Evolutionary Dynamics of the Spatial Prisoner's Dilemma

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Chapter in the book by M.M. Peixoto et al. (eds.), Dynamics, Games and Science II, Springer Proceedings in Mathematics 2, DOI 10.1007/978-3-642-14788-3 25.

Introduction

The Royal Swedish Academy of Sciences has decided to award the Bank of Sweden Prize in **Economic Sciences in Memory of Alfred Nobel, 2005**, jointly to

Robert J. Aumann
Center for Rationality, Hebrew University of Jerusalem, Israel and

Thomas C. Schelling
Department of Economics and School of Public Policy, University of Maryland, College Park, MD, USA,

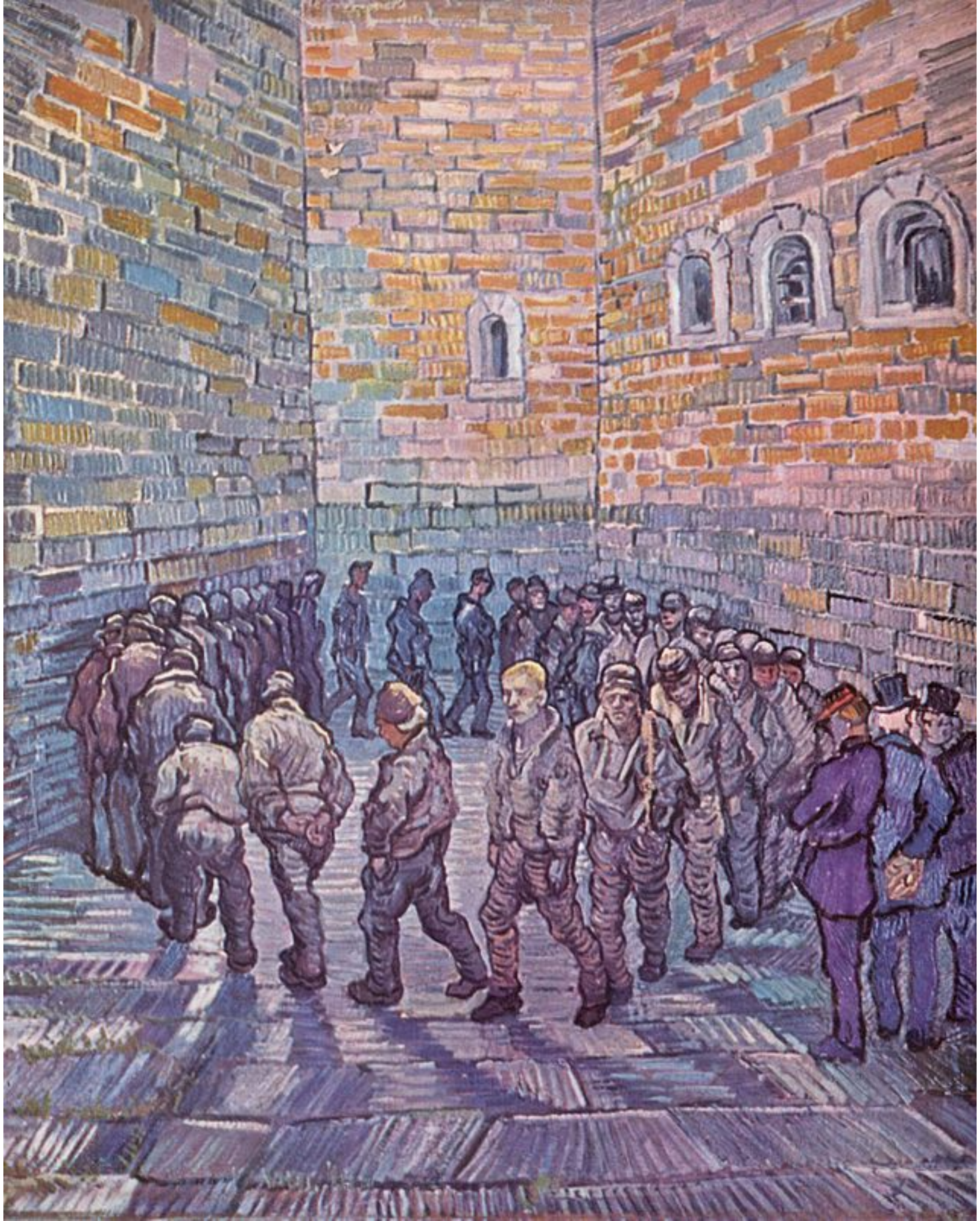
"for having enhanced our understanding of conflict and cooperation through game-theory analysis".

http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2005/press.html

Alexander Pope: ***"a little learning is a valuable thing and it is too much learning that is dangerous".***

Source:

Kraines, D., Kraines, V.: The threshold of cooperation among adaptive agents: Pavlov and the stag hunt. In: ECAI '96: Proceedings of the Workshop on Intelligent Agents III, Agent Theories, Architectures, and Languages, pp. 219–231, London, UK. Springer, Berlin (1997)



Vincent van Gogh,
The Round of the Prisoners, 1890,
[Pushkin Museum, Moscow]

Mathematics, Physics and Economics

Mathematics:

