Optimal Trading Ratios for Pollution Permit Markets[*]

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Abstract

We analyze a novel method for improving the efficiency of pollution permit markets by optimizing the exchange of emissions through trade. Under full-information, it is optimal for emissions to exchange according to the ratio of marginal damages. Under asymmetric information, we show that marginal damage trading ratios are generally not optimal. We calculate the optimal trading ratios for a global carbon market and for a regional nitrogen market. The gains from using optimal trading ratios rather than marginal damage trading ratios range from substantial to trivial, which suggests the need for careful consideration of asymmetric information when designing permit markets.

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Incentive-based environmental regulations, such as permit markets or emissions taxes, have typically been designed to minimize the costs of achieving emissions targets.[1] Focusing on reducing abatement costs simplifies program implementation by eliminating the need to quantify damages from emissions of pollution. However, advances in air and water quality modeling now make it feasible to estimate damages precisely and thereby to incorporate them into program design. This suggests that regulators should turn from the narrow criterion of minimizing abatement costs to the more general criterion of efficiency that accounts for both abatement costs and damages (Muller and Mendelsohn 2009).

In this paper we demonstrate a novel method for improving the efficiency of pollution permit markets by optimizing the way in which emissions are exchanged through trade. In our model, there is asymmetric information, and sources of pollution are differentiated by the number of permits they are required to hold for each unit of emissions. When sources trade permits, these differentiated requirements govern the exchange of emissions, and hence they are typically called trading ratios. Several recent studies have shown that selecting trading ratios equal to the ratio of expected marginal damages can substantially increase efficiency relative to the one-for-one trading found in many permit markets (Williams III, 2002; Farrow et al., 2005; Muller and Mendelsohn, 2009; Henry et al., 2011; and Fowlie and Muller, 2013.) Taking this as a point of departure, we ask if further efficiency improvements are possible. The rather surprising answer is yes. We characterize the optimal trading ratios and show that they generally depart from the marginal damage trading ratios.

The main reason for this result is the presence of asymmetric information about the costs of reducing pollution between the sources and the regulator that designs the market. Indeed, in a first-best environment with full information, the optimal trading ratios are equal to the ratios of marginal damages. However, permit markets are generally employed to allow firms to respond flexibly to private information about their abatement costs. This information is typically not available to the regulator when the regulator designs the program. In such a second-best environment, the regulator must account for the damages from pollution as well as the uncertainty about abatement costs when selecting the optimal trading ratios.

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[1]In practice, these regulations have proven quite successful (Carlson, et al., 2000; Ellerman, et al., 2000; Keohane, 2006; Fowlie, et al., 2012).

To understand how this leads to a divergence between the optimal trading ratios and marginal damage ratios, consider uniformly mixed pollution such as greenhouse gas (GHG) emissions. In this case, marginal damages are equal across sources, and the marginal damage trading ratios actually imply one-for-one trading. But one-for-one trading is generally not the most efficient structure. Due to the asymmetric information, the regulator cannot set the aggregate permit endowment (i.e. the "cap") at the *ex post* optimal level. The cap is either too tight, in the case costs are higher than expected, or is too loose, in the case costs are lower than expected. Using trading ratios that are not one-for-one enables the regulator to partially circumvent this problem. By giving relatively favorable trading ratios to sources whose emissions are positively correlated with the market price of permits, the regulator can, in effect, allow increased emissions when costs are high and require decreased emissions when costs are low. These optimal trading ratios improve efficiency relative to one-for-one trading by allowing flexibility in emissions even though the number of permits is fixed at the cap. The regulator obtains this *ex ante* efficiency gain by tolerating an *ex post* efficiency loss due to the fact that the marginal abatement costs are not equal across sources.

The importance of determining optimal trading ratios is buttressed by three observations. First, regulators have begun to incorporate trading ratios into a variety of existing and proposed permit markets.[2] The NOx Budget Program uses trading ratios to restrict banking through a "flow control" provision. The Clean Air Interstate Rule (CAIR) uses trading ratios to reduce uniformly the allowed emissions for each permit. The Cross State Air Pollution Rule (CSPAR), which would have replaced CAIR but was invalidated by the courts, would have used trading ratios through an "assurance provision" to reduce uniformly the emissions per permit if emissions exceed a threshold. The failed Waxman-Markey legislation addressing U.S. GHG emissions would have used trading ratios to implement costly borrowing. Despite this growing use of trading ratios, optimal implementation of trading ratios has not been studied.

Second, regulators are currently grappling with how to regulate non-uniformly mixed pollution. Early pollution permit trading programs could yield large gains by simply reducing

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[2]See Holland and Moore (forthcoming) for more details on each of these programs.

the overall level of emissions.[3] Because low-cost emissions reduction opportunities may have already been realized, current programs must more carefully target emissions reductions to high-damage areas in order to pass a cost-benefit test.[4] Permit markets with trading ratios are well suited for this task. For example, in their analysis of the celebrated SO_2 Acid Rain cap-and-trade program established by the 1990 Clean Air Act Amendments, Henry et al. (2011) argue that utilizing marginal damage trading ratios rather than one-for-one trading would improve the efficiency of the market. Optimal trading ratios offer the possibility of even greater efficiency improvements.

Third, proposed markets to limit GHG emissions would swamp existing permit markets in size and scope. Ellerman and Buchner (2007) compare the Acid Rain Program, which is the largest existing non-GHG program, with the European Union Emissions Trading Scheme (EU ETS) and note that the EU ETS is much larger even though it covers a smaller fraction of total emissions than the Acid Rain Program.[5] To address global climate change effectively, similarly sized programs would need to be implemented throughout the world and then linked together. The massive scale of such programs implies that efficiency gains from using optimal trading ratios could be quite large in absolute terms, even if they are small in relative terms.

Given these observations, it is not sufficient to just delineate the optimal trading ratios, we must also investigate the practical importance of using the optimal trading ratios rather than marginal damage trading ratios. We accomplish this through the numerical analysis of two pollution problems. The first is a multi-country carbon emission market, and the second is a nitrogen trading market for several rivers in North Carolina and Virginia. In both cases, we show that the optimal trading ratios lead to efficiency improvements relative to marginal damage trading ratios. The magnitude of these improvements varies from significant to trivial, depending in particular on the regulators' uncertainty about abatement

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[3]The Acid Rain Program established by the 1990 Clean Air Act Amendments for regulating SO_2 emissions; the leaded gasoline trading program; and the RECLAIM program regulating NOx emissions in southern California each had one-for-one trading over broad regions and a declining level of allowed emissions.

[4]For example, the US EPA recently attempted to modify the Acid Rain Program with CAIR and later CSPAR, which was then struck down by the courts. The new programs attempt to account for spatial heterogeneity in damages mainly by prohibiting trades across regions.

[5]The EU ETS covers approximately 11,500 sources, compared to about 3,000 for the U.S. SO2 program, and the prepolicy emissions in the EU ETS were over two billion metric tons of CO2, versus sixteen million (short) tons of SO2 in the U.S. program. In addition, the value of the allowances distributed under the EU ETS is about $41 billion versus about $5 billion under the U.S. SO2 program.

costs. These results suggest that regulators should give careful consideration to the structure of asymmetric information when designing future permit markets.

Our analysis combines two prominent strands of the literature on incentive based regulations. The first strand follows the seminal work of Montgomery (1972) who introduced the idea of trading ratios in permit markets. Montgomery recognized that, if damage from pollution differs across sources, then emissions licenses should not simply trade one-for-one. His proposed trading rules are consistent with marginal damage trading ratios.[6] More recent work estimates the marginal damage trading ratios for several prominent non-uniformly mixed pollution problems (Muller and Mendelsohn, 2009; Henry et al., 2011; Fowlie and Muller, 2013). The second strand of literature follows Weitzman (1974), who introduced the idea of informational asymmetries in permit markets. Since the parameters of permit markets must be set potentially years in advance, the regulator lacks information which will be available to market participants when they make abatement decisions. This asymmetric information has important implications for the choice of policy instruments and the resulting literature on "prices vs. quantities" is vast. But here we are interested in a different question: What happens to Montgomery's trading ratios when we apply Weitzman's fundamental insight about asymmetric information? There has not been a systematic study of this issue.[7]

The authors who come closest to disentangling the relationship between trading ratios and asymmetric information are Fowlie and Muller (2013). In analyzing a model with quadratic abatement costs and linear damages, they observe that, under asymmetric information, the marginal damage trading ratios may not perform as well as simple one-for-one trading. This suggests, of course, that there may be a completely different set of trading ratios that dominate either benchmark. But they do not pursue this line of inquiry. To replicate their result, we construct a simple numerical example in which one-for-one trading does indeed dominate the marginal damage trading ratios. We go on to calculate the optimal trading

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[6]Montgomery proposed trading at the ratio of the transfer coefficients. The ratio of the transfer coefficients is exactly the ratio of marginal damages *holding ambient concentrations at the other sites constant.*

[7]A few authors have chipped away at the edges. Yates and Cronshaw (2001) and Feng and Zhao (2006) determine the optimal intertemporal trading ratio in models with a specific damage function. Rabotyagov and Feng (2010) observe that the trading ratios may not be equal to the transfer coefficients, but their focus is on cost-effectiveness, rather than efficiency.

ratios and show that they perform better than either the marginal damage trading ratios or one-for-one trading.

We also contribute to two other environmental regulation literatures. The first is the literature on hybrid incentive-based mechanisms, which attempt to mitigate variance in permit prices through, for example, price floors or ceilings supported by injections or withdrawals of permits (Roberts and Spence, 1976).[8] Optimal trading ratios offer an alternative, and potentially complementary, mechanism to mitigate permit price variance. The optimal trading ratios allow aggregate emissions to adjust to the permit price, thereby reducing its variance. The second is the literature on environmental taxes. It is well-known that emissions trading and environmental taxes can each correct environmental externalities by pricing the externality. Optimal trading ratios imply that, because of asymmetric information, trading ratios should not simply reflect expected marginal damages. Similarly, we show that source-specific taxes generally should not equal expected marginal damages but should be adjusted to reflect abatement cost uncertainty.[9]

Section 1 presents the model and derives the main results, which characterize the slope of the regulators objective when using marginal damages trading ratios. These results demonstrate that marginal damage trading ratios are not optimal, show how marginal damage trading ratios should be adjusted to improve efficiency, and provide a first-order approximation of the efficiency gains from optimal trading ratios. Section 2 analyzes a special case of the model in which the abatement costs and damages have the familiar linear-quadratic form. We provide necessary and sufficient conditions for the optimality of marginal damage trading ratios and present closed form solutions for the regulator's objective studied in Section 1. In Section 3, we present a simple two-source example of the linear-quadratic model. This enables us to illustrate the intuition for the main results graphically and numerically. Section 4 presents some preliminary calculations estimating the gains from optimal trading ratios for two hypothetical emissions trading markets: a global carbon trading market and a regional nitrogen trading market. Section 5 concludes.

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[8]For recent contributions to this literature see Fell and Morgenstern, 2010; Hasegawa and Salant, 2011; Grüll and Taschini, 2011; and Stocking, 2012.

[9]This generalizes Chavez and Stranlund (2009) as they obtain their result in a model with quadratic functions. Also Weitzman (1974) uses the optimal source-specific taxes to derive the comparative advantage of prices vs. quantities formula in a quadratic model, but does not present the actual values for these taxes.

1 Model

There are n regulated sources of pollution. The description of a source varies depending on the particular application of the model. For example, a source may correspond to a single facility, or it may correspond to a large group of firms within the same sector of a given country's economy. The abatement costs for source i are $C_i(e_i; \theta_i)$, where e_i is the emissions from source i and θ_i is a parameter that influences costs. Because abatement costs are in terms of emissions, we define marginal abatement costs as $MAC_i \equiv -\frac{\partial C_i}{\partial e_i}$. We assume costs are convex in emission reductions, so that $-\frac{\partial C_i}{\partial e_i} > 0$ and $C_i'' \equiv \frac{\partial^2 C_i}{\partial e_i^2} > 0$. From the point of view of source i, the cost parameter θ_i is known when the abatement decision is made. From the point of view of the regulator, θ_i is random variable with positive expected value $\bar{\theta}_i$ and non-negative variance σ_i^2. We use the expression "cost shocks" to refer to various realizations of these random variables. Let $E = (e_1, e_2, \ldots, e_n)$ denote the vector of emissions.

These emissions cause damages, which are specified by a convex damage function $D(E)$. The marginal damage from source i is $MD_i \equiv \frac{\partial D}{\partial e_i} > 0$. We say a damage function is *regular* if it can be written as

$$D(E) = F\left(\sum \alpha_i e_i\right) \tag{1}$$

for some convex function F and set of positive α_i's. Two familiar special cases of regular damage functions are uniformly mixed pollution, in which $\alpha_i = 1$ for every i, and constant marginal damage, in which F is linear.

The regulator uses a permit market to ameliorate the damages from pollution. We assume this permit market is competitive. Each source is given an endowment of permits w_i and the aggregate endowment is $w = \sum w_i$. The sources face possibly different constraints on the number of permits they must hold for each unit of emissions. These constraints are described by a source-specific variable r_i that is chosen by the regulator. In particular, if source i emits e_i units of pollution then they must hold $r_i e_i$ permits. The ratio of r_j to r_i reflects the rate at which emissions of source i can be converted to emissions of source j through the trade of permits between the two sources.[10] If the ratio $\frac{r_i}{r_j}$ is the same for every

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[10]Suppose source i decreases emissions by one unit. Then it can sell r_i permits to source j, which in turn can increase emissions by $\frac{r_i}{r_j}$.

i and j, then we have one-for-one trading of emissions. Following the literature, we refer to the r_i's as *trading ratios.*

The choice variables for the regulator are nominally the trading ratios and the permit endowments. However, because we assume the permit market is competitive, the market equilibrium only depends on w and is independent of the distribution of the w_i.[11] Moreover, the permit market equilibrium is unchanged if the trading ratios and the aggregate endowment are all multiplied by the same constant. Without loss of generality, then, we can normalize the aggregate endowment as convenient. In our theoretical analysis we normalize w to be equal to one.

