{min}^*(p, \alpha)$ for all $\alpha \in \Phi$ using subjective data. We formulate the intersection method proposed by Goedhart *et al.* (1977), using minimum needs income subjective data.

3. MINQ-BASED DATA AND THE POVERTY LINE: THE INTERSECTION METHOD

The derivation of the poverty line using subjective questions was first proposed by Goedhart *et al.* (1977). They introduce two approaches: *Leyden Poverty Line* (LPL) based on multi-level question, and *Subjective Poverty Line* (SPL) based on a one-level question. Even though Flik and Van Praag (1991) argue that LPL is theoretically superior to the SPL, we suggest that SPL may be preferred because it is less restrictive and can easily be integrated with any other study which requires specification of the utility function of income.

3.1. Subjective Poverty Line

The SPL is based on the answer to the minimum needs question (MINQ): "what is the minimum income that you would have to have to make ends meet?" Even though for a given group of households the answers to MINQ vary due to "misperception," Goedhart et al. (1977) argued that there is a systematic relationship between answers to MINQ and an actual income⁸. We further call it perception error. In particular, those with actual income above their minimum income overestimate the poverty threshold, whereas those with actual income below their minimum income underestimate the poverty threshold. Therefore, according to their argument only those whose actual income equals minimum needs income would answer MINQ correctly, which in turn becomes a definition of the poverty line. The intuition is that at this level of income the household does not have any savings nor debt during the survey period, but any shortfall from this level would push them into poverty.

Unfortunately because most samples do not include households whose actual income equals the answer to MINQ, or their numbers in the population may be very limited, it is not possible to directly use answers to MINQ in formulation of the subjective poverty line. However, Goedhart et al. (1977) shows that by modeling the perception error in MINQ one can use data from the entire survey to estimate the correct answer to MINQ (true minimum needs income). The objective of their approach is to find such actual income level where households would not make a perception error when reporting MINQ. The technique is called the *intersection method* because graphically the poverty line is at the point of intersection between the line representing actual answers to MINQ conditional on actual income, and the line representing

⁸ In the discussion of subjective utility evaluations Roberts (1997) suggests that households may make "mistakes" when comparing themselves to those that are different from them, say poor vs. non-poor, but their opinion may correctly reflect objective comparisons when they compare themselves to households which are similar, say all households that have just enough income to meet their needs.

hypothetical answers to MINQ as if they would always equal to the actual income, 45 degree line (see figure 1). A formal presentation of the intersection method follows.

[Figure 1 about here]

3.2. Intersection method

Suppose that for all households in the subpopulation with α characteristics, there exists unique true minimum needs income (poverty line) which is unobservable, defined as $y_{min}^*(\alpha)$. What is observable are the answers to MINQ which are “distorted by the fact that (respondent’s) actual income is not equal to his minimum income” (Goedhard et al. 1977, p. 514). Thus, to represent actual answers to MINQ we define minimum needs income perception function (IPF) of subpopulation α , $f_p(y, \alpha)$, $f_p: [0, y_{max}(\alpha)] \rightarrow [0, y_{max}(\alpha)]$, where $y_{max}(\alpha)$ is the observed maximum income of the group α ; which depends on actual income y , assumed to be continuous in y with $\frac{\Delta f_p(y, \alpha)}{\Delta y} > 0$. The difference between IPF and $y_{min}^*(\alpha)$ is the perception error which is systematic, such that poor households have $f_p(y, \alpha) - y_{min}^*(\alpha) < 0$, and non-poor households have $f_p(y, \alpha) - y_{min}^*(\alpha) > 0$.

It has to be noted that the perception error is different than random error, which is present when estimating IPF using survey data. If we denote $y_{min}(\alpha)$ as the reported answer to MINQ in survey data, the random error is the difference: $y_{min}(\alpha) - f_p(y, \alpha)$. Therefore, the full decomposition of the errors becomes (see Figure 1):

$$y_{min}(\alpha) - y_{min}^*(\alpha) = [f_p(y, \alpha) - y_{min}^*(\alpha)] + [y_{min}(\alpha) - f_p(y, \alpha)] \quad (5)$$

total reported error = perception (systematic) error + random error

Throughout the paper we ignore random error (assume it is equal to zero) because it is data-specific and the derivation of SPL does not depend on the properties of the random error. In other words, modeling random error is the subject of econometric specification, which is not the focus of the present paper.