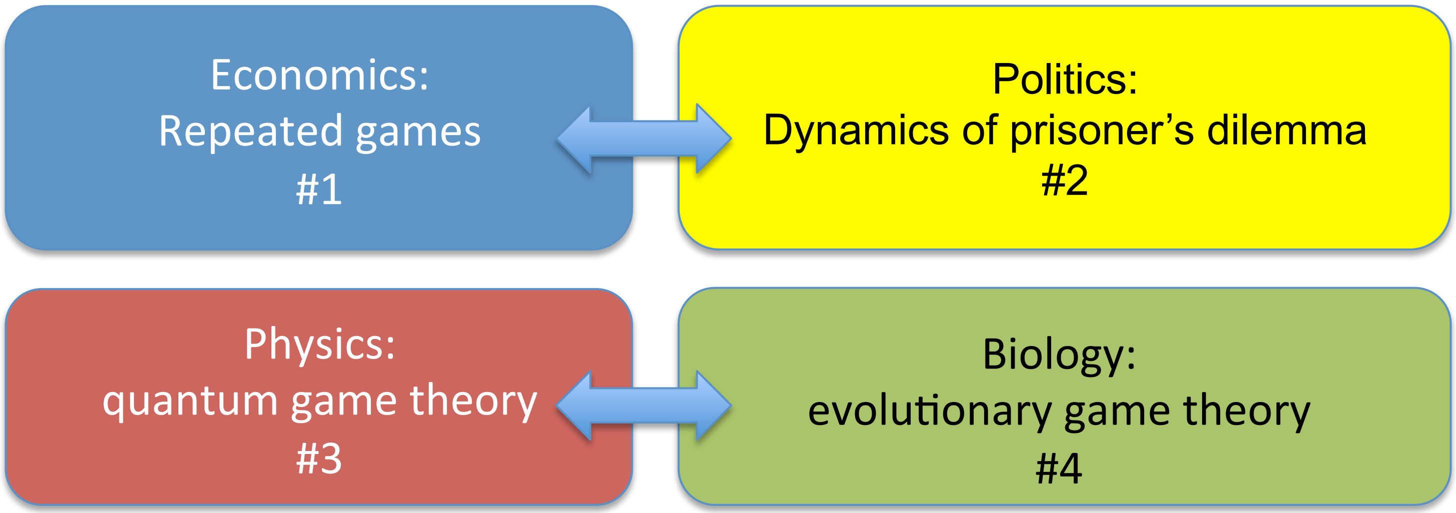
- John von Neumann and Oscar Morgenstern, Theory of Games and Economic Behavior, Princeton University Press, 1944.
- Fudenberg D. The Theory of Learning in Games. Cambridge, MA: MIT Press; 1998.
- *J. Watson, Strategy: An Introduction to Game Theory*, third edition, New York: W. W. Norton and Company, 2013.

Physics:

Is economics the next physical science?
Doyne Farmer, Martin Shubik and Eric Smith, Physics Today, 2005.

Evolutionary Games on Graphs,
Gyorgy Szabo, Gabor Fath, Physics Reports (2007)

Game theory for applications



- *#1. Repeated games where the payoffs and monitoring structure are unknown, D. Fudenberg and Y. Yamamoto, Econometrica, Vol. 78, No. 5 (September, 2010), 1673–1710.*

- *#2. Robert Axelrod, "The Evolution of Strategies in the Iterated Prisoner's Dilemma," 1987.*

- *#3. Quantum Stackelberg duopoly with incomplete information, C.-F. Lo, D. Kiang, Physics Letters A 346 (2005) 65–70.*

- *#4. Emergence of cooperation and evolutionary stability in finite populations, M. A. Nowak, A. Sasaki, C. Taylor & D. Fudenberg, Nature, vol.428, 2004.*

What is a Game?

- There are many types of games, board games, card games, video games, field games (e.g. football), etc.
- In this course, our focus is on games where:
  - There are 2 or more *players*.
  - There is some choice of action where *strategy* matters.
  - The game has one or more *outcomes*, e.g. someone wins, someone loses.
  - The outcome depends on the strategies chosen by all players; there is *strategic interaction*.
- What does this rule out?
  - Games of pure chance, e.g. lotteries, slot machines. (Strategies don't matter).
  - Games without strategic interaction between players, e.g. Solitaire.

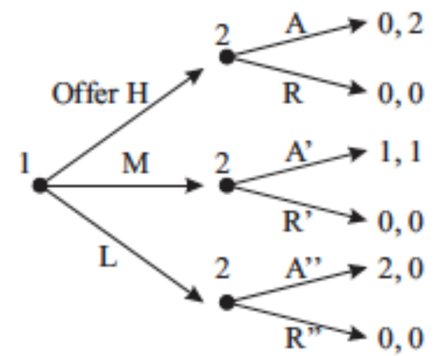
Why Do Economists Study Games?

- Games are a convenient way in which to model the strategic interactions among economic agents.
- Many economic issues involve strategic interaction.
  - Behavior in imperfectly competitive markets, e.g. Coca-Cola versus Pepsi.
  - Behavior in auctions, e.g. Investment banks bidding on U.S. Treasury bills.
  - Behavior in economic negotiations, e.g. trade.
- Game theory is not limited to Economics.

Strategic Behavior in Elections and Markets

Representing games

To describe a game, formally specify: (1) the list of players, (2) the possible actions for each player, (3) their knowledge (what each player knows when he/she takes an action), (4) how actions lead to outcomes, and (5) the players' preferences over outcomes.



Extensive Form Representation: A tree, featuring

Nodes – where actions are taken or the game ends
Branches – actions
Labels – player on the move (for decision nodes), actions (for branches)
Payoff numbers – represent preferences
Information sets – represent the players' information

1 \ 2	AA'A"	AA'R"	AR'A"	AR'R"	RA'A"	RA'R"	RR'A"	RR'R"
H	0, 2	0, 2	0, 2	0, 2	0, 0	0, 0	0, 0	0, 0
M	1, 1	1, 1	0, 0	0, 0	1, 1	1, 1	0, 0	0, 0
L	2, 0	0, 0	2, 0	0, 0	2, 0	0, 0	2, 0	0, 0

Strategy and Normal Form

Strategy – a complete contingent plan for a player in a game

Normal Form Representation:

a description of strategy spaces and payoffs.

For games with two players and a finite number of strategies, the normal form can be written as a table with appropriate labels.

Notation:

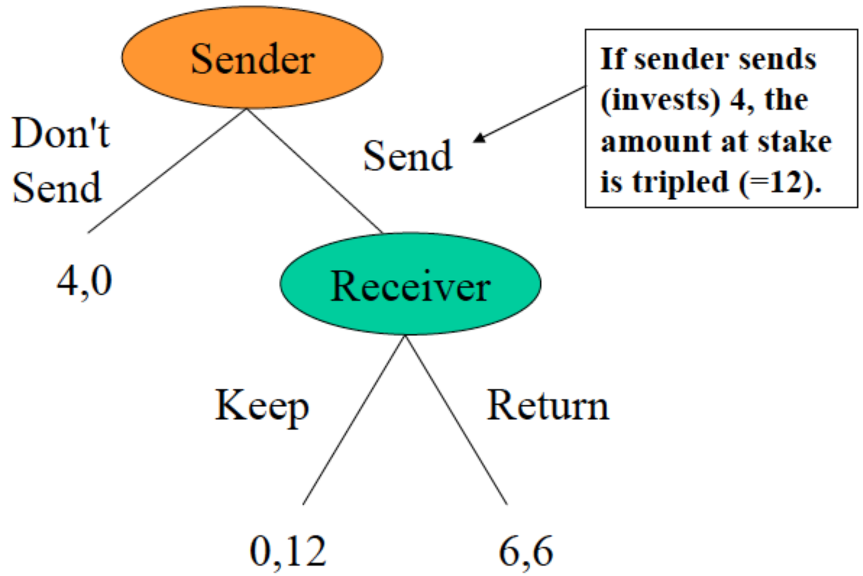
S_i player i's strategy set
s_i individual strategy for player i
s_{-i} strategies of players other than i

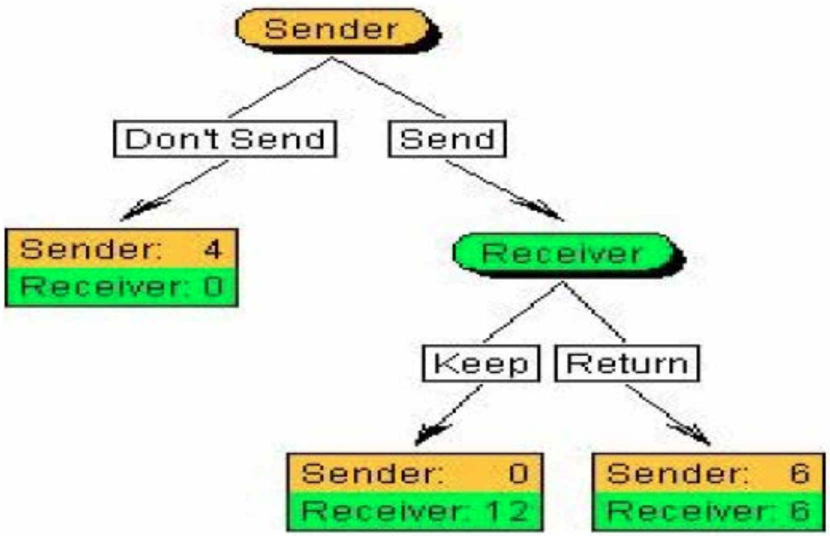
$S = S_1 \times \cdots \times S_n$ set of strategy profiles
$s = (s_1, s_2, \ldots, s_n)$ individual strategy profile
$s = (s_i, s_{-i})$

$u_i : S \to \mathbf{R}$ payoff function for player i

Investment game

Sequential Move Game





- Players choose actions in a particular sequence are sequential move games.
- Player is either the sender or the receiver.
- If player is the receiver, wait for the sender's decision.

Nash equilibrium

A situation in which neither of the players can improve his payoff by a unilateral change of strategy is a Nash equilibrium.

John F. Nash, Equilibrium Points in n-Person Games, PNAS, 36 (1950) 48-49.

Once a Nash equilibrium has been reached no player has a reason to deviate from his strategy- even if another state would provide a higher payoff for both players.

A strategy profile is called a **subgame perfect Nash equilibrium** if it specifies a Nash equilibrium in every **subgame** of the original game.

Monopoly manufacturer/monopoly retailer

M produces at a cost \$10 per unit.
M sells to R, who then sells to consumers.
The inverse demand curve is $p = 200 - q/100$.

The game runs as follows: (1) M chooses a price x to offer to R.
(2) R observes x and then chooses how many units q to purchase.
(3) M obtains profit $u_M = q(x - 10)$; R obtains $(200 - q/100)q - xq$.

Calculating the subgame-perfect Nash equilibrium:

Note that there are an infinite number of information sets for R, each is identified by a number x, and each initiates a subgame.

Calculate the equilibrium of these subgames, by finding R's optimal q as a function of x... $q^*(x) = 10000 - 50x$.

M can anticipate this from R, so M's payoff of choosing x is $q^*(x)(x - 10) = (10000 - 50x)(x - 10)$. M's optimum is... $x^* = 105$.

Joel Watson, Strategy: An Introduction to Game Theory, New York: W. W. Norton and Company, 2007.

Cournot Duopoly

Normal Form:

$n = 2$	two players.
$S_1 = S_2 = [0, \infty)$	strategy spaces. Denote i's strategy q_i.
$u_i(q_i, q_j) = (1 - q_i - q_j)q_i$	payoff functions. (Demand: $p = 1 - Q$, zero production cost.)

Suppose player i has the belief μ_j about the strategy of the other player (j). Then think of q_j as a random variable distributed according to θ_j.

If player i selects q_i then his/her expected payoff is

$$u_i(q_i, \theta_j) = E[(1 - q_i - q_j)q_i \mid q_j \sim \theta_j]$$

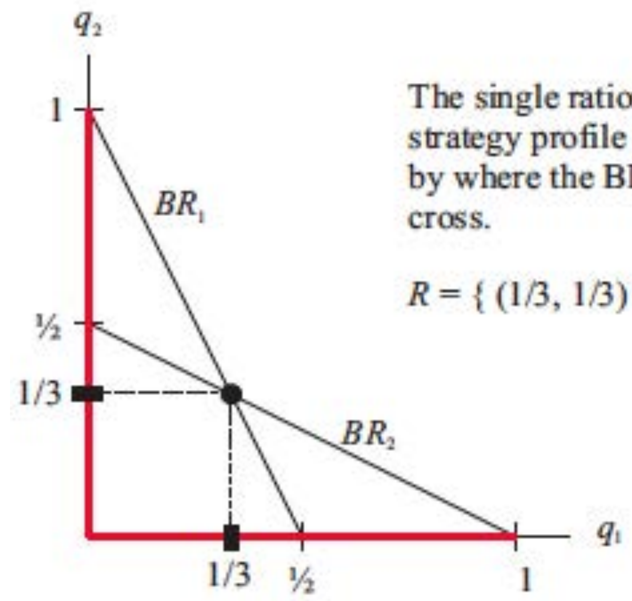
$$= E[q_i - q_i^2 - q_i q_j \mid q_j \sim \theta_j]$$

$$= q_i - q_i^2 - q_i \bar{q}_j, \text{ where } \bar{q}_j \text{ is the expected } q_j.$$

In other words, we can write $u_i(q_i, \theta_j) = u_i(q_i, \bar{q}_j)$ and just think of player j as choosing $\bar{q}_j$ for sure.

Player i's best response is $BR_i(\bar{q}_j) = (1 - \bar{q}_j) / 2$.

Note: Regardless of player i's belief, his/her best response is always in the interval $[0, 1/2]$. Thus strategies above 1/2 are dominated.



The single rationalizable strategy profile is identified by where the BR functions cross.

$R = \{ (1/3, 1/3) \}$

Joel Watson, Strategy: An Introduction to Game Theory, New York: W. W. Norton and Company, 2007.

Repeated Cournot duopoly

Stage game: Players select quantities q_1 and q_2.
Assume a zero-cost production technology.
Demand is given by $p = 24 - q_1 - q_2$.
Player i's payoff is $(24 - q_1 - q_2)q_i$.

Stage Nash: ... $q_1^* = q_2^* = 8$ ... each gets a payoff of 64.

Collusion: Share monopoly quantity; each produce 6 and get a payoff of 72.

Deviation gain: The best way to deviate when $q_j = 6$ is to produce $q_i = 9$, which gives a payoff of 81.

Grim trigger: In a given period, choose $q_i = 6$ as long as both players selected 6 in the past; otherwise, revert to the stage Nash quantity 8.

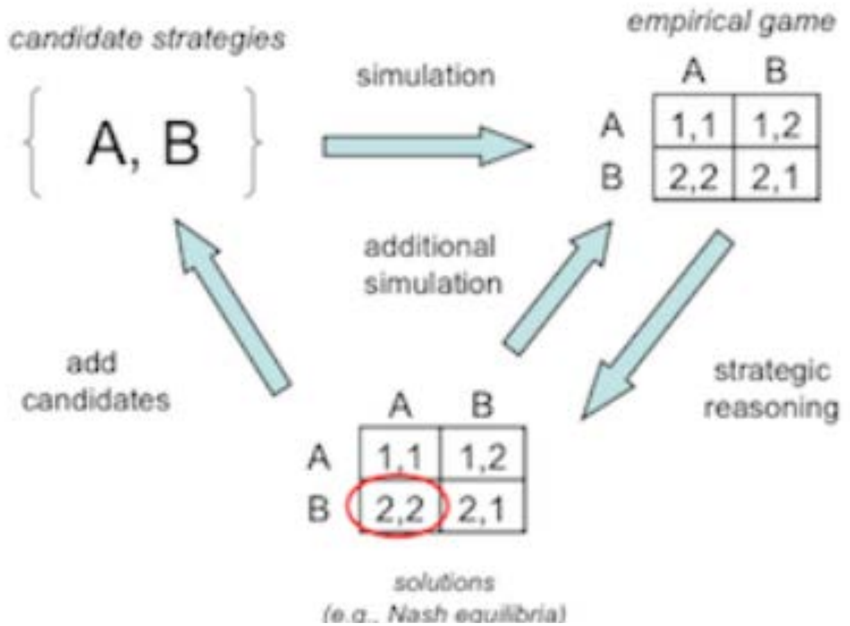
In a repeated game, players interact by playing a stage game in each of a number of periods.

Their payoffs for the repeated game are the sum of stage-game payoffs in the individual periods (sometimes discounted).

Joel Watson, Strategy: An Introduction to Game Theory, New York: W. W. Norton and Company, 2007.

Empirical game theory

Building models of games using simulation or other empirical evidence.

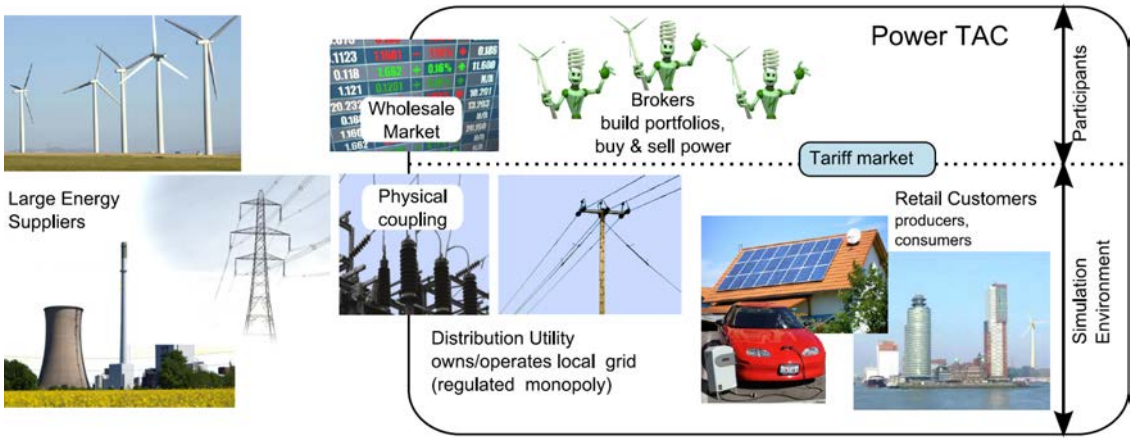


Trading Agent Competition
Supply Chain Management game.

http://www.powertac.org

http://tradingagents.org/

http://tac.sics.se/page.php?id=1



Robust Bayesian Methods for Stackelberg Security Games, C Kiekintveld, J Marecki, M Tambe, *Ninth International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2010.*

Designing games to conduct experiments online

http://www.comlabgames.com

Examples of Auction Games

1. First Price Sealed Bid Auction
2. English Private Value Auction
3. Dutch Private Value Multi Unit Auction
4. Strategic Equivalence First and Dutch auction – common value auction

Examples of Market Games

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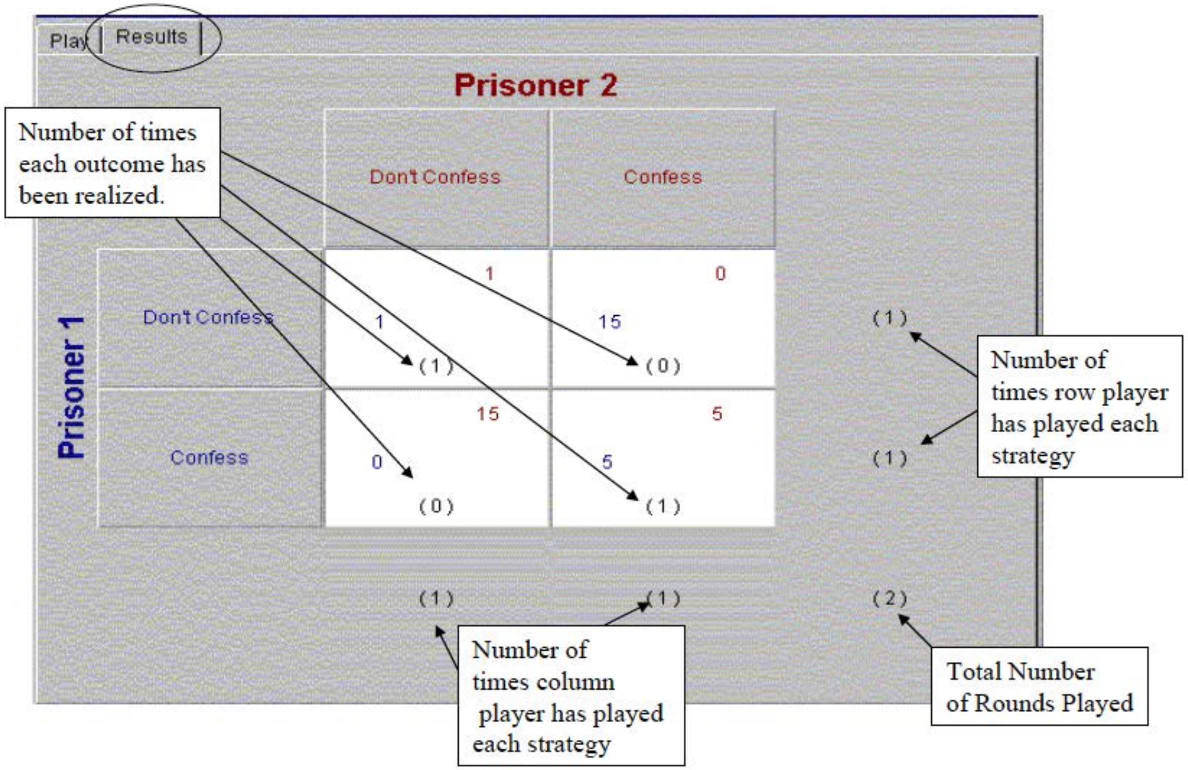
1. Telecommunication providers
2. Telecommunication providers with changing demand factors
3. Currency exchange – basic model
4. Currency exchange – injection of currency
5. Currency exchange – injection/deletion of currency

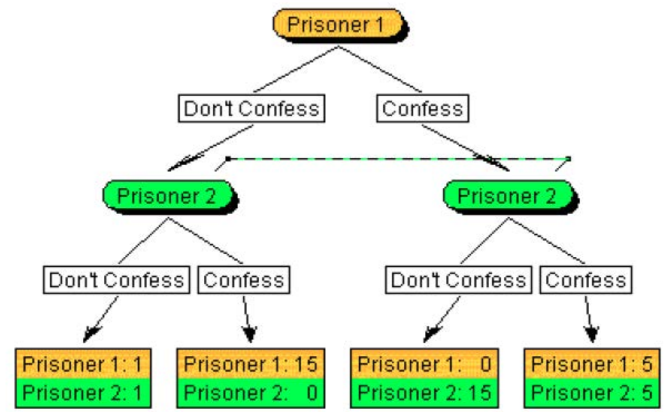
The Prisoners' Dilemma Game

Introduced by Albert W. Tucker Princeton University:
Tucker, A.W., Kuhn, H.W.: Contributions to the Theory of Games I. Annals of Mathematics Studies, no. 24. Princeton University Press, Princeton (1950)

- Two players, prisoners 1, 2.
- Each prisoner has two possible actions.
  - Prisoner 1: Don't Confess, Confess
  - Prisoner 2: Don't Confess, Confess
- Players choose actions simultaneously without knowing the action chosen by the other.
- Payoff consequences quantified in prison years.
- Fewer years=greater satisfaction=>higher payoff.
    - Prisoner 1 payoff first, followed by prisoner 2 payoff.

The Prisoners' Dilemma visualization

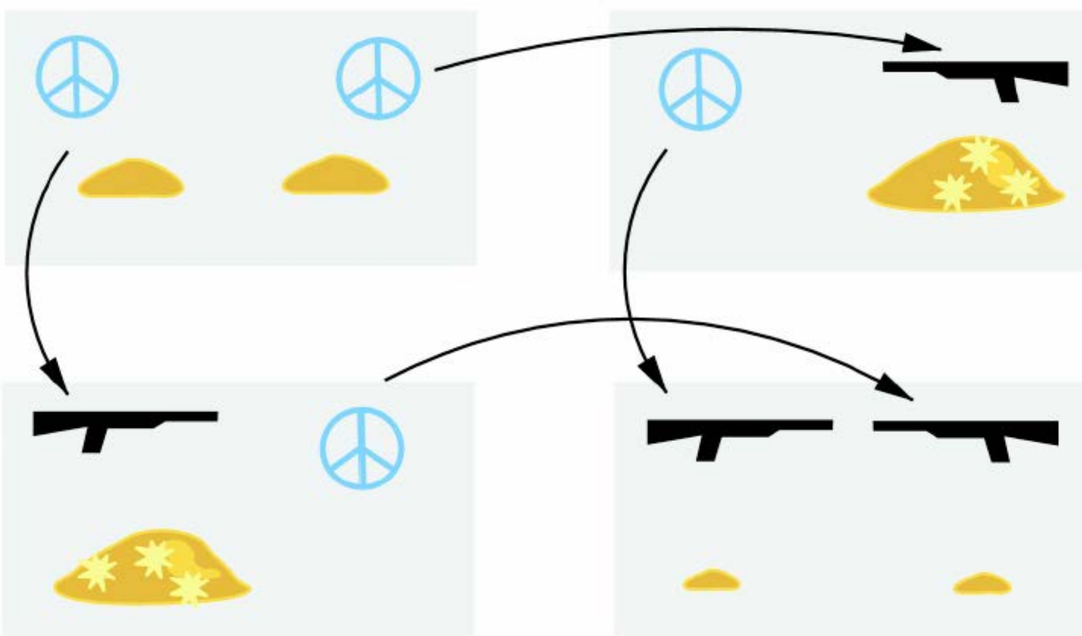




Prisoners' Dilemma is an example of a Non-Zero Sum Game, players interests are not always in direct conflict, so that there are opportunities for both to gain. For example, when both players choose Don't Confess in the Prisoners' Dilemma.

The Nash equilibrium for Prisoner's dilemma (1: defect/2:defect).

The Prisoners' Dilemma



win	lose much
win much	lose

Gain of the resource v,
Cost of injury c,
Assumption is that v>c

In Prisoner's dilemma it is always best to defect, no matter which strategy the coplayer will choose.

	2 : cooperate	2 : defect
1 : cooperate	v/2	0
1 : defect	v	(v-c)/2

The repeated Prisoner's Dilemma

	C	D
C	R	S
D	T	P

The game is a prisoner's Dilemma if T>R>P>S.
The temptation to defect, **T**, exceeds the reward for mutual cooperation, **R**, which is greater than the punishment, **P**, for mutual defection, which trumps the sucker's payoff, **S**.
It is assumed that **R>(T+P)/2**.

<u>Direct reciprocity</u>: the game is repeated several times between the same two players. Imagine the game is <u>repeated m times</u> and consider two strategies Always Defect(ALLD) and Tit-for-Tat (TFT)

	TFT	ALLD
TFT	mR	S+(m-1)P
ALLD	T+(m-1)P	mP

TFT can resist invasion by ALLD if m>(T-P)/(R-P).

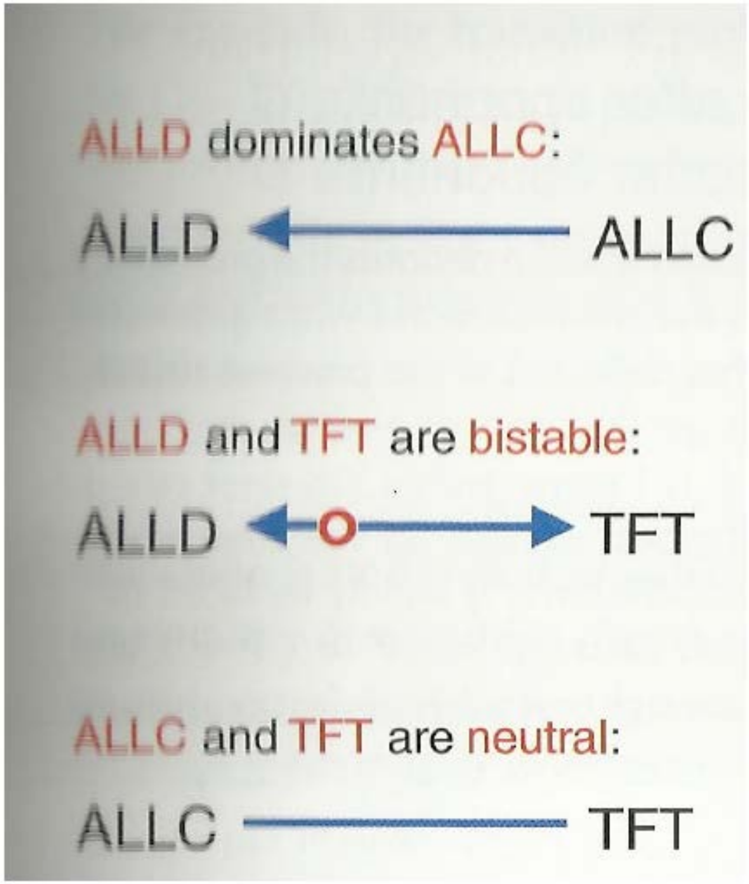
- TFT starts with cooperation and then does whatever the opponent did in previous round.
- Playing against TFT is like playing the mirror image of yourself shifted by one round.
- TFT was invented by Anatol Rapoport.

Strategies in the repeated Prisoner's Dilemma

ALLC: C C C C C C C . . .
ALLD: D D D D D D D . . .

ALLD: D D D D D D D . . .
TFT: C D D D D D D . . .

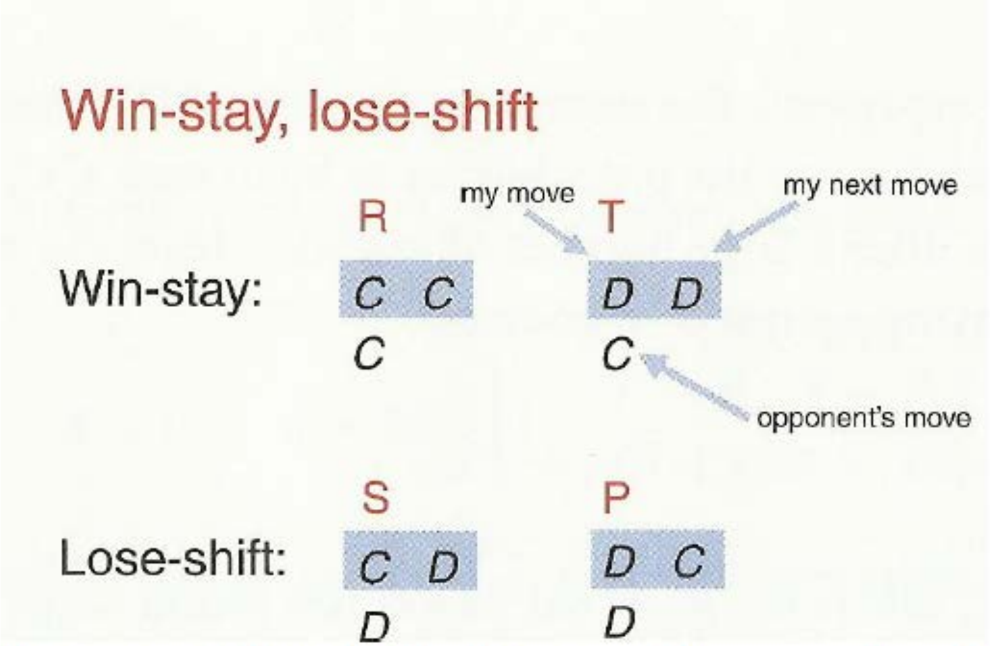
TFT: C C C C C C C . . .
TFT: C C C C C C C . . .



Notations for the strategies:
ALLC= always cooperate;
ALLD= always defect;
TFT= tit-for-tat.

In a game between ALLD and TFT, ALLD received a slightly higher payoff than TFT, but two TFT players receive a much higher payoff still.

Pavlovian agent: Win-stay, lose-shift strategy



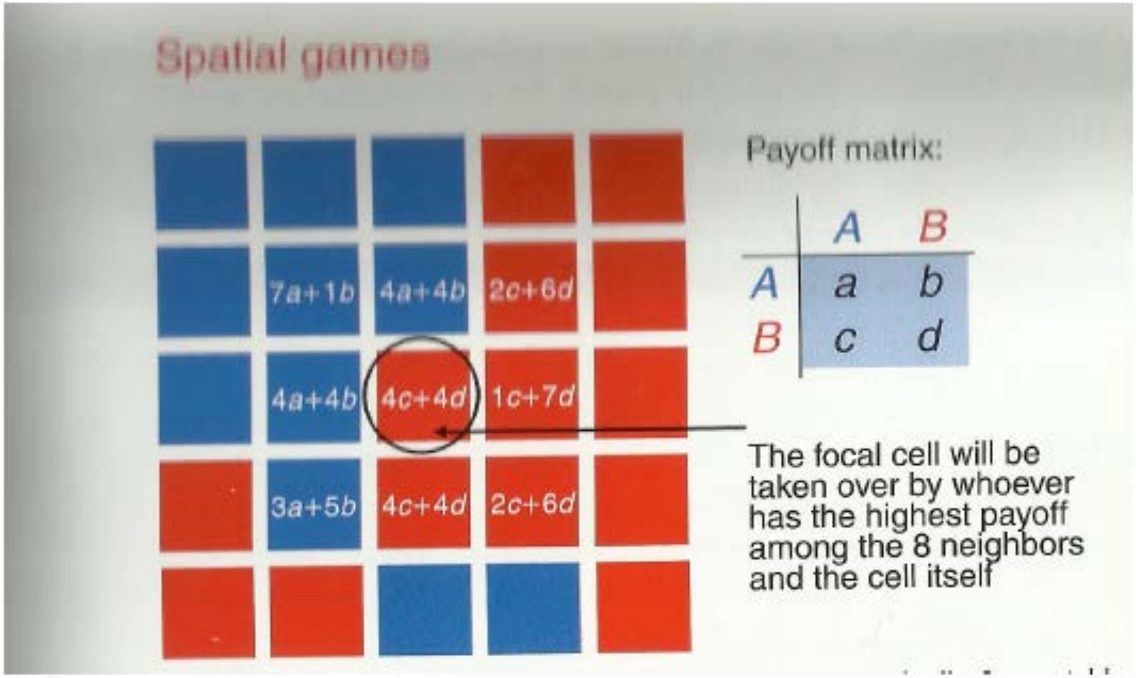
The payoff R is obtained for mutual cooperation, **CC**. In this case WSLS will cooperate again.
The larger payoff T is obtained for **DC**.
In this case WSLS will defect again.
The payoffs R and T are considered a "win" and therefore WSLS "stays" with its current move.
The payoff S is obtained for **CD**. In this case WSLS will shift from D to C.
The payoff P is obtained for mutual defection, in this case WSLS will shift from D to C.

- The strategy cooperates if the previous move was CC or DD and defects if the previous move was CD or DC.
- Note that the strategy repeats its previous move whenever it has received a high payoff, T or R.
- Thus the strategy follows the simple principle Win-stay, lose-shift (WSLS)

Win-stay lose-shift strategy is better then Tit-or-Tat because: TFT is weak in the presence of mistakes, mistakes imply that TFT can be
invaded and even dominated by many other strategies.

What are spatial games?

Spatial games arises from consideration of evolutionary game dynamics on spatial grids. The analysis brings together game theory and cellular automata.



A cooperator is someone who pays a cost, **c** , for another individual to receive a benefit, **b** .

Shown are the square lattice and the **Moore neighborhood**, where each cell has 8 neighbors. The fate of each cell depends on the state of all 25 cells in the 5 x5 square that is centered around.

Wolfram, S.: Computation Theory of Cellular Automata. Princeton University Press, Princeton (1984)

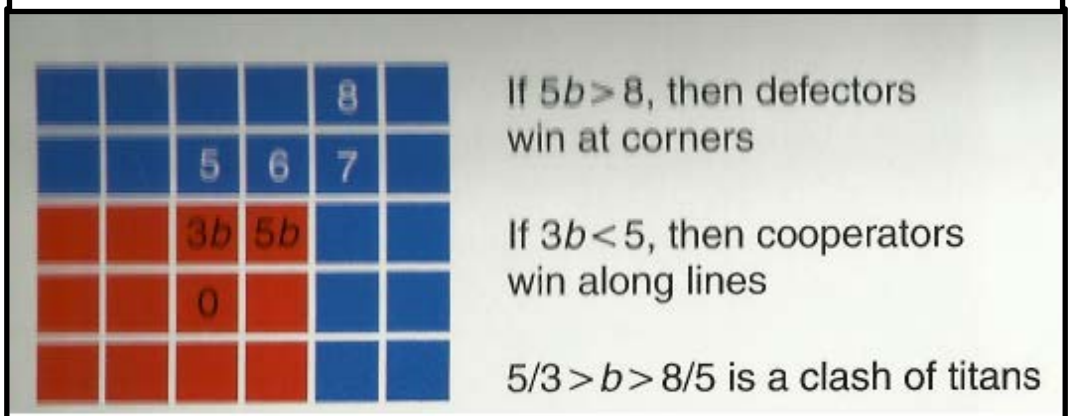
Spatial reciprocity

The spatial game between cooperators C and defectors D lead to a fascinating new mechanism for the evolution of cooperation "spatial reciprocity". The Prisoner's Dilemma payoff matrix is the following

$$\begin{pmatrix} & C & D \\ C & 1 & 0 \\ D & b & \epsilon \end{pmatrix} \qquad (1)$$

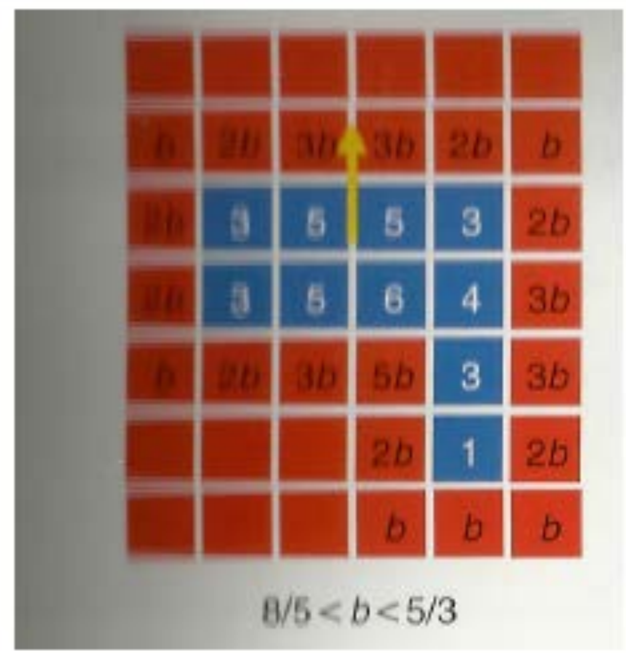
If two cooperators interact, both receive one point. If a defector meets a cooperator, the defector gets payoff $b > 1$, while the cooperator gets payoff zero. The interaction between two defectors leads to very small positive payoff ϵ. For the analysis of the game we consider variation on parameter b and set $\epsilon \to 0$.

The corner and line condition



A cooperator is someone who pays a cost, **c** , for another individual to receive a benefit, **b** .

A "walker" is a structure of 10 cooperators. It moves into direction indicated by the yellow arrow.

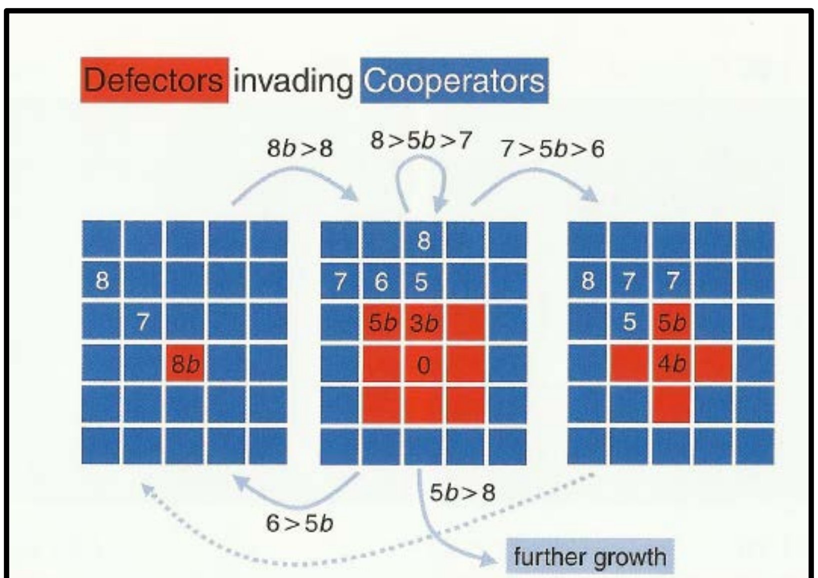


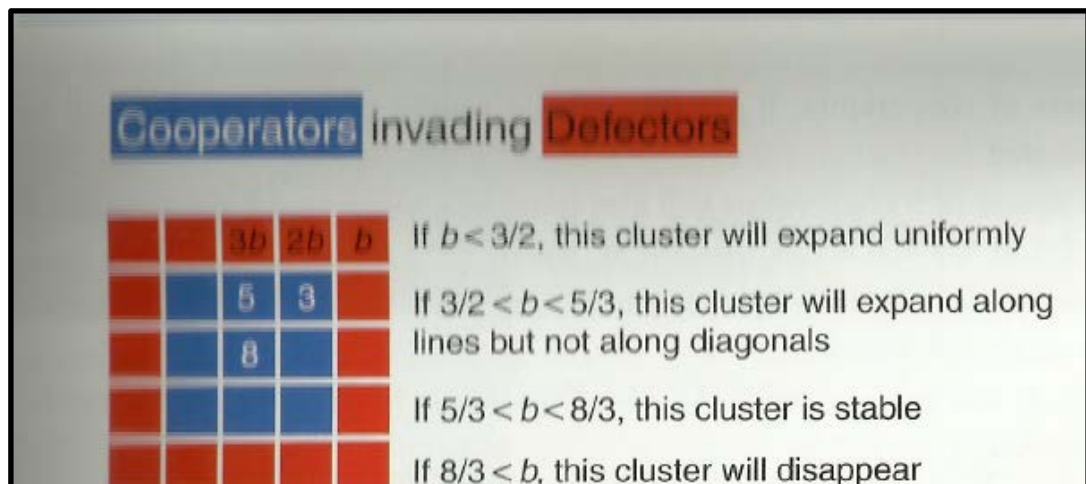
Companies and Strategies

Vincent van Gogh,
[The Potato Eaters,](#)
[1885, Van Gogh Museum](#)

- The companies can be **restaurants in the city of Porto** and their business is to sell **"francesinhas"**, the typical dish of this Portuguese city.

- The neighbors can be seen as the nearest restaurants that competes in the same physical area or district.





Spatial dynamics