Given a price p for permits, source i selects emissions to minimize the sum of abatement costs and expenditures in the permit market. Source i's problem is

$$\min_{e_i} C_i(e_i; \theta_i) + p(r_i e_i - w_i).$$

The first-order condition for e_i is

$$-\frac{\partial C_i}{\partial e_i} = r_i p. \tag{2}$$

An immediate consequence of this equation is that, if two sources have different trading ratios, then their marginal abatement costs will not be equal, i.e., the regulation is not cost-effective. Under our normalization of the aggregate permit endowment, the permit market clearing equation is

$$\sum_i r_i e_i = 1. \tag{3}$$

The permit market equilibrium, conditioned on the regulator's choice of trading ratios, is summarized by equations (2) and (3). This is a system of $n+1$ equations and $n+1$ unknowns (each of the e_i and p). It is useful to describe the solution to these equations as a function of the vector of trading ratios R and the vector of cost parameters Θ. Thus we have $e_i(R; \Theta)$, $E(R; \Theta)$, and $p(R; \Theta)$.

To compute the *optimal trading ratios*, the regulator selects values for the trading ratios to minimize the expected sum of abatement costs and damages. Thus the regulator's problem

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[11]The analysis is unchanged if the regulator distributes permits with any non-distortionary method such as through auctioning.

is to choose R to minimize

$$\mathcal{W} \equiv \mathbb{E}\left[\sum_i C_i(e_i(R;\Theta);\theta_i) + D\left(E(R;\Theta)\right)\right].$$

The corresponding first-order condition for r_j is

$$\frac{\partial \mathcal{W}}{\partial r_j} = \mathbb{E}\left[\sum_i \left(\frac{\partial C_i}{\partial e_i} + \frac{\partial D}{\partial e_i}\right)\frac{\partial e_i}{\partial r_j}\right] = 0 \text{ for } j = 1, 2, \ldots n. \tag{4}$$

This implies that, on average, the marginal abatement costs are equal to marginal damages, where the average is weighted by the $\frac{\partial e_i}{\partial r_j}$'s. There is not a simple closed form solution to the first-order conditions, even in a standard case in which the abatement cost functions and the damage function are quadratic.

Now consider the intuitive, but ultimately inferior, approach to selecting the trading ratios based on the ratio of expected marginal damages. This corresponds to selecting r_1 and r_j such that:

$$\frac{r_j}{r_1} = \frac{\mathbb{E}[\frac{\partial D}{\partial e_j}]}{\mathbb{E}[\frac{\partial D}{\partial e_1}]} \text{ for every } j \neq 1. \tag{5}$$

The system of equations defined by (2), (3), and (5) has $2n$ equations and $2n$ unknowns (e_i, p, and r_j for $j \neq 1$). Let $e_i(r_1;\Theta)$, $E(r_1;\Theta)$, $p(r_1;\Theta)$, and $r_j(r_1;\Theta)$ be the solutions to these equations as a function of r_1 and the Θ vector. The regulator's problem in this case is to find the value for r_1 that minimizes total expected costs:

$$\min_{r_1} \mathbb{E}\left[\sum_i C_i(e_i(r_1;\Theta);\theta_i) + D(E(r_1;\Theta))\right].$$

The first-order condition for r_1 is

$$\mathbb{E}\left[\sum_i \left(\frac{\partial C_i}{\partial e_i} + \frac{\partial D}{\partial e_i}\right)\frac{\partial e_i}{\partial r_1}\right] = 0. \tag{6}$$

The first-order condition implies that the regulator sets marginal abatement costs equal to marginal damages on average where the average is weighted by the $\frac{\partial e_i}{\partial r_1}$'s.[12] Let $\tilde{r}_1$ be the

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[12]Although (6) appears similar to (4), note that there is no reason that (4) should should hold for every

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solution to (6) and let $\tilde{r}_j \equiv r_j(\tilde{r}_1; \Theta)$. We refer to the vector $\tilde{R} = (\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n)$ as the *marginal damage trading ratios*.

If the damage function is regular, then the condition defining the marginal damage trading ratios simplifies considerably. Combining (1) with (5) implies

$$\frac{r_j}{r_1} = \frac{\alpha_j}{\alpha_1}.$$

For example, if pollution is uniformly mixed, then the marginal damage trading ratios are all equal to a common value and hence imply one-for-one trading.

The main result of our paper is to show that the marginal damage trading ratios will generally not be equal to the optimal trading ratios. This may seem a bit surprising, so let us first give intuition for why it is indeed true before turning to a more formal analysis.[13] Building on our discussion of this point in the Introduction, once again focus on the the special case of uniformly mixed pollution. Here marginal damages are the same across sources, so one might expect that the optimal trading ratios would be equal across sources as well. To see why such one-for-one trading is, in fact, not generally optimal for uniformly mixed pollution, consider the market equilibrium condition (3). Evaluating this at the solution $e_i(R; \theta)$ gives

$$\sum_i r_i e_i(R; \Theta) = 1. \tag{7}$$

Now suppose for the moment the market is indeed designed with one-for-one trading and let r be the common value for the trading ratios. It follows from (7) that the sum of emissions is equal to the constant $\frac{1}{r}$, i.e., equal to the effective permit endowment. In general, however, when the trading ratios differ between sources, the sum of emissions will not be constant, and moreover, it will vary according to the realized values of Θ. This suggests that permit markets that do not use one-for-one trading have an interesting and under-appreciated feature. In these markets, sources in aggregate emit more (or less) pollution depending on the actual values of their abatement cost functions, even though the aggregate permit endowment is fixed.

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j when (6) holds.

[13]Additional Appendix D gives a graphical analysis of why marginal damage trading ratios are optimal only under full information and why optimal trading ratios differ from them under asymmetric information.

The regulator, in turn, can use this feature to improve the performance of the permit market. Because of the uncertainty about abatement costs, the regulator does not know the efficient quantity of pollution. Loosely speaking, when aggregate marginal abatement costs are high, the efficient quantity of pollution is large. When the aggregate marginal abatement costs are low, the efficient quantity of pollution is small. The regulator can engender a similar relationship between emissions and abatement costs by optimally selecting the trading ratios.

Now return to the formal analysis of the general case of an arbitrary damage function. To show that the optimal trading ratios will generally not be equal to the marginal damage trading ratios, we utilize the structure of the regulator's problem as well as the characteristics of the marginal damage trading ratios to evaluate the derivative of the regulator's objective function $\mathcal{W}$ at the marginal damage trading ratios. This gives us our main result (all proofs are in the Appendix).

Proposition 1. *The derivative of the regulator's objective function $\mathcal{W}$ with respect to r_j, evaluated at the marginal damage trading ratios $\tilde{R}$, is given by*

$$\frac{\partial \mathcal{W}}{\partial r_j}\bigg|_{\tilde{R}} = COV(p, e_j) + \sum_i COV\left(\tilde{r}_j \frac{\partial D}{\partial e_i} - \tilde{r}_i \frac{\partial D}{\partial e_j}, A^{-1}\frac{a_i p}{\tilde{r}_i C_j''}\right)$$

$$- COV\left(A^{-1}\sum_i \frac{a_i}{\tilde{r}_i}\frac{\partial D}{\partial e_i}, e_j\right) + \mathbb{E}\left[\left(p - A^{-1}\sum_i \frac{a_i}{\tilde{r}_i}\frac{\partial D}{\partial e_i}\right)\right]\mathbb{E}\left[e_j\right]$$

where the covariances and the expectations are also evaluated at $\tilde{R}$, $a_i \equiv r_i^2/C_i''$, and $A \equiv \sum_i a_i$.

Proposition 1 shows that the derivative of $\mathcal{W}$ with respect to r_j, evaluated at the marginal damage trading ratios, can be written as the sum of $n+2$ covariances plus an additional term which is the product of two expected values. If the overall sum of these terms is positive, then the derivative is positive, and the objective function can be improved by decreasing the trading ratio r_j below the marginal damage trading ratio $\tilde{r}_j$. If this sum is negative, then the derivative is negative, and the objective function can be improved by increasing r_j above the marginal damage trading ratio $\tilde{r}_j$. If this sum is equal to zero, then the derivative is equal to zero, and the first-order-condition (4) is satisfied at the point $\tilde{R}$. In this case, the optimal

trading ratio for source j is in fact equal to the marginal damage trading ratio $\tilde{r}_j$. We will analyze the properties of the derivative in Proposition 1 through a variety of special cases and numerical examples. But at this point, it is important to stress that there is no reason, in general, that the overall sum should be equal to zero. In other words, the marginal damage trading ratios are generally not optimal. Furthermore, to a first-order approximation, the efficiency gain in moving from a marginal damage trading toward an optimal trading ratio is given by the magnitude of the derivative in Proposition 1. This magnitude depends on the marginal damages, the marginal damage trading ratios, and the uncertainty about price and emissions generated by the uncertainty about the abatement cost functions.

An additional implication of Proposition 1 is that the optimal trading ratios lead to *ex post* inefficiency. Once again this is perhaps most clearly illustrated with the case of uniformly mixed pollution. If trading ratios are not one-for-one, then by Equation (2), the marginal costs are not equal. Aggregate abatement costs could be reduced by increasing abatement from a low-cost source and decreasing abatement from a high-cost source. The regulator tolerates this (second-order) loss in *ex post* efficiency to obtain the (first-order) gain in *ex ante* efficiency from using the optimal trading ratios.

Next consider a special case in which damage functions are regular as defined in (1). For this special case, the derivative in Proposition 1 simplifies considerably.

Corollary 1. *Suppose that the damage function is regular. Then the derivative of the regulator's objective function* $\mathcal{W}$ *with respect to* r_j, *evaluated at the marginal damage trading ratios* $\tilde{R}$, *is given by*

$$\left. \frac{\partial \mathcal{W}}{\partial r_j} \right|_{\tilde{R}} = COV\left(p, e_j\right)|_{\tilde{R}}.$$

For regular damage functions, the optimal trading ratios are equal to the marginal damage trading ratios if and only if $COV(p, e_j)$, evaluated at $\tilde{R}$, is equal to zero for every j. Although both the cases of uniformly mixed pollution and linear damages have received considerable attention in the literature, Corollary 1 appears to be a novel insight. In Section 3 we give simple numerical examples in which the covariance is indeed not equal to zero, for both uniformly mixed and linear damages.

Corollary 1 shows that, for regular damage functions, the optimality of marginal damage

trading ratios depends on $COV(p, e_j)$. In particular, the regulator should decrease r_j below $\tilde{r}_j$, i.e., offer source j a favorable trading ratio, if and only if $COV(p, e_j)$ is positive. Since the permit price is driven by the marginal abatement costs, the permit price will be high when marginal abatement costs are high. This is precisely the situation in which the total permit endowment should be relaxed. By giving favorable trading ratios to the source whose emissions are large when the permit price is high, the regulator can, in essence, relax the aggregate emissions constraint in the event of high prices and hence improve efficiency. This result is illustrated graphically in Additional Appendix D.

Having shown that the optimal trading ratios will generally not be equal to the marginal damage trading ratios, we now turn to characterizing the optimal trading ratios.

Proposition 2. *The optimal trading ratios imply*

$$\mathbb{E}[p] = \mathbb{E}\left[A^{-1} \sum_i a_i \frac{1}{r_i} \frac{\partial D}{\partial e_i}\right]. \tag{8}$$

The expression $\frac{1}{r_i} \frac{\partial D}{\partial e_i}$ is equal to source i's marginal damage divided by its trading ratio, which we interpret as the "normalized marginal damage". Proposition 2 shows that the expected price is equal to the expected weighted average of the normalized marginal damages, where the weight a_i is inversely proportional to the second derivative of the marginal abatement cost functions. This generalizes the intuition that price should reflect marginal damage. We can further characterize the optimal trading ratios by taking the expectation of (2) which gives $\mathbb{E}[p] = \mathbb{E}[-\frac{\partial C_i}{\partial e_i}]/r_i$. Putting this together with (8) and defining "normalized marginal abatement costs" analogously reveals that the expected normalized marginal abatement costs are equal to the expected weighted average of the normalized marginal damages for each source. This generalizes the intuition that marginal abatement costs should equal marginal damages.

1.1 Optimal Source-Specific Taxes

We have shown that marginal damage trading ratios are generally not optimal. The regulator can improve the performance of the market by adjusting the trading ratios such that expected

price is equal to an expected weighted average of marginal damages where the weights depend on the second derivative of the abatement cost functions. This raises the question as to whether pricing mechanisms, such as pollution taxes, should be set to expected marginal damages or whether they too should be adjusted under asymmetric information.

Suppose t_i is the tax per unit of emissions for source i, and T is the vector of source-specific taxes. As is well-known, the source will equate its marginal abatement costs and the tax, so the first-order condition for e_i in the source's cost minimization problem is

$$-\frac{\partial C_i}{\partial e_i} = t_i. \tag{9}$$

Let the solution to this equation be $e_i(T; \Theta)$. The regulator selects the source-specific taxes to minimize the expected sum of abatement costs and damages. Thus the regulator's problem is to choose T to minimize

$$W \equiv \mathbb{E}\left[\sum_i C_i(e_i(T; \Theta); \theta_i) + D\left(E(T; \Theta)\right)\right].$$

The first-order condition of the regulator's objective with respect to t_j is[14]

$$\frac{\partial W}{\partial t_j} = \mathbb{E}\left[\sum_i \left(\frac{\partial C_i}{\partial e_i} + \frac{\partial D}{\partial e_i}\right)\frac{\partial e_i}{\partial t_j}\right] = \mathbb{E}\left[\left(-t_j + \frac{\partial D}{\partial e_j}\right)\left(\frac{-1}{C_j''}\right)\right] = 0 \tag{10}$$

Solving for t_j implies that

$$t_j = \frac{\mathbb{E}\left[\frac{\partial D}{\partial e_j}/C_j''\right]}{\mathbb{E}\left[1/C_j''\right]} = \mathbb{E}\left[\frac{\partial D}{\partial e_j}\right] + \frac{COV\left(\frac{\partial D}{\partial e_j}, 1/C_j''\right)}{\mathbb{E}\left[1/C_j''\right]}. \tag{11}$$

Since in general there is no reason the covariance term in (11) should equal zero, it is generally not the case that optimal source-specific taxes should equal expected marginal damages. We see that optimal source-specific taxes should indeed be adjusted by a factor that depends on

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[14]The second equality follows from (9) and from differentiating (9), which implies $\frac{\partial e_j}{\partial t_j} = \frac{-1}{C_j''}$ and $\frac{\partial e_i}{\partial t_j} = 0$ for $i \neq j$.

the second derivative of the abatement cost function.[15]

The optimal source-specific taxes in (11) are related to the theory of optimal taxation first studied by Ramsey (1927). In optimal Ramsey taxation, larger taxes are applied to more inelastic goods. Note that $1/C''$ is related to the abatement cost elasticity.[16] Thus if marginal damages are high when the abatement cost elasticity is high, then the second term in (11) is positive and the optimal source-specific tax exceeds expected marginal damages (i.e., is larger in the inelastic good). This intuition is illustrated graphically in the Additional Appendix C.

2 A Linear-Quadratic Example

Additional insight into the the structure of the optimal trading ratios, the marginal damage trading ratios, and the differences between them can be gleaned from an example with specific functional forms. In this example, the abatement cost function

$$C_i(e_i; \theta_i) = \frac{\lambda_i}{2} \left(\frac{\theta_i}{\lambda_i} - e_i \right)^2 \tag{12}$$

is quadratic and the marginal abatement cost function

$$-\frac{\partial C_i}{\partial e_i} = \theta_i - \lambda_i e_i \tag{13}$$

is linear. We interpret θ_i as the intercept and λ_i as the slope of the marginal abatement cost function. It is convenient to collect the λ_i into the diagonal matrix $\mathbf{\Lambda}$. We assume that the random variables θ_i are independent.

The example features a quadratic damage function as well. We have

$$D(E) = W^t E + \frac{1}{2} E^t \mathbf{V} E, \tag{14}$$

where W is a vector with entries ω_i and $\mathbf{V}$ is a symmetric matrix with entries v_{ij}. Marginal

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[15]In the special case of linear damages, $\partial D / e_i$ is non-stochastic so the second term in (11) is zero and the optimal tax is simply marginal damages.

[16]The abatement cost elasticity is $(1/C'')(P/e)$.

damages are given by

$$\frac{\partial D}{\partial e_i} = \omega_i + \sum_k v_{ik} e_k.$$

Some special cases are worth noting. First, if **V** is the zero matrix, then damages are linear and marginal damages constant. Second, if $\omega_i = \omega$ for every i and $v_{ij} = v$ for every i and j, then pollution is uniformly mixed.

A distinct advantage of the linear-quadratic example is that we can obtain simple closed-form expressions for p and e_j. These, in turn, enable us give an explicit expression for $COV(p, e_j)$. We state this as the first of several results for the linear-quadratic example. (The proofs are in Additional Appendix A.)

Result 1. *In the linear-quadratic example,*

$$COV(p, e_j) = \frac{A^{-1} r_j}{\lambda_j} \left(\frac{\sigma_j^2}{\lambda_j} - A^{-1} \sum_i a_i \frac{\sigma_i^2}{\lambda_i} \right). \tag{15}$$

It follows that, if the damage function is regular, then the optimal trading ratios are equal to the marginal damage trading ratios if and only if $\frac{\sigma_j^2}{\lambda_j}$ is the same for every source j.

We see that, for regular damage functions, the difference between the optimal trading ratios and marginal damage trading ratios depends on whether or not the abatement cost functions exhibit a specific type of homogeneity. In particular, if the ratio of the variance of the cost parameter σ_j^2 to the slope of the marginal abatement cost function λ_j is the same across all sources, then the optimal trading ratios are equal to the marginal damage trading ratios. If, however, the ratios of the variance to the slope vary across sources, then the optimal trading ratios are different from the marginal damage trading ratios.

Building on Result 1, we can quantify the efficiency gains from moving from marginal damage trading ratios to the optimal trading ratios. The slope of the regulator's objective at the marginal damage trading ratios gives a first-order approximation of these efficiency gains. For regular damage functions, this first-order approximation for a small change in r_j is given by (15). Thus the relative gain from a small change in r_j is larger if σ_j^2/λ_j is further from the weighted average of the σ_i^2/λ_i's. If we adjust all the r_j's from the marginal damage trading ratios toward the optimal trading ratios, the first-order approximation of

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the gain will be larger if the σ_j^2/λ_j's are further from their weighted average, intuitively, if the dispersion of the σ_j^2/λ_j's is larger. A special case of regular damage functions illustrates this intuition most clearly.

Result 2. *In the linear-quadratic example, suppose that pollution is uniformly mixed and that $\lambda_i = 1$ for every i. To a first-order approximation, the efficiency advantage of the optimal trading ratios relative to the marginal damage trading ratios is*

$$\frac{1}{\tilde{r}\sqrt{n}}\sqrt{\left(\frac{1}{n}\right)\sum_i^n\left(\sigma_j^2 - \frac{1}{n}\sum_i\sigma_i^2\right)^2}.$$

The square root term in this expression corresponds to the standard deviation of the list of numbers $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$. So for this special case, the efficiency advantage of the optimal trading ratios relative to the marginal damage trading ratios is increasing in the standard deviation of the variances of the cost parameters. It is decreasing in the number of sources, but at a slow rate, even with our assumption that the uncertainty is uncorrelated across sources.

In summary, for regular damage functions, heterogeneity of abatement costs (through differences in the ratio of variance of cost uncertainty to slope of marginal abatement cost) leads to a wedge between the optimal trading ratios and the marginal damage trading ratios. The greater the degree of this heterogeneity, the greater the efficiency advantage of the optimal trading ratios.

Next consider arbitrary damage functions. To study these, we simplify the linear-quadratic case by eliminating the abatement cost heterogeneity that was critical in our discussion of regular damage functions. Accordingly, we have

$$\sigma_i^2 = \sigma \text{ and } \lambda_i = \lambda \text{ for every } i. \tag{16}$$

Under this restriction, we can characterize the regulator's objective as follows.

Result 3. *In the linear-quadratic example, suppose that (16) holds. Then we have*

$$\mathcal{W} = \sum C_i(\mathbb{E}[e_i]; \bar{\theta}_i) + D(\mathbb{E}[E]) + \frac{\sigma^2}{2\lambda^2}\left(\lambda + \sum_i v_{ii} - \frac{R^t \mathbf{V} R}{R^t \mathbf{I} R}\right), \tag{17}$$

where $\mathbf{I}$ *is the identity matrix.*

As we might expect, the quadratic functions yield a mean-variance structure for the regulator's objective. Analyzing extreme cases allows us to further characterize the optimal trading ratios. First, suppose that the variance σ^2 is close to zero. In this case, the regulator's objective is approximately deterministic. It follows that the optimal trading ratios are proportional to the marginal damages evaluated at the optimal emissions standards (Yates 2002). Now suppose that the variance is large. Then the regulator's objective is approximately equal to the $\frac{\sigma^2}{2\lambda^2}$ term in (17). Thus the regulator wants to select the values for r_i to *maximize*

$$\frac{R^t \mathbf{V} R}{R^t \mathbf{I} R}.$$

This is the well-known problem of maximizing the ratio of quadratic forms. The solution follows from Kaiser and Rice (1973). Let ν be the largest eigenvalue of $\mathbf{V}$. The optimal vector R is equal to the eigenvector of $\mathbf{V}$ corresponding to this eigenvalue. In less extreme cases for the variance, the optimal trading ratios reflect a trade-off between these two benchmarks. As the variance increases, the optimal trading ratios approach the eigenvector of $\mathbf{V}$. As the variance decreases, the optimal trading ratios approach the marginal damages evaluated at the optimal emission standards.

We can also give a condition under which the optimal trading ratios are equal to the marginal damage trading ratios.

Result 4. *In the linear-quadratic example, suppose equation (16) holds,* $W = 0$*, and* $\mathbf{V}$ *is invertible. Then the optimal trading ratios are equal to the marginal damage trading ratios if and only if* $\mathbb{E}[\Theta]$ *is an eigenvector of* $\mathbf{V}$*.*

Result 4 is similar in structure to Result 1. If the parameters of the abatement cost functions satisfy a particular condition, then the optimal trading ratios are equal to the marginal damage trading ratios. But this time the condition is defined with respect to

the vector of expected values, rather than being a condition on the variances. Under the conditions of Result 4, the marginal damage trading ratios, the expected emissions, and the expected cost parameters all lie on the same ray from the origin. This ray is also an eigenvector of $\mathbf{V}$. It turns out that this eigenvector maximizes the quadratic form in (17), which effectively eliminates concerns about uncertainty. Hence the optimal trading ratios, which in general differ from the marginal damage trading ratio on account of such uncertainty, offer no improvement relative to the marginal damage trading ratios in this case.[17]

Taken as a whole, the results for the quadratic example reinforce and enhance our findings from the general model. It is possible that the optimal trading ratios can be equal to the marginal damage trading ratios, but this will generally not occur. The efficiency gains from using the optimal trading ratios will depend in a complicated manner on distributions of both the expected value and variances of the random variables in the cost functions as well as the interaction of these distributions with the properties of the damage function.

3 Numerical Calculations

In this section we use special cases of the linear-quadratic model to illustrate Corollary 1 with graphs and numerical calculations.[18]

3.1 Uniformly Mixed

We start with uniformly mixed pollution. Consider a simple example with two sources. Source 1's marginal abatement costs are known with certainty. Source 2's cost shock can either be high (H) or low (L) with equal probability. For convenience, we normalize the total permit endowment such that the marginal damages trading ratios are unity, i.e., $\tilde{R} = (1, 1)$.

Figure 1 shows the marginal abatement costs for each source, the two aggregate (or "market") marginal abatement costs corresponding to the high and low outcome, and marginal damages.[19] The first-best outcome occurs at the intersection of the appropriate market

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[17] In Figure 7 in Additional Appendix D, these points are not all on the same ray, so the marginal damage trading ratios are not optimal.

[18] The code used to determine these results is available upon request.

[19] The market marginal abatement cost is found by horizontal summation of the sources' marginal abate-

marginal abatement cost and marginal damages. The numerical values for emissions, prices, and marginal abatement costs are given in Panel A of Table 1. Notice that the first-best outcome features variance in both prices and aggregate emissions: when costs are low, prices and aggregate emissions are low and when costs are high, prices and aggregate emissions are high.

Figure 1 and Panel B of Table 1 show the results for the marginal damage (one-for-one) trading ratios. Because of asymmetric information, the first-best outcome is not obtained. In the event of the high cost shock, the total permit endowment is too small, marginal abatement costs exceed marginal damages, and there is a deadweight loss relative to the first-best outcome. This deadweight loss is indicated by the upper triangle. In the event of a low cost shock, the total endowment is too large, marginal abatement costs are less than marginal damages, and the deadweight loss is the lower triangle. The marginal damage trading ratios feature variance in price, but aggregate emissions are unchanged across the two shocks.

The covariance between prices and emissions is of particular interest. Inspection of either the data in Panel B of Table 1 or the relationship between the points in Figure 1 reveals that $COV(p, e_j)|_{\tilde{R}} \neq 0$ for either source. In fact, the covariance of emissions and prices are negative for Source 1, but positive for Source 2. Applying Corollary 1 shows that efficiency can be increased by increasing Source 1's trading ratio, but decreasing Source 2's trading ratio.

The optimal trading ratios are illustrated in Figure 2 and Panel C of Table 1. The optimal trading ratio for Source 1 is larger than the marginal damage trading ratio. It follows that the cost of emissions are higher for Source 1 and so its emissions are lower for both cost shocks. This is reversed for Source 2. Relative to the marginal damage trading ratios, the optimal trading ratios lead to a decrease in the variance in prices and an increase in the variance in aggregate emissions. Moreover, aggregate emissions are larger in the case of a high cost shock, but smaller in the case of a low cost shock. Thus the optimal trading ratios provide a closer match to the features of the first-best outcome than the marginal damage trading ratios. Correspondingly, as shown in Panel B and C of Table 1, deadweight loss

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ment costs.

decreases when optimal trading ratios are used, but it does not disappear.[20] The optimal trading ratios cannot duplicate the first-best outcome.

Table 1: Numerical example: uniformly mixed pollution

Panel A: First-Best

	MAC_1	price	MAC_2	e_1	e_2	$e_1 + e_2$
Low Cost	13.33	n.a.	13.33	6.67	1.67	8.34
High Cost	16.67	n.a.	16.67	3.33	8.33	11.67

Panel B: Marginal damage trading ratios.
$r_1 = r_2 = 1$; DWL=2.08

	MAC_1	price	MAC_2	e_1	e_2	$e_1 + e_2$
Low Cost	12.5	12.5	12.5	7.5	2.5	10
High Cost	17.5	17.5	17.5	2.5	7.5	10

Panel C: Optimal trading ratios
$r_1 = 1.03$; $r_2 = 0.98$; DWL=1.92

	MAC_1	price	MAC_2	e_1	e_2	$e_1 + e_2$
Low Cost	12.89	12.55	12.23	7.11	2.77	9.87
High Cost	17.89	17.41	16.97	2.11	8.03	10.14

[1] The example is parameterized by $MAC_1 = 20 - e_1$; $MAC_2^L = 15 - e_1$; $MAC_2^H = 25 - e_1$; $MD = 5 + e_1 + e_2$ where high and low costs occur with equal probability. Total permits are normalized to 10.

3.2   Linear Damages

Consider another example of Corollary 1. The two sources and their abatement costs are identical to those used in the example above, but damages are linear and differ across the two sources. Source 1 has low marginal damages ($MD_1 = 10$) and Source 2 has high marginal damages ($MD_2 = 12$). This example is also consistent with the model employed by Fowlie and Muller (2013).

Table 2 illustrates the results for the marginal damage trading ratios, one-for-one trading, and the optimal trading ratios. From (5), the marginal damage trading ratios satisfy $r_2 =$

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[20]Because the market is not cost effective *ex post*, the market marginal abatement cost is not simply the horizontal sum of the source marginal abatement costs. Thus the deadweight loss cannot be simply illustrated in Figure 2.

$12/10 * r_1$. Panel A of Table 2 shows that, under marginal damage trading ratios, the value for r_1 is 0.92, so that $r_2 = 1.10$. Thus the low damage source (Source 1) pays a relatively low effective price for its emissions and the high damage source (Source 2) pays a relatively high effective price for its emissions. The marginal damage trading ratios hold damages constant across the two cost shocks, but allow aggregate emissions to vary.

Interestingly, one-for-one trading actually performs better than marginal damage trading, even though pollution is not uniformly mixed in this example. As shown in Panel B of Table 2, under one-for-one trading, the damages are not held constant across the cost shocks, but the aggregate emissions are held constant. This leads to a lower deadweight loss than marginal damage trading ratios, which verifies Fowlie and Muller's observation that such an outcome is possible in their model.

The optimal trading ratios have a lower deadweight loss than either of the other schemes. Panel C of Table 2 shows calculations for the the optimal trading ratios. Since $COV(p, e_1) < 0$, the optimal trading ratio for source 1 is greater than the marginal damage trading ratio (0.96 vs. 0.93). On the other hand, since $COV(p, e_2) > 0$, the optimal trading ratio for source 2 is lower than the marginal damage trading ratio (1.04 vs 1.10). Under the optimal trading ratios, neither the aggregate emissions nor the damages are constant across the cost shocks. This flexibility improves efficiency.

4   Applications

We have established that the optimal trading ratios will generally be different from the marginal damage trading ratios, even for uniformly mixed pollution. We now illustrate potential policy implications of this observation by considering two permit trading applications.

4.1   Uniformly Mixed: Carbon Trading

Consider a stylized global carbon trading market. Ackerman and Bueno (2011) determine simple two-parameter functions that characterize the cost of reducing carbon emissions for

Table 2: Numerical example: linear damages with $MD_1 = 10, MD_2 = 12$

Panel A: Marginal damage trading ratios.
$r_1 = 0.93$; $r_2 = 12/10r_1 = 1.10$; DWL=7.38

	MAC_1	price	MAC_2	e_1	e_2	$e_1 + e_2$	Damages
Low Cost	7.54	8.21	9.05	12.5	5.95	18.45	196.0
High Cost	12.5	13.6	15	7.54	10.0	17.54	196.0

Panel B: One-for-one trading.
$r_1 = 1$; $r_2 = 1$; DWL=7.25

	MAC_1	price	MAC_2	e_1	e_2	$e_1 + e_2$	Damages
Low Cost	8.5	8.5	8.5	11.5	6.5	18	193
High Cost	13.5	13.5	13.5	6.5	11.5	18	203

Panel C: Optimal trading ratios.
$r_1 = 0.96$; $r_2 = 1.04$; DWL=7.06

	MAC_1	price	MAC_2	e_1	e_2	$e_1 + e_2$	Damages
Low Cost	8.10	8.40	8.67	11.9	6.24	18.14	193.9
High Cost	13.08	13.56	14.16	6.92	10.8	17.72	199.3

[1] The example is parameterized by $MAC_1 = 20 - e_1$; $MAC_2^L = 15 - e_1$; $MAC_2^H = 25 - e_1$; where high and low costs occur with equal probability. Total permits are normalized to 18, which would be the optimal emissions with 1:1 trading.

various geographic regions of the world.[21] To apply these to our model, we interpret our sources as regions and write Ackerman and Bueno's functions in terms of emissions rather than emission reductions. This gives

$$-\frac{\partial C_i}{\partial e_i} = \frac{a_i(b_i(1 + \theta_i) - e_i)}{e_i},$$

where a_i and b_i are constants determined by Ackerman and Bueno. We interpret $b_i(1 + \theta_i)$ as the stochastic business-as-usual (BAU) emissions. For simplicity we model the random variable θ_i with a three point symmetric distribution with zero expectation so that θ_i takes on the values $\{-k_i, 0, k_i\}$ with probabilities $\{\rho_i, 1 - 2\rho_i, \rho_i\}$. For example, there is a probability ρ_i that BAU emissions increase by k_i percent over their expected value. We also assume that the θ_i are independent across regions. To complete the model we specify the marginal damage function as

$$\frac{\partial D}{\partial e_i} = \beta \sum (e_i - b_i) + s,$$

where β (the slope of marginal damage) comes from Newell and Pizer (2003) and s (the social cost of carbon at expected BAU) comes from IWGSSC (2010).

For tractability, we focus on the industrial sectors of the four regions with the largest emissions: China, Europe, South/South East Asia, and the U.S. The results are given in Tables 3 and 4. For a given set of parameters ρ_i an k_i, we calculate the total expected costs (expected sum of abatement costs and damages) under the optimal trading ratios and the marginal damage trading ratios.

In Table 3, we consider symmetric abatement cost shocks (the tail probabilities ρ_i and the percentage change in BAU emissions k_i are the same across regions). Because China has the largest BAU emissions, shocks to Chinese abatement costs drive the carbon price. Hence Chinese emissions covary positively with price under marginal damages trading ratios, and Corollary 1 implies that efficiency can be improved by lowering China's trading ratio. Indeed, China's optimal trading ratios are below one in each scenario, whereas the trading ratios for the other regions exceed one in each scenario. For the largest uncertainty ($k_i = 0.5$

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[21]These functions are particularly easy to work with, but are not uncontroversial. A similar analysis could be done with any integrated assessment model.

Table 3: Carbon Trading with Optimal Trading Ratios: Symmetric Scenarios

	$\rho_i = 50\%$				$k_i = 0.2$	
	$k_i = 0.5$	$k_i = 0.33$	$k_i = 0.2$	$k_i = 0.1$	$\rho_i = 25\%$	$\rho_i = 10\%$
Optimal trading ratios						
China	0.877	0.946	0.981	0.995	0.990	0.996
Europe	1.082	1.037	1.013	1.003	1.007	1.003
S/SE Asia	1.133	1.055	1.019	1.005	1.010	1.004
U.S.A.	1.063	1.030	1.011	1.003	1.006	1.002
St. Dev. Price						
Marginal Damage TR	38.534	25.694	15.418	7.709	10.902	6.895
Optimal TR	37.649	25.463	15.371	7.704	10.886	6.891
Expected Price						
Marginal Damage TR	74.022	74.022	74.022	74.022	74.022	74.022
Optimal TR	73.838	73.997	74.023	74.023	74.024	74.023
Total Cost						
First Best	305.699	305.699	305.699	305.699	305.699	305.699
Marginal Damage TR	321.374	312.554	308.148	306.309	306.921	306.188
Optimal TR	320.665	312.424	308.132	306.308	306.917	306.187
Deadweight Loss						
Marginal Damage TR	15.674	6.855	2.449	0.610	1.222	0.488
Optimal TR	14.966	6.725	2.432	0.609	1.218	0.488
Percent Reduction	4.5%	1.9%	0.7%	0.16%	0.33%	0.13%

[1] The permit endowment is set so that the marginal damage trading ratios are 1. This represents approximately a 50% reduction from BAU emissions.

[2] Costs and deadweight loss (DWL) in billions of dollars. Prices in 2007 dollars per ton carbon

and $\rho_i = 50\%$), optimal trading ratios reduce the deadweight loss by $0.5 billion or about 5%. For lower levels of uncertainty, the gains from optimal trading ratios are more modest.

In Table 4, we consider asymmetric cost shocks. Here China's abatement costs are uncertain and the other regions' abatement costs are known. The gains from using optimal trading ratios are more dramatic than in the symmetric case of Table 3. With a high level of uncertainty about China's abatement costs ($k_{\text{China}} = 0.5$), optimal trading ratios reduce the deadweight loss by about 22% or around $2 billion per year.

Table 4: Carbon Trading with Optimal Trading Ratios: Asymmetric Scenarios

	$k_{\text{China}} = 0.5$	$k_{\text{China}} = 0.33$	$k_{\text{China}} = 0.2$	$k_{\text{China}} = 0.1$
Optimal trading ratios				
China	0.798	0.903	0.965	0.991
Europe	1.146	1.071	1.026	1.007
S/SE Asia	1.154	1.073	1.027	1.007
U.S.A.	1.152	1.072	1.027	1.007
St. Dev. Price				
Marginal Damage TR	30.426	20.285	12.171	6.085
Optimal TR	27.113	19.312	11.966	6.060
Expected Price				
Marginal Damage TR	74.023	74.022	74.022	74.022
Optimal TR	73.635	73.968	74.031	74.028
Total Cost				
First Best	305.699	305.699	305.699	305.699
Marginal Damage TR	315.270	309.934	307.220	306.079
Optimal TR	313.194	309.516	307.166	306.076
Deadweight Loss				
Marginal Damage TR	9.571	4.235	1.521	0.380
Optimal TR	7.495	3.817	1.467	0.377
Percent Reduction	21.7%	9.9%	3.6%	0.9%

[1] The permit endowment is set so that the marginal damage trading ratios are 1. This represents approximately a 50% reduction from BAU emissions.

[2] Costs and deadweight loss (DWL) in billions of dollars. Prices in 2007 dollars per ton carbon.

[3] For each column, the tail probabilities are $\rho_{\text{China}} = 50\%$ for China and $\rho_{\text{ROW}} = 0\%$ for the other three regions.

4.2 Non-Uniformly Mixed: Nitrogen Trading

Our non-uniformly mixed policy application considers the trading of Nitrogen emission per-
mits between waste-water treatment plants (WWTP) located in North Carolina and Vir-
ginia. The data for this application is described in detail in Yates et al (2013) and Doyle
et al (2013). Briefly, there are 51 waste-water treatment plants spatially distributed along a
river system that connects to a coastal estuary (see Figure 3). In the context of our model,
each WWTP corresponds to a source of pollution. The abatement costs and damages are
consistent with linear-quadratic example and, furthermore, $\lambda_i = \lambda$ for every i. The value for
λ is obtained from engineering cost estimates for reducing pollution at a generic WWTP.

There is a distinct advantage to using the linear-quadratic structure for policy applica-
tions of this type. The equations characterizing equilibrium in the permit market are linear
in emissions, so that expected total costs of a given policy are a functions of the first and
second powers of the random variables θ_i. Thus we do not need to make explicit distribu-
tional assumptions about the random variables. Rather we just need to specify the mean
and variance of the distributions. The expected value for the cost parameters, $\bar{\theta}_i$, are based
on the size of the WWTP. The variances, σ_i^2, are scaled proportionately to the expected
values, so that a single parameter η captures the "percent error" in the random variables.[22]

Damages are measured at 96 sites along the river system and in the estuary. The damage
function is given by

$$D(E) = \frac{1}{2}(\mathbf{A}E + Y)^t \mathbf{B}(\mathbf{A}E + Y).$$

In this expression, E is the vector of emissions of nitrogen from the 51 WWTP, Y is the
vector of background levels of Nitrogen from non-point sources at the 96 measurement sites,
$\mathbf{B}$ is a diagonal matrix with entries b_{jj} (interpreted as the slope of marginal damage at
site j), and $\mathbf{A}$ is a transfer matrix that maps emissions from the WWTP through the river
system to the measurement sites. The elements of Y and $\mathbf{A}$ are determined by matching
the location of the WWTP and measurement sites to the output of the USGS maintained
SPARROW model (Hoos and McMahon 2009). For simplicity we assume that the $b_{jj} = b$,

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[22]The standard deviation of each random variable is $\frac{\eta}{100}\left(\frac{\mathbb{E}[\theta_i]}{2}\right)$, so that it is very likely that a realization of
the random variable lies within η percent of the expected value (for a normal random variable the probability
is 0.95).

and that b is within the range determined by Yates et al (2013).

We determine the optimal trading ratios and the marginal damage trading ratios as a function of the parameters η and b. The parameter η measures the magnitude of the uncertainty about abatement costs. The parameter b measures the magnitude of the severity of damages from emissions. The results for various parameter combinations are shown in Table 5. The ratio of the largest trading ratio to the smallest is consistently larger for the optimal trading ratios than for the marginal damage trading ratios. The optimal trading ratios decrease the price dispersion relative to the marginal damage trading ratios. They also increase the dispersion in total emissions. Now consider the parameter combination in the first column of Table 5. For this case, all 51 marginal damage trading ratios and all 51 optimal trading ratios are shown in Figure 3.

The percent reduction in deadweight loss from using optimal trading ratios rather than marginal damage trading ratios varies quite a bit according to the values for the parameters. The percentage reduction is generally substantial and greater than in the carbon example. As we would expect from our analysis of the quadratic example, the distributions for the uncertainty parameters σ_i^2 and $\bar{\theta}_i$ play a critical role in determining these reductions. There are a few WWTP that have large expected values (and hence large variances) relative to the other WWTP.[23] In Additional Appendix E, we show that simply taking the WWTP with the largest expected value and artificially reducing its expected value to be equal to the average expected value (and hence its variance to be equal to the average variance as well) changes the reduction in deadweight loss from 75 percent to 28 percent. Furthermore, if we assume that all WWTP have the same expected values and variances, then the reduction in deadweight loss is much less than 1 percent, but it does not equal zero.

5   Conclusion

We analyze a model of asymmetric information between a regulator and sources of pollution and show that optimal policies are not based simply on expected marginal damages. In

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[23]These WWTP have large output, and it is assumed that the expected value is proportional to output, and the variance is proportional to expected value.

Table 5: Nitrogen Trading with Optimal Trading Ratios

Uncertainty Percent Error	$\eta = 10$			$\eta = 5$		
Slope of MD	$b = 30$	$b = 60$	$b = 90$	$b = 30$	$b = 60$	$b = 90$
Trading ratios						
Max Marginal Damage	3.553	3.580	3.607	3.553	3.580	3.607
Min Marginal Damage	0.063	0.064	0.066	0.063	0.064	0.066
Max Optimal	6.553	5.088	4.598	5.009	4.214	3.956
Min Optimal	0.015	0.035	0.048	0.035	0.055	0.061
St. Dev. Price						
Marginal Damage TR	2.171	2.157	2.143	1.085	1.079	1.072
Optimal TR	0.525	0.829	1.080	0.420	0.669	0.820
Expected Price						
Marginal Damage TR	2.737	5.327	7.777	2.739	5.328	7.778
Optimal TR	1.635	3.907	6.280	2.024	4.618	7.179
St. Dev. Total Emissions						
Marginal Damage TR	0.038	0.038	0.038	0.019	0.019	0.019
Optimal TR	0.055	0.048	0.044	0.024	0.020	0.019
Expected Total Emissions						
Marginal Damage TR	4.842	4.780	4.791	4.842	4.780	4.719
Optimal TR	4.846	4.783	4.722	4.844	4.781	4.720
Total Cost						
First Best	9.773	19.240	28.419	9.771	19.236	28.412
Marginal Damage TR	9.870	19.336	28.512	9.796	19.260	28.436
Optimal TR	9.798	19.281	28.471	9.782	19.253	28.432
Deadweight Loss						
Marginal Damage TR	0.097	0.095	0.094	0.024	0.024	0.023
Optimal TR	0.024	0.040	0.053	0.010	0.016	0.019
Percent Reduction	74.9%	57.8%	43.8%	56.9%	31.4%	18.6%

[1] The permit endowment is set to the sum of the optimal pollution standards.
[2] Costs and deadweight loss (DWL) in millions of dollars per year.
[3] Prices in dollars per pound.
[4] Emissions in millions of pounds per year.

the context of pollution permit markets, we find that optimal trading ratios (trading ratios which maximize *ex ante* efficiency) generally depart from marginal damage trading ratios. The gains from optimal trading ratios depend on the sum of various covariances and expected values. In simple cases, such as uniformly mixed pollution or linear damages, the gain is simply determined by the covariance of the price and a source's emissions. In particular, if a source's emissions covary positively with the market price of permits, then the regulator can improve efficiency by giving the source a relatively favorable trading ratio. Intuitively, this favorable trading ratio allows additional emissions—despite a fixed cap—in precisely the case when the cap is set too tight from an *ex post* perspective. In the context of an emissions tax, our results imply that the regulator can improve *ex ante* efficiency by setting source-specific taxes according to a Ramsey-like rule which adjusts the expected marginal damages to account for the covariance of marginal damages with the slope of the marginal abatement costs.

Our theoretical analysis shows that it is possible for a regulator to improve the efficiency of pollution permit markets by using optimal trading ratios. However, whether the regulator should implement optimal trading ratios depends crucially on whether the benefits of optimal trading ratios are sufficient to offset any additional regulatory costs which might arise from a more complicated regulatory scheme. To estimate the magnitude of possible benefits, we compare optimal trading ratios to marginal damage trading ratios in two policy environments: a global carbon trading market and a nitrogen trading market for watersheds in North Carolina and Virginia. The results from these calculations show that the benefits vary from significant to trivial depending the characteristics of the regulator's uncertainty about abatement costs.

We did not estimate the additional regulatory costs, but in many respects, the optimal trading ratios are no more costly to implement than the marginal damage trading ratios, especially for non-uniformly mixed pollution. Both require the regulator to estimate marginal damages by analyzing models of emission transport through the relevant physical space in conjunction with models mapping emissions into harm to humans and ecosystems. Both require moving away from the intuitively appealing and easy to explain cost-effectiveness criterion. And both give the regulator discretion to give differential regulatory requirements

to the various sources of pollution, thereby potentially opening the door for the sources to lobby or litigate for a more favorable treatment. The only additional cost of optimal trading ratios would appear to be the cost of estimating the parameters for the random variables in the abatement cost functions.

Guided by Weitzman (1974), the variance of abatement cost uncertainty has traditionally been viewed as the reason why price and quantity instruments may perform differently. Weitzman shows that the superior instrument can be determined by comparing the relative slopes of the marginal abatement cost and the marginal damage functions. Once the superior instrument has been determined, regulators simply focus on expected costs and damages. On the contrary, our analysis suggests a more fundamental role for the variance of abatement cost uncertainty. In particular, this variance should be incorporated into the design of the policy instrument itself, not just inform the choice between policy instruments.

Appendix

Proof of Proposition 1.

From (2) and (3), the equilibrium for the optimal trading ratios is defined by the $n-1$ equations $\frac{\partial C_i}{\partial e_i}/r_i = \frac{\partial C_j}{\partial e_j}/r_j$ for each $i \neq j$ and by the equation $\sum_i r_i e_i = 1$. Differentiating the $n-1$ equations with respect to r_j gives

$$\frac{C_i''}{r_i}\frac{\partial e_i}{\partial r_j} = \frac{-\frac{\partial C_j}{\partial e_j}}{r_j^2} + \frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j} = \frac{p}{r_j} + \frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j} \qquad \text{for each} \qquad i \neq j \qquad (18)$$

where the first equation follows from differentiating and the second equation follows from the definition of (2). Differentiating $\sum_i r_i e_i = 1$ with respect to r_j implies that

$$\sum_i r_i \frac{\partial e_i}{\partial r_j} + e_j = 0 \qquad (19)$$

which implies that

$$-e_j = \sum_i r_i \frac{\partial e_i}{\partial r_j} = \sum_{i \neq j} \frac{r_i^2}{C_i''}\frac{p}{r_j} + \sum_i \frac{r_i^2}{C_i''}\frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j}$$

$$= \frac{p}{r_j}\sum_i \frac{r_i^2}{C_i''} - \frac{pr_j}{C_j''} + \frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j}\sum_i \frac{r_i^2}{C_i''}$$

where the first equality follows from rearranging (19), the second equality follows from substituting in (18), and the third equality follows from algebra. Solving this equation implies that

$$\frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j} = \left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1}\left(\frac{r_j p}{C_j''} - e_j\right) - \frac{p}{r_j} \qquad (20)$$

which implies from (18) that

$$\frac{C_i''}{r_i}\frac{\partial e_i}{\partial r_j} = \left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1}\left(\frac{r_j p}{C_j''} - e_j\right) \qquad \text{for each} \qquad i \neq j. \qquad (21)$$

Substituting (2) into the derivative of the regulator's objective with respect to r_j as shown

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in (4) gives

$$\frac{\partial \mathcal{W}}{\partial r_j} = \mathbb{E}\left[\sum_i -pr_i\frac{\partial e_i}{\partial r_j} + \sum_i \frac{\partial D}{\partial e_i}\frac{\partial e_i}{\partial r_j}\right]$$

$$= \mathbb{E}\left[pe_j + \sum_i \frac{\partial D}{\partial e_i}\left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1}\left(\frac{r_jp}{C_j''} - e_j\right)\frac{r_i}{C_i''} - \frac{\partial D}{\partial e_j}\frac{p}{C_j''}\right]$$

$$= \mathbb{E}\left[pe_j + A^{-1}\left[\frac{p}{C_j''}\left\{\sum_i \frac{r_jr_i}{C_i''}\frac{\partial D}{\partial e_i} - \sum_i \frac{r_i^2}{C_i''}\frac{\partial D}{\partial e_j}\right\} - e_j\sum_i \frac{r_i}{C_i''}\frac{\partial D}{\partial e_i}\right]\right]$$

$$= \mathbb{E}\left[A^{-1}\sum_i\left[\frac{pr_i}{C_j''C_i''}(r_j\frac{\partial D}{\partial e_i} - r_i\frac{\partial D}{\partial e_j})\right] + \left(p - A^{-1}\sum_i \frac{r_i}{C_i''}\frac{\partial D}{\partial e_i}\right)e_j\right]$$

$$= \mathbb{E}\left[\sum_i A^{-1}\left[\frac{a_ip}{r_iC_j''}(r_j\frac{\partial D}{\partial e_i} - r_i\frac{\partial D}{\partial e_j})\right] + \left(p - A^{-1}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right)e_j\right] \tag{22}$$

where the second equality follows from subsituting (19), (20), and (21), and the rest follow from algebra and the definition of a_i and A.

Next recall that that $COV(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$. Applying this formula repeatedly to (22) and noting that $\tilde{r}_j\mathbb{E}\left[\frac{\partial D}{\partial e_i}\right] = \tilde{r}_i\mathbb{E}\left[\frac{\partial D}{\partial e_j}\right]$ by the definition of marginal damage trading ratios establishes that:

$$\left.\frac{\partial \mathcal{W}}{\partial r_j}\right|_{\tilde{R}} = \sum_i COV\left(\tilde{r}_j\frac{\partial D}{\partial e_i} - \tilde{r}_i\frac{\partial D}{\partial e_j}, A^{-1}\frac{a_ip}{\tilde{r}_iC_j''}\right) + \mathbb{E}\left[\left(p - A^{-1}\sum_i \frac{a_i}{\tilde{r}_i}\frac{\partial D}{\partial e_i}\right)e_j\right].$$

Applying the covariance formula to the expected value term on the right gives us the equation in the proposition. ∎

Before proving Corollary 1, we first prove a Lemma about the marginal damage trading ratios that holds provided damages are regular.

Lemma 1. *Suppose that damages are regular. For the marginal damage trading ratios, the regulator selects $\tilde{R}$ such that*

$$\mathbb{E}\left[p - A^{-1}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right] = 0$$

where $a_i \equiv r_i^2/C_i''$ and $A \equiv \sum_i a_i$

Proof of Lemma 1.

From (2) and (3), the equilibrium for marginal damage trading ratios is defined by the $n-$ 1 equations $\frac{\partial C_i}{\partial e_i}/r_i = \frac{\partial C_1}{\partial e_1}/r_1$ for each $i \neq 1$ and by the equation $\sum_i r_i e_i = 1$. Differentiating the $n-1$ equations with respect to r_1 gives

$$-\frac{\frac{\partial C_i}{\partial e_i}}{r_i^2}\frac{r_i}{r_1} + \frac{C_i''}{r_i}\frac{\partial e_i}{\partial r_1} = -\frac{\frac{\partial C_1}{\partial e_1}}{r_1^2} + \frac{C_1''}{r_1}\frac{\partial e_1}{\partial r_1}.$$

(To derive this equation, we have used the fact that (5) and (1) imply that $\partial r_i/\partial r_1 = r_i/r_1$). By substituting in p this implies that

$$\frac{C_i''}{r_i}\frac{\partial e_i}{\partial r_1} = \frac{C_1''}{r_1}\frac{\partial e_1}{\partial r_1} \tag{23}$$

for each $i \neq 1$. Differentiating $\sum_i r_i e_i = 1$ implies that

$$\sum_i r_i \frac{\partial e_i}{\partial r_1} + \sum_i \frac{r_i}{r_1} e_i = 0 \tag{24}$$

which implies that

$$\frac{-1}{r_1} = \sum_i r_i \frac{\partial e_i}{\partial r_1} = \sum_i r_i \frac{C_1'' r_i}{r_1 C_i''} \frac{\partial e_1}{\partial r_1} = \frac{\partial e_1}{\partial r_1} \frac{C_1''}{r_1} \sum_i \frac{r_i^2}{C_i''}$$

where the first equality comes from $\sum_i r_i e_1 = 1$ and rearranging (24), the second equality follows from substituting in (23), and the third equality follows from algebra. Solving this equation shows that

$$\frac{\partial e_1}{\partial r_1} = \frac{-1}{C_1''}\left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1} = \frac{-a_1}{r_1^2} A^{-1}$$

which implies

$$\frac{\partial e_i}{\partial r_1} = \frac{r_i}{r_1}\frac{-1}{C_i''}\left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1} = \frac{-a_i}{r_1 r_i} A^{-1} \tag{25}$$

from (23).

Substituting (2) into (6), the first-order condition for r_1, gives

$$0 = \mathbb{E}\left[\sum_i -pr_i\frac{\partial e_i}{\partial r_1} + \sum_i \frac{\partial D}{\partial e_i}\frac{\partial e_i}{\partial r_1}\right]$$

$$= \mathbb{E}\left[\frac{p}{r_1} - \sum_i \frac{\partial D}{\partial e_i}\frac{a_i}{r_i r_1}A^{-1}\right] \qquad (26)$$

where the second equality follows from subsituting in (24) and (25). Multiplying through by r_1 establishes the result. ∎

Proof of Corollary 1. From Lemma 1, it follows that the formula in Proposition 1 can be written as

$$\left.\frac{\partial \mathcal{W}}{\partial r_j}\right|_{\tilde{R}} = COV(p, e_j) - COV\left(A^{-1}\sum_i \frac{a_i}{\tilde{r}_i}\frac{\partial D}{\partial e_i}, e_j\right) + \sum_i COV\left(\tilde{r}_j\frac{\partial D}{\partial e_i} - \tilde{r}_i\frac{\partial D}{\partial e_j}, A^{-1}\frac{a_i p}{\tilde{r}_i C_j''}\right).$$

Because the damage function is regular, we have

$$\frac{\tilde{r}_j}{\tilde{r}_1} = \frac{\alpha_j}{\alpha_1}.$$

It follows from (3) that $\sum \alpha_i e_i = \sum \frac{\alpha_1}{\tilde{r}_1}\tilde{r}_i e_i = \frac{\alpha_1}{\tilde{r}_1}$ is a constant. Next consider the marginal damage function

$$\frac{\partial D}{\partial e_i} = F'(\sum \alpha_i e_i)\alpha_i = F'\left(\frac{\alpha_1}{\tilde{r}_1}\right)\alpha_i.$$

This is non-stochastic, and so (5) implies that $\frac{\partial D}{\partial e_i}/\tilde{r}_i = \frac{\partial D}{\partial e_1}/\tilde{r}_1$ for every i. Substituting these expressions into the partial derivative above gives

$$\left.\frac{\partial \mathcal{W}}{\partial r_j}\right|_{\tilde{R}} = COV(p, e_j) - COV\left(A^{-1}\sum_i \frac{a_i}{\tilde{r}_1}\frac{\partial D}{\partial e_1}, e_j\right) + \sum_i COV\left(0, A^{-1}\frac{a_i p}{\tilde{r}C_j''}\right)$$

$$= COV(p, e_j) - COV\left(\frac{\frac{\partial D}{\partial e_1}}{\tilde{r}_1}A^{-1}\sum_i a_i, e_j\right) = COV(p, e_j)$$

where the third equality follows since $\tilde{r}_1$ and $\frac{\partial D}{\partial e_1}$ are non-stochastic. ∎

Proof of Proposition 2.

Start with derivative of regulator's objective function. Expanding (22) gives

$$\frac{\partial \mathcal{W}}{\partial r_j} = \mathbb{E}\left[A^{-1}\frac{pr_j}{C_j''}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i} - A^{-1}\frac{p}{C_j''}\frac{\partial D}{\partial e_j}\sum a_i + pe_j - A^{-1}e_j\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i} \right] = 0.$$

Simplifying and then collecting terms gives

$$\frac{\partial \mathcal{W}}{\partial r_j} = \mathbb{E}\left[A^{-1}\left(\frac{pr_j}{C_j''} - e_j\right)\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i} - \frac{p}{C_j''}\frac{\partial D}{\partial e_j} + pe_j \right] = 0.$$

Now take the weighted sum of the first-order conditions :

$$\sum r_i\frac{\partial \mathcal{W}}{\partial r_i} = \mathbb{E}\left[A^{-1}\left(p\sum_i \frac{r_i^2}{C_i''} - \sum_i r_ie_i\right)\left(\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right) - p\sum_i \frac{r_i}{C_i''}\frac{\partial D}{\partial e_i} + p\sum r_ie_i \right] = 0.$$

Using (3) and the definition of a_i it follows that

$$\mathbb{E}\left[A^{-1}\left(p\sum_i a_i - 1\right)\left(\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right) - p\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i} + p \right] = \mathbb{E}\left[(p - A^{-1} - p)\left(\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right) + p \right] = 0,$$

from which the desired result follows directly.

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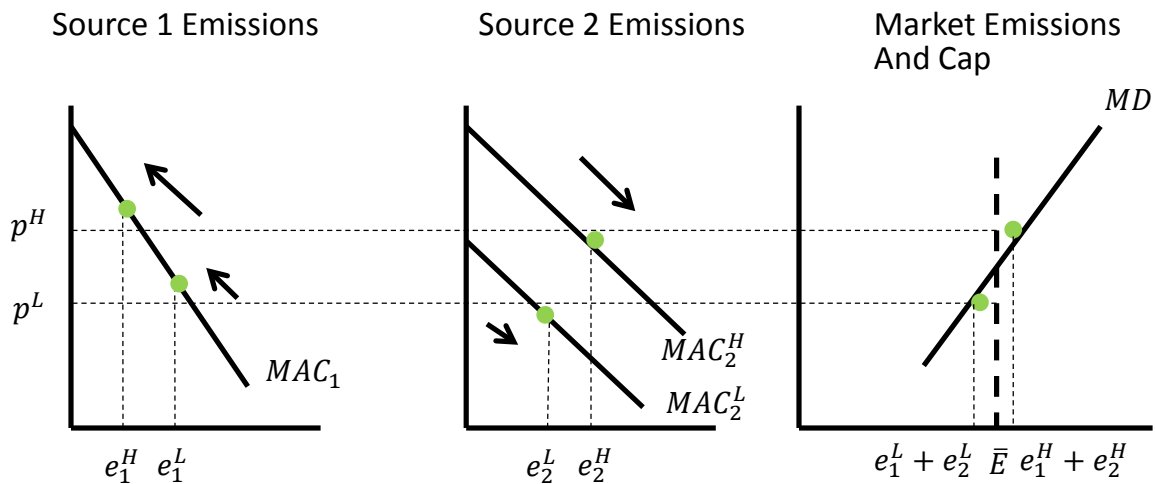
Figures

Figure 1: Marginal Damage Trading Ratios with Uniformly Mixed Pollution
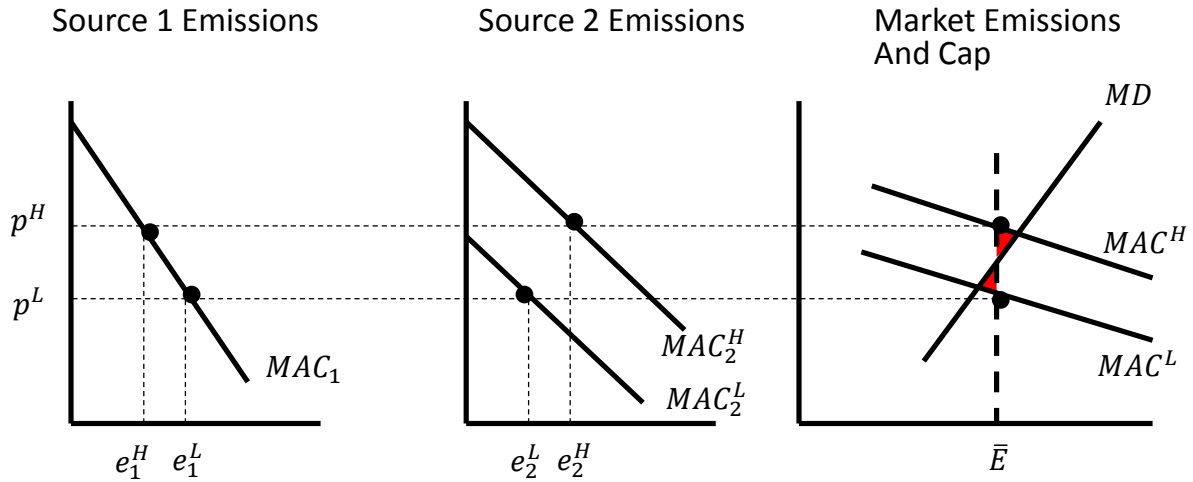
The x-axis is in units of emissions (e.g., in tons) and the y-axis measure value per unit (e.g., dollar per ton).

Figure 2: Optimal Trading Ratios with Uniformly Mixed Pollution

The x-axis is in units of emissions (e.g., in tons) and the y-axis measure value per unit (e.g., dollar per ton).

Figure 3: Location of WWTP; Marginal Damage and Optimal Trading Ratios
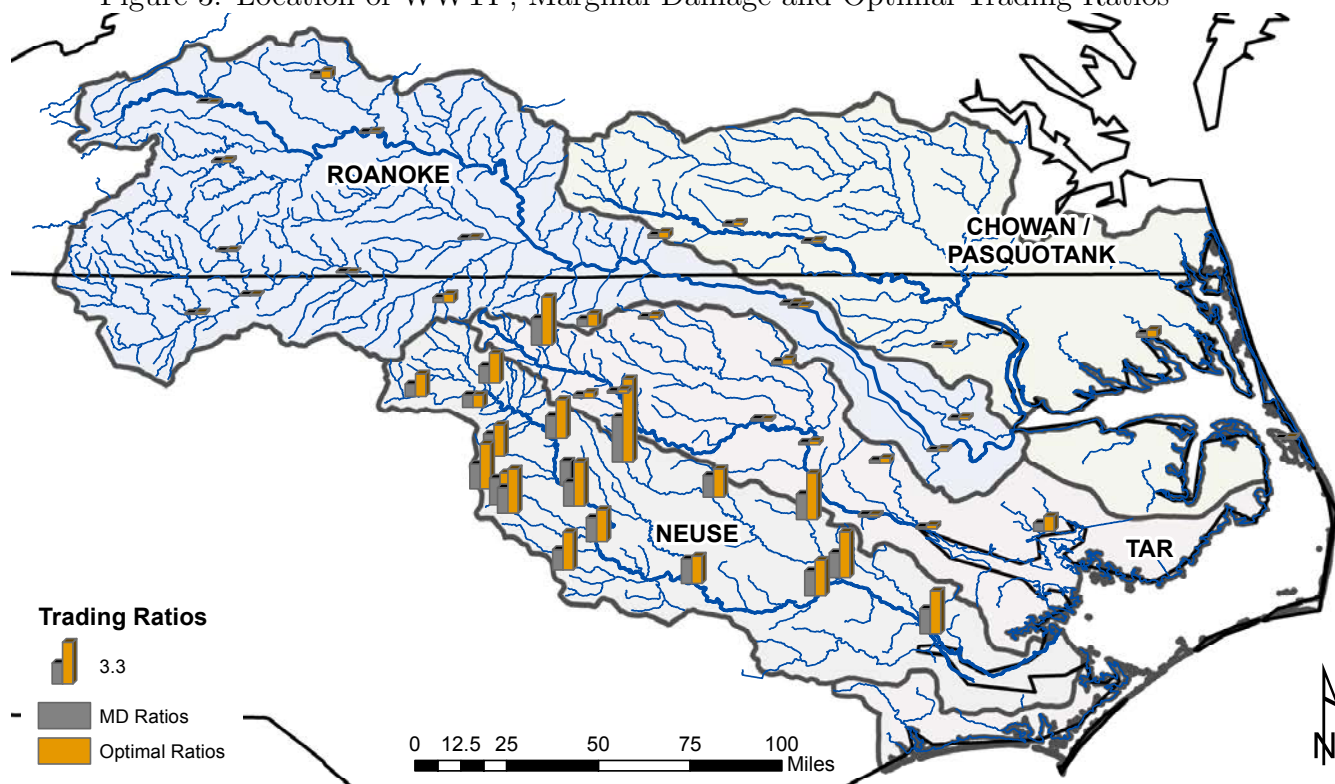
For Online Publication: Additional Appendices

A   Proofs of Results for the Linear-Quadratic Example

Preliminary Calculations

In analyzing the linear-quadratic model, it is first useful to derive a number of formulas. First, note that $a_i = r_i^2/C_i'' = r_i^2/\lambda_i$ is non-stochastic so $A = \sum_i r_i^2/\lambda_i$ is also non-stochastic. We can solve for emissions and prices by combining (2) and (13) which gives

$$e_j = \frac{\theta_j - r_j p}{\lambda_j}. \tag{27}$$

Substituting this into (3) and solving for p gives

$$p = A^{-1}\left(\sum_i \frac{r_i \theta_i}{\lambda_i} - 1\right). \tag{28}$$

This allows us to calculate the variance of the price, $VAR(p)$ as

$$VAR(p) = (A^{-1})^2 \sum_i VAR\left(\frac{r_i \theta_i}{\lambda_i}\right) = (A^{-1})^2 \sum_i a_i \frac{\sigma_i^2}{\lambda_i}. \tag{29}$$

Several covariances are also useful. Using (28) and the fact that the random variables are independent gives

$$COV(p, \theta_j) = A^{-1}COV(\sum_i \frac{r_i \theta_i}{\lambda_i}, \theta_j) = A^{-1}r_j \frac{\sigma_j^2}{\lambda_j}. \tag{30}$$

The covariance of emissions is

$$COV(e_j, e_k) = \frac{1}{\lambda_j \lambda_k}COV(\theta_j - r_j p, \theta_k - r_k p)$$

$$= \frac{1}{\lambda_j \lambda_k}\left[COV(\theta_j, \theta_k) - r_j COV(p, \theta_k) - r_k COV(\theta_j, p) + r_j r_k VAR(p)\right]$$

$$= \frac{1}{\lambda_j \lambda_k}\left[COV(\theta_j, \theta_k) - r_j r_k A^{-1}\left(\frac{\sigma_j^2}{\lambda_j} + \frac{\sigma_k^2}{\lambda_k}\right) + r_j r_k (A^{-1})^2 \sum_i a_i \frac{\sigma_i^2}{\lambda_i}\right]$$

$$= \frac{1}{\lambda_j \lambda_k}\left[COV(\theta_j, \theta_k) - r_j r_k A^{-1}\left(\frac{\sigma_j^2}{\lambda_j} + \frac{\sigma_k^2}{\lambda_k} - A^{-1}\sum_i a_i \frac{\sigma_i^2}{\lambda_i}\right).\right] \tag{31}$$

With these in hand, we turn to proving the results for the linear-quadratic example.

Proof of LQ Result 1. To prove the first part, we have

$$COV(p, e_j) = \frac{1}{\lambda_j}\left(COV(p, \theta_j) - COV(p, r_j p)\right) = \frac{1}{\lambda_j}\left(COV(p, \theta_j) - r_j VAR(p)\right)$$

$$= \frac{A^{-1}r_j}{\lambda_j}\left(\frac{\sigma_j^2}{\lambda_j} - A^{-1}\sum_i a_i \frac{\sigma_i^2}{\lambda_i}\right), \tag{32}$$

where the first equality comes from substituting (27) for e_j, the second equality comes from algebra, and the third equality comes from substituting (30) for $COV(p, \theta_j)$ and (29) for $VAR(p)$.

To prove the second part, notice that Equation 32 implies that the covariances, evaluated at any R, will all be equal to zero if and only if $\frac{\sigma_j^2}{\lambda_j}$ is equal to the same constant for every j. The desired result now follows from Corollary 1. ■

Proof of Result 2. Consider the first-order Taylor series expansion of $\mathcal{W}$ at the point $\tilde{R}$. We have

$$\mathcal{W}(R) \approx \mathcal{W}(\tilde{R}) + \nabla W \cdot (R - \tilde{R}),$$

where $\nabla W = (\frac{\partial \mathcal{W}}{\partial r_1}, \frac{\partial \mathcal{W}}{\partial r_2}, \ldots, \frac{\partial \mathcal{W}}{\partial r_n})$ is the gradient. It is well known that the vector $-\nabla W/|\nabla W|$ points in the direction of maximum decrease in the function $\mathcal{W}$. So we have

$$\mathcal{W}(R) - \mathcal{W}(\tilde{R}) \approx -\nabla W \cdot (\nabla W/|\nabla W|) = -|\nabla W|$$

gives the first-order approximation to the efficiency advantage of the optimal trading ratios relative to the marginal damage trading ratios. From Corollary 1 and Result 1 we have

$$\left.\frac{\partial \mathcal{W}}{\partial r_j}\right|_{\tilde{R}} = COV(p, e_j) = \frac{A^{-1}\tilde{r}_j}{\lambda_j}\left(\frac{\sigma_j^2}{\lambda_j} - A^{-1}\sum_i a_i \frac{\sigma_i^2}{\lambda_i}\right).$$

It follows that

$$|\nabla W| = \sqrt{\sum_i^n \left(\frac{A^{-1}\tilde{r}_j}{\lambda_j}\right)^2 \left(\frac{\sigma_j^2}{\lambda_j} - A^{-1}\sum_i a_i \frac{\sigma_i^2}{\lambda_i}\right)^2}.$$

This expression holds for all regular damage functions. Now we use the additional information in the statement of the result to further simplify it. Because pollution is uniformly mixed we have $\tilde{r}_j = \tilde{r}$ for every j. Combining this with $\lambda = 1$ implies that $a_i = \tilde{r}^2$ and $A^{-1} = \frac{1}{n\tilde{r}^2}$. So the expression above simplifies to

$$|\nabla W| = \sqrt{\sum_i^n \left(\frac{1}{n\tilde{r}}\right)^2 \left(\sigma_j^2 - \frac{1}{n}\sum_i \sigma_i^2\right)^2},$$

and further to

$$|\nabla W| = \frac{1}{\tilde{r}\sqrt{n}}\sqrt{\left(\frac{1}{n}\right)\sum_i^n\left(\sigma_j^2 - \frac{1}{n}\sum_i\sigma_i^2\right)^2},$$

from which the result follows directly. ∎

Proof of LQ Result 3.