The objective of the intersection method is to use answers to MINQ in order to find the unobserved minimum needs income, $y_{min}^*(\alpha)$. The ingenuity of the approach is that we can find such income even if there are no households in the data for whom $y = y_{min}^*(\alpha)$ (otherwise the solution is obvious). Following the specification in Kapteyn, Kooreman, and Willemse (1988), the method is based on the existence of IPF such that $y_{min}^*(\alpha)$ is the unique solution to:

$$y_{min}^*(\alpha) = f_p(y_{min}^*(\alpha), \alpha) \quad (6)$$

where for $y < y_{min}^*(\alpha)$ we have that $y < y_{min}(\alpha)$ and for $y > y_{min}^*(\alpha)$ we have that $y > y_{min}(\alpha)$. Because there is no random error it means that for $y > y_{min}^*(\alpha)$ we have $y < f_p(y, \alpha)$ and for $y < y_{min}^*(\alpha)$ we have $y > f_p(y, \alpha)$. Solution to (6) can be presented in Figure 1, where vertical axis represents values of $f_p(y, \alpha)$ (assumed linear for demonstration purposes) conditional on actual income y (horizontal axis). The 45 degree line includes all the points where the condition $y = y_{min}^*(\alpha)$ is satisfied. Therefore intersection of 45 degree line with the function $f_p(y, \alpha)$ is the solution to the problem in (6).

In practice, because of existence of random error, one needs econometric model to estimate IPF. However, the unique feature of SPL is that the functional form of IPF is irrelevant as long as it is monotonically increasing in both y , and its distance from actual income is always smaller than the distance between actual income and true minimum

needs income, $|y - f_p(y, \alpha)| < |y - y_{min}^*(\alpha)|$. We want to note that it is a very important distinction between SPL and LPL because the functional form of IPF in LPL is the double-log specification, which is derived from the assumed functional form of WFI.

3.3. Formalization of the intersection method

In the following we propose a formalization to justify the existence of the SPL which are implied by the methods originally presented in Goedhart et al. (1977). The focus is on demonstrating that those methods do not restrict the underlying utility function to any particular functional form and thus ordinal utility specification is acceptable. In other words, the intersection method is consistent with OLC. The second assumption deals with the properties of the IPF.

Assumption 2: *Minimum Income Perception.* $\forall y \in \mathbb{R}_+$ and $\forall \alpha \in \Phi$:

$$y \geq y_{min}^*(\alpha) \Rightarrow y \geq f_p(y, \alpha) \geq y_{min}^*(\alpha) \quad (7a)$$

and

$$y \leq y_{min}^*(\alpha) \Rightarrow y \leq f_p(y, \alpha) \leq y_{min}^*(\alpha) \quad (7b)$$

(The \Leftarrow is obvious and is omitted). It says that those households with actual income below their minimum income underestimate the poverty threshold, whereas those with actual income above their minimum income overestimate the poverty threshold.

The intuition is that poor households at the very least must be aware that they are poor ($y \leq f_p(y, \alpha)$) and they always underestimate the degree of their poverty ($f_p(y, \alpha) \leq y_{min}^*(\alpha)$).

A direct implication of assumption 1 is that for any particular income value $y \in \mathbb{R}_+$, one of the following expressions is correct:

$$y > f_p(y, \alpha) > y_{min}^*(\alpha), \quad (8a)$$

$$y = f_p(y, \alpha) = y_{min}^*(\alpha) \quad (8b)$$

$$y < f_p(y, \alpha) < y_{min}^*(\alpha) \quad (8c)$$

The interpretation of assumption 1 is that even though IPF can take very flexible functional forms, it has to be restricted to guarantee unique solution to (6), it cannot be multiple intersection points.

Proposition 1: $y_{min}^*(\alpha)$ always exists.