To evaluate the regulator's objective, we begin by evaluating expected abatement costs. From (12), expected abatement costs for i are

$$\mathbb{E}[C_i(e_i;\theta_i)] = \frac{\lambda}{2}\mathbb{E}\left[\left(\frac{\theta_i}{\lambda} - e_i\right)^2\right]$$

$$= \frac{\lambda}{2}VAR\left(\frac{\theta_i}{\lambda} - e_i\right) + \frac{\lambda}{2}\left(\mathbb{E}\left[\frac{\theta_i}{\lambda} - e_i\right]\right)^2$$

$$= \frac{\lambda}{2}VAR\left(\frac{r_ip}{\lambda}\right) + \frac{\lambda}{2}\left(\frac{\bar{\theta}_i}{\lambda} - \mathbb{E}[e_i]\right)^2$$

$$= \frac{r_i^2\sigma^2}{2\lambda\sum r_i^2} + C_i(\mathbb{E}[e_i];\bar{\theta}_i). \tag{33}$$

where the third equality follows from (27) and the fourth from (29).

Now carry out a similar manipulation of the damage function

$$D(E) = \frac{1}{2}\sum_i\sum_j v_{ij}e_ie_j + \sum\omega_ie_i.$$

We have

$$\mathbb{E}[D(E)] = \frac{1}{2}\sum_i\sum_j v_{ij}\mathbb{E}[e_ie_j] + \sum\omega_i\mathbb{E}[e_i]$$

$$= \frac{1}{2}\sum_i\sum_j v_{ij}\left(COV[e_i,e_j] + \mathbb{E}[e_i]\mathbb{E}[e_j]\right) + \sum\omega_i\mathbb{E}[e_i]$$

$$= \frac{1}{2}\sum_i\sum_j v_{ij}COV[e_i,e_j] + D(\mathbb{E}[E])$$

$$= \frac{1}{2\lambda^2}\sum_i\sum_j v_{ij}\left(COV[\theta_i,\theta_j] - \frac{r_ir_j\sigma^2}{\sum r_i^2}\right) + D(\mathbb{E}[E])$$

$$= \frac{1}{2\lambda^2}\left(\sigma^2\sum_i v_{ii} - \frac{\sigma^2}{\sum r_i^2}\sum_i\sum_j v_{ij}r_ir_j\right) + D(\mathbb{E}[E])$$

$$= \frac{\sigma^2}{2\lambda^2}\left(\sum v_{ii} - \frac{R^t\mathbf{V}R}{R^t\mathbf{I}R}\right) + D(\mathbb{E}[E]) \tag{34}$$

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where the fourth equality follows from (31). In this expression $\mathbf{I}$ is the identity matrix and R^t is the transpose of R.

Summing (33) over i and adding it to (34) establishes the result. ■

Proof of Result 4.

We start with two preliminary observations. First, we can write (5) in vector form as

$$R = k\mathbf{V}\mathbb{E}[E]$$

for some constant k. Under (16), we can write simplify the expression for e_j in (27) and substitute into the above, giving

$$R = \frac{k}{\lambda}\mathbf{V}(\mathbb{E}[\Theta] - R\mathbb{E}[p]).$$

This can be written as

$$R + \frac{k}{\lambda}\mathbb{E}[p]\mathbf{V}R = \frac{k}{\lambda}\mathbf{V}\mathbb{E}[\Theta]$$

and

$$(\mathbf{I} + \frac{k}{\lambda}\mathbb{E}[p]\mathbf{V})R = \frac{k}{\lambda}\mathbf{V}\mathbb{E}[\Theta]. \tag{35}$$

Second, we note that the eigenvectors of a matrix $\mathbf{X}$ are the same as the eigenvectors of the matrices $\mathbf{X}^{-1}$, $(\mathbf{I} + q\mathbf{X})$, and $(\mathbf{I} + q\mathbf{X})^{-1}$ where q is a scalar constant.

Proceeding to the main proof, we now show that if $\mathbb{E}[\Theta]$ is an eigenvector of $\mathbf{V}$, then the optimal trading ratios are equal to the marginal damage trading ratios. Let Z be the eigenvector of $\mathbf{V}$ and let ζ be its eigenvalue. Since $\mathbb{E}[\Theta]$ is an eigenvector of $\mathbf{V}$, (35) implies that

$$(\mathbf{I} + \frac{k}{\lambda}\mathbb{E}[p]\mathbf{V})R = \frac{k}{\lambda}\zeta\mathbb{E}[\Theta].$$

Solving for R gives

$$R = \frac{k\zeta}{\lambda}(\mathbf{I} + \frac{k}{\lambda}\mathbb{E}[p]\mathbf{V})^{-1}\mathbb{E}[\Theta] = k_1\mathbb{E}[\Theta],$$

where the second equality follows since $\mathbf{V}$ and $(\mathbf{I} + q\mathbf{V})^{-1}$ have the same eigenvectors.

Having established that R_t is proportional to $\mathbb{E}[\Theta]$, we know that R_t is an eigenvalue of $\mathbf{V}$, and is also proportional to Z. Because $\mathbb{E}[\Theta]$ is positive, we know that the eigenvector Z is positive. Furthermore, because $\mathbf{V}$ is symmetric, all eigenvectors of $\mathbf{V}$ are orthogonal. So Z is the only non-negative eigenvector of $\mathbf{V}$. It follows from the Frobenius-Perron Theorem that ζ is at least as large as any other eigenvalue of $\mathbf{V}$.

Now consider the regulator's objective (17). For the marginal damage trading ratios, the regulator selects R to minimize this expression, subject to the constraint that $R = k\mathbf{V}\mathbb{E}[E]$ For any R_t that satisfies the constraint, we have established that R_t is an eigenvalue of $\mathbf{V}$. Thus for these R_t, the σ^2 term in the regulator's objective function is equal to a constant. So marginal damage trading ratio $\tilde{R}$ minimizes the expected value term. This implies that $\tilde{R}$ is proportional to the marginal damages evaluated at the optimal emissions standards (which maximize the expected value term for *any* R). Now once again appealing to Kaiser

and Rice (1973), because ζ is at least as large as any other eigenvalue of $\mathbf{V}$, it follows that $\tilde{R}$ also minimizes the σ^2 term for *any* R. Since $\tilde{R}$ maximizes both terms of (17), there cannot be any other value for R that leads to a greater value for the sum of these terms. Hence the optimal trading ratios are equal to the marginal damage trading ratios.

Next we show that if the optimal trading ratios are equal to the marginal damage trading ratios, then $\mathbb{E}[\Theta]$ is an eigenvector of $\mathbf{V}$. From Proposition 1 we have

$$\left.\frac{\partial \mathcal{W}}{\partial r_j}\right|_{\tilde{R}} = COV(p, e_j) + \sum_i COV\left(\tilde{r}_j \frac{\partial D}{\partial e_i} - \tilde{r}_i \frac{\partial D}{\partial e_j}, A^{-1} \frac{a_i p}{\tilde{r}_i C_j''}\right)$$

$$- COV\left(A^{-1} \sum_i \frac{a_i}{\tilde{r}_i} \frac{\partial D}{\partial e_i}, e_j\right) + \mathbb{E}\left[\left(p - A^{-1} \sum_i \frac{a_i}{\tilde{r}_i} \frac{\partial D}{\partial e_i}\right)\right] \mathbb{E}[e_j]$$

From (16) and Result 1, we know that $COV(p, e_j) = 0$ for every j. Thus the first term in this equation is zero. All the covariances in the second term are zero as well, because for the quadratic model marginal damages are $\frac{\partial D}{\partial e_j} = \sum_k v_{j,k} e_k$, and, in addition, a_i is non-stochastic. So the second term can be written as the sum of covariances of e_j and p, which of course are equal to zero. Now focus on the third term

$$\text{Term 3} = -COV\left(A^{-1} \sum_i \frac{a_i}{\tilde{r}_i} \frac{\partial D}{\partial e_i}, e_j\right) = -\frac{A^{-1}}{\lambda} \sum_i \tilde{r}_i COV\left(\frac{\partial D}{\partial e_i}, e_j\right)$$

Substituting in the expression for marginal damage gives

$$\text{Term 3} = -\frac{A^{-1}}{\lambda} \sum_i \tilde{r}_i \sum_k v_{i,k} COV(e_k, e_j) = -\frac{A^{-1}}{\lambda} \sum_i \tilde{r}_i \sum_k v_{i,k} \frac{1}{\lambda^2}\left(COV[\theta_k, \theta_j] - \frac{\tilde{r}_k \tilde{r}_j \sigma^2}{\sum \tilde{r}_i^2}\right).$$

where the second equality follows from (16) and (31). Now, because the random variables are independent, the $COV(\theta_k, \theta_j)$ will be equal to σ^2 when $k = j$ and 0 otherwise. So we have

$$\text{Term 3} = -\frac{A^{-1}}{\lambda^3} \sum_i \tilde{r}_i \left(v_{i,j}\, \sigma^2 - \sum_k v_{i,k} \frac{\tilde{r}_k \tilde{r}_j \sigma^2}{\sum \tilde{r}_i^2}\right) = -\frac{A^{-1}\sigma^2}{\lambda^3}\left(\sum_i \tilde{r}_i v_{i,j} - \frac{\tilde{r}_j}{\sum \tilde{r}_i^2} \sum_i \sum_k v_{i,k} \tilde{r}_i \tilde{r}_k\right).$$

Because the optimal trading ratios are equal to the marginal damage trading ratios, it must be the case that

$$\left.\frac{\partial \mathcal{W}}{\partial r_j}\right|_{\tilde{R}} = 0 \text{ for every } j.$$

We can write this system of n equations in terms of the n variables r_i using matrix-vector notation. This gives

$$-\frac{A^{-1}\sigma^2}{\lambda^3}\left(\mathbf{V}\tilde{R} - \tilde{R}\frac{\tilde{R}^t \mathbf{V}\tilde{R}}{\tilde{R}\mathbf{I}\tilde{R}}\right) + \mathbb{E}\left[\left(p - A^{-1} \sum_i \frac{a_i}{\tilde{r}_i} \frac{\partial D}{\partial e_i}\right)\right] \mathbb{E}[E] = 0.$$

By Proposition 2, we know that first expectation is equal to zero. It follows that $\tilde{R}$ is an eigenvector of V.

Now, since $\tilde{R}$ is an eigenvector of V, it is also an eigenvector of $(\mathbf{I} + q\mathbf{V})$. Thus from (35) we have

$$\gamma R = \frac{k}{\lambda}\mathbf{V}\mathbb{E}[\Theta]$$

which implies

$$\mathbb{E}[\Theta] = \frac{\gamma\lambda}{k}\mathbf{V}^{-1}R = \frac{\gamma\eta\lambda}{k}R,$$

where the second equality follows the fact that $\mathbf{V}$ and $\mathbf{V}^{-1}$ have the same eigenvectors. This means that $\mathbb{E}[\Theta]$ is proportional to R, and hence $\mathbb{E}[\Theta]$ is an eigenvector of $\mathbf{V}$ as well. ∎

B   Carbon Trading Computation

Ackerman and Bueno (2011) describes the adaptation of the well-known McKinsey abatement cost curves for 2030 (from McKinsey & Company) for use in their integrated assessment model (IAM) of global climate change named the Climate and Regional Economics of Development (CRED) model. The McKinsey curves are controversial for their estimates of substantial quantities of negative cost abatement opportunities. Ackerman and Bueno circumvent this difficulty by fitting a simple two-parameter functional form through the positive cost portion of the McKinsey abatement cost curves. The functional form is $a_i A/(b_i - A)$ where A is abatement and a_i and b_i are the fitted parameters. The function goes through the origin by assumption.

Figure 4 shows an example of the fitted functional form and the McKinsey abatement cost curve for the industrial sector in S/SE Asia. Note that the curve asymptotes to b_i (the vertical line on the right). Thus b_i can be thought of as BAU emissions, i.e., the maximum possible abatement. Note also that a_i is the marginal abatement costs when emissions (equivalently abatement) are half of b_i.

We use the equations by converting them to functions of emissions (rather than abatement) and by introducing stochastic BAU emissions. Thus the marginal abatement cost curve is

$$-\frac{\partial C_i}{\partial e_i} = \frac{a_i(b_i(1 + \theta_i) - e_i)}{e_i}.$$

Integrating the marginal abatement cost curve yields the total abatement costs

$$C_i(e_i; \theta_i) = a_i e_i - a_i b_i(1 + \theta_i)\ln(e_i)$$

Note that abatement costs would be infinite if emissions were zero. Also note that the integration yields an unspecified constant of integration, so we calculate deadweight loss as the difference between the first-best, full-information outcome and the policy of interest.

The appeal of this functional form is twofold. First, the function form is simple, and we can introduce stochasticity in a transparent way. Second, because $-\frac{\partial C_i}{\partial e_i}$ asymptotes to zero, we need not worry about the boundary condition of nonnegative emissions. As long as the price is positive, we safely have an interior solution and the second order conditions are

satisfied.

To reduce the dimensions of the problem, we focus on the industrial sectors in the four regions with the largest estimated b_i's. These four regions are reported in Table 6. Table 6 reports the fitted coefficients for 2030 along with the actual emissions from 2006-2008. The b_i are reasonable approximations of BAU emissions.

The calculations are done in Mathematica. With four regions and three independent outcomes of each random variable, there are eighty-one possible states of the world to evaluate. To calculate the first-best, full-information outcome, we calculate the optimal emissions cap for each of the eighty-one states. Because marginal damage trading ratios for a uniformly mixed pollutant imply one-for-one trading, the marginal damage trading ratios can be calculated by optimizing the emissions cap that minimizes the sum of expected abatement costs and damages.

Calculation of the optimal trading ratios requires optimization over a four-dimensional vector of trading ratios. We first calculate the equilibrium that would result from trading for a given vector of trading ratios. We then optimize the trading ratio vector to minimize the sum of expected abatement costs and damages.

Figure 4: McKinsey Marginal Abatement Cost Curve and Ackerman-Bueno Approximation
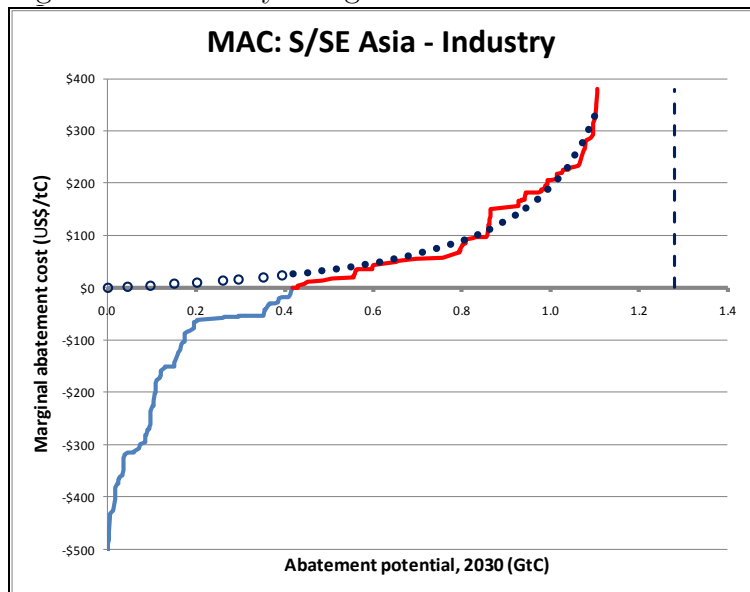
Source: Ackerman and Bueno (2011)

C   Illustrating Optimal Source-Specific Taxes

Figure 5 illustrates optimal source-specific environmental taxes from (11). The figure illustrates marginal damages and "high" and "low" marginal abatement costs for a single source where the high and low costs occur with equal probability. If marginal abatement costs in the low-cost state are MAC_1^L, then the slope of the marginal abatement costs are equal across the two states; the COV term in (11) is zero; and the regulator should set the source-

Table 6: Coefficients for Industry Marginal Abatement Cost Curves Coefficients and Fossil-Fuel Emissions

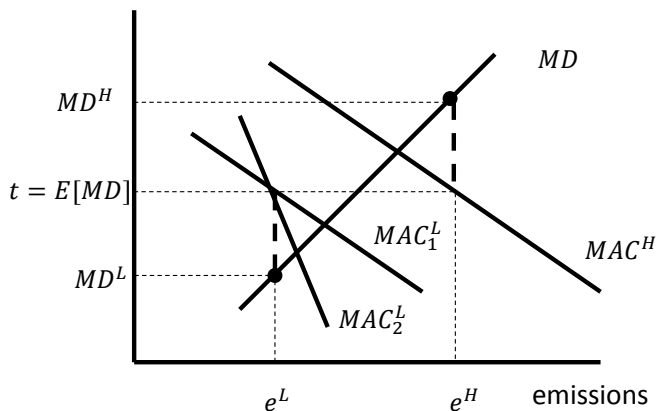|	a_i	b_i	Emissions
China	66.39	2.64	2.02
Europe	101.91	0.83	1.19
S/SE Asia	53.90	1.28	0.56
U.S.A.	64.74	1.39	1.72

Coefficients from Ackerman and Bueno (2011).
The a_i coefficient is the marginal abatement cost when emissions are half of BAU in $ per ton C. The b_i coefficient is the maximal possible abatement, which can be interpreted as BAU in GtC. Emissions (from fossil fuels and cement manufacture) are from the World Bank database averaged across 2006-08 in GtC.

specific tax equal to expected marginal damages. As illustrated, the resulting emissions in the high-cost state are e^H and in the low-cost state are e^L.

However, if marginal abatement costs in the low-cost state are MAC_2^L, then the slope of the marginal abatement cost curve is greater in the low-cost state, and the COV term in (11) is positive. In this case, the regulator can increase efficiency by setting the source-specific tax above the expected marginal damage. In fact, in the extreme case in which MAC_2^L is perfectly inelastic, the regulator could attain the first best by setting the source specific tax such that $MD = MAC^H$.

Figure 5: Optimal Source-Specific Taxes

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D   Illustrating Optimal Trading Ratios

The intuition of optimal trading ratios is illustrated in Figures 6, 7, and 8. Figure 6 shows convex iso-damage and iso-cost curves for two sources of emissions. Points further from the origin have higher damages. Abatement costs are minimized at Point A, the unregulated emissions vector, and increase at points further away from Point A. The sum of abatement costs and damages is minimized somewhere along the locus of tangencies of the iso-cost and iso-damage curves. The separating hyperplane theorems imply that a regulator can implement the efficient emissions vector by trading under an emissions cap. An emissions cap which implements (e_1^*, e_2^*) is illustrated in Figure 6. The slope of the emissions cap budget, r_1/r_2, is the trading ratio and reflects trading between the two sources. By the implicit function theorem, the slope of the emissions cap budget should equal the ratio of marginal damages. This is the theoretical basis for marginal damage trading ratios.

With uncertain abatement costs, the theoretical basis for marginal damage trading ratios no longer holds. Figure 7 interprets the solid iso-cost circles as the expectation and the dashed iso-cost circles as realizations of the costs. The illustrated emissions trading cap represents the marginal damages trading ratios. As illustrated abatement costs are lower when reaching the emissions trading cap under the upper-left abatement cost realization. Thus, we call this the low-cost state and its unregulated emissions vector is labeled A^L. On the other hand, abatement costs are higher when reaching the emissions trading cap under the lower-right abatement cost realization. Thus, we call this the high-cost state and its unregulated emissions vector is labeled A^H.

Since the price of permits reflects the marginal abatement costs, the permit price is high in the high-cost state, and emissions from Source 1 are positively correlated with the permit price in Figure 7. Since this is the marginal damages trading ratio cap, the first term of the equation in Proposition 1 is positive, which would imply that the regulator may be able to improve efficiency by decreasing r_1. However, in general there are additional terms in the equation in Proposition 1. In the special case of regular damages—as defined in (1)—the slope of the regulator's objective is given by $COV(p, e_i)$.

The case of regular damages is illustrated in Figure 8. With regular damages, the iso-damages curves are parallel lines with slope α_1/α_2. Thus the marginal damages trading ratios hold damages constant. However, as illustrated the regulator can increase efficiency by tightening the cap in the low-cost state and loosening the cap in the high-cost state. This is the result in Corollary 1. Since $COV(p, e_1) > 0$, the regulator can improve efficiency by reducing r_1, i.e., by flattening the emissions trading budget line as illustrated by the lighter shaded line.

E   Sensitivity Analysis: Nitrogen Trading Example

In Table 7 we show the results when we vary the expected value (and variance) of the WWTP's.

Table 7: Nitrogen Trading with Optimal Trading Ratios: Sensitivity to Distribution of Cost Parameters

	Baseline $\eta = 10$ $b = 30$	Convert all to average	Convert largest to average
Trading ratios			
Max Marginal Damage	3.553	5.025	4.755
Min Marginal Damage	0.063	0.077	0.098
Max Optimal	6.553	5.048	6.878
Min Optimal	0.015	0.077	0.012
St. Dev. Price			
Marginal Damage TR	2.171	0.844	0.816
Optimal TR	0.525	0.843	0.465
Expected Price			
Marginal Damage TR	2.737	2.103	1.749
Optimal TR	1.635	2.101	1.347
St. Dev. Total Emissions			
Marginal Damage TR	0.038	0.025	0.037
Optimal TR	0.055	0.025	0.040
Expected Total Emissions			
Marginal Damage TR	4.842	4.847	4.107
Optimal TR	4.846	4.848	4.110
Total Cost			
First Best	9.773	8.440	6.443
Marginal Damage TR	9.870	8.461	6.462
Optimal TR	9.798	8.461	6.457
Deadweight Loss			
Marginal Damage TR	0.097	0.021	0.020
Optimal TR	0.024	0.021	0.014
Percent Reduction	74.9%	.003 %	28.7%

[1] The permit endowment is set to the sum of the optimal pollution standards.

[2] Costs and deadweight loss (DWL) in millions of dollars per year.

[3] Prices in dollars per pound.

[4] Emissions in millions of pounds per year.

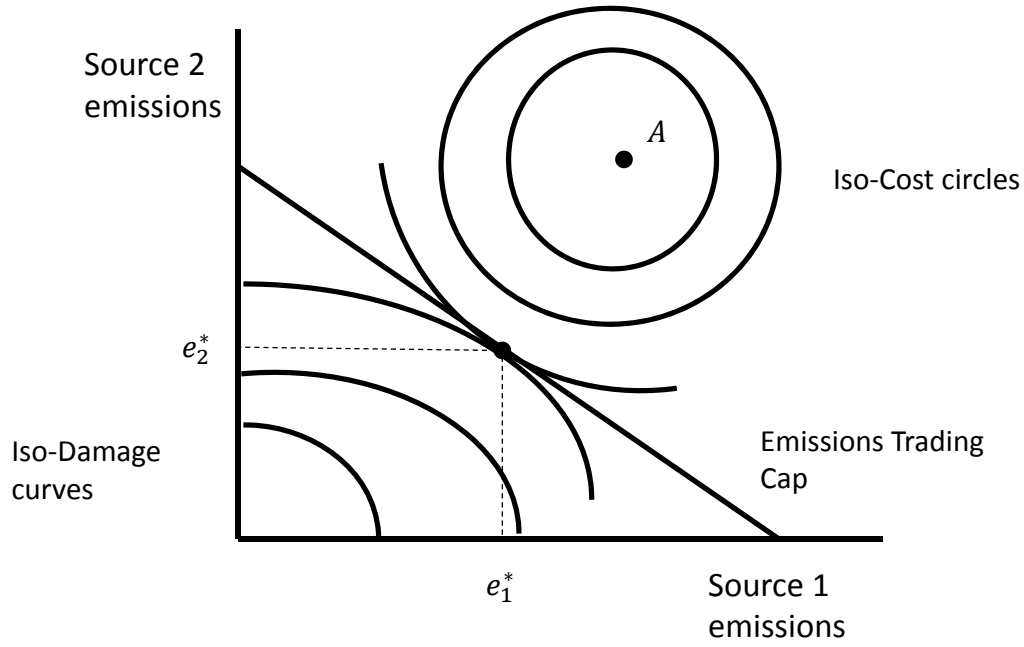Figure 6: Optimal Trading Ratios Across Two Sources: No Uncertainty

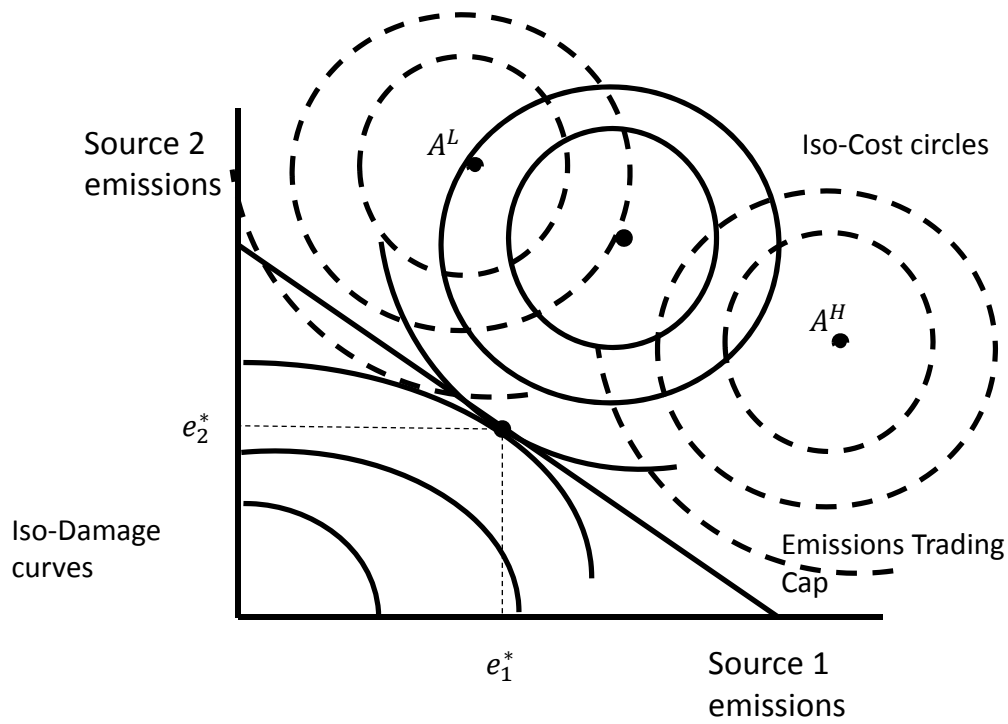Figure 7: Optimal Trading Ratios Across Two Sources: Uncertain Marginal Abatement Costs

Figure 8: Optimal Trading Ratios Across Two Sources: Regular Damages

Source 2
emissions

A^L

Iso-Cost circles

A^H

e_2^*

Iso-Damage
curves

Emissions Trading
Cap

e_1^*

Source 1
emissions