Proof: Define the perception function $f_p: [0, y_{max}(\alpha)] \rightarrow [0, y_{max}(\alpha)]$ as a continuous mapping of a convex bounded set in itself. Brouwer's fixed-point theorem guarantees that there exist at least one fixed-point y_0 for all α such that $f_p(y_0, \alpha) = y_0$. Then $y_0 = f_p(y_0, \alpha) = y_{min}^*(\alpha)$.

Moreover, if we assume $0 \leq \frac{\Delta f_p(y, \alpha)}{\Delta y} < 1, \forall y \in \mathbb{R}_+$ and $\forall \alpha \in \Phi$; $y_{min}^*(\alpha)$ is unique.

It implies that under this condition we can identify the poverty line $y_{min}^*(\alpha)$ for every needs group α , from subjective MINQ-based data; and obtain the underlying equivalence scales by substituting the $y_{min}^*(\alpha)$ values in equation (last one in section 2). Recall that by assumption 1, at the poverty lines, two households with different needs α and $\bar{\alpha}$ attain same (ordinal and comparable) utility:

$$V(p, y_{min}^*(\alpha), \alpha) = V(p, y_{min}^*(\bar{\alpha}), \bar{\alpha}) \quad (9)$$

Note that if $\alpha < \bar{\alpha}$ then $y_{min}^*(\alpha) < y_{min}^*(\bar{\alpha})$ due to $\Delta V/\Delta y > 0$ and $\Delta V/\Delta \alpha < 0$. Thus, the method imposes higher poverty lines for households with higher needs. The equivalence scale for group $\bar{\alpha}$ with respect to group α at the poverty threshold is just $y_{min}^*(\bar{\alpha})/y_{min}^*(\alpha)$. See Figure 2 for an illustration.

Moreover, if you restrict the class of the utility functions to those under the Independence of Base (IB) or the Equivalent-Scale Exactness (ESE) proposed by Lewbel (1989) and Blackorby and Donaldson (1991,1993), respectively, you can extend those equivalence scales to the rest of the population.

3. CONCLUSION

Uniform equivalence scales, which assume the same adjustment of incomes across households with different utility levels, are convenient practical simplification used in empirical studies. However, such approach imposes IB/ESE condition which implies Ordinal Full Comparability (OFC) of utility profiles. In turn, it limits the possibilities of preferences that can be used. We demonstrate that an approach based on using single utility level to calculate equivalence scale requires much weaker condition. In particular, we focus only on households living at the poverty level income. Such approach only needs to satisfy the axiom of Ordinal and Local Comparability (OLC) that allows for non-comparable preferences except for a single utility level.

One concern with using equivalence scale calculated for a single subgroup is that it ignores the information about the rest of the population. In other words, one may argue

that such approach does not allow for generalization required to perform income comparisons across households with different utility levels. However, if the preferences satisfy IB/ESE then a single equivalence scale is automatically correct for the entire population. This conclusion relates to the fact that preference ordering satisfying OFC also satisfies OLC. The benefit of OLC, however, is that it does not require particular IB/ESE preferences.

Even if one would reject IB/ESE and argue that there are equivalence scales at each level of utility, there is still a benefit of using minimum needs income approach because often times welfare policy focuses on particular subgroup of the population. Consider Rawls social welfare functional which is primarily concerned about the welfare of poor. Therefore, the information about the most efficient way to improve the situation of the households at the bottom of the income distribution is particularly useful, even if it ignores the rest of the population.

An additional benefit of minimum needs income equivalence scales is the ability to construct a test for IB/ESE. One could ask multiple subjective questions regarding levels of incomes for households to “meet their minimum needs,” “meet their needs to be middle class,” or “meet their needs so that they regard themselves wealthy,” etc. The answer to each question provides information about unique utility level. With the use of the intersection method it should be possible to calculate multiple equivalence scales and test whether they are the same or different.

Finally, minimum needs income approach can provide important identifying information which is inherently lacking when estimating equivalence scales using the demand system. Blundell and Lewbel (1989) showed limits of information provided by such approach and suggested that availability of correct equivalence scale at a selected reference utility level is sufficient for effective use of the demand data (see lemma on p.

52). In fact, they mention the possibility of using subjective data (see subsection (iii) on p. 57), and our example of MINQ-based approach provides such alternative which can be easily integrated with the demand system because of its minimal requirements imposed on the utility function.

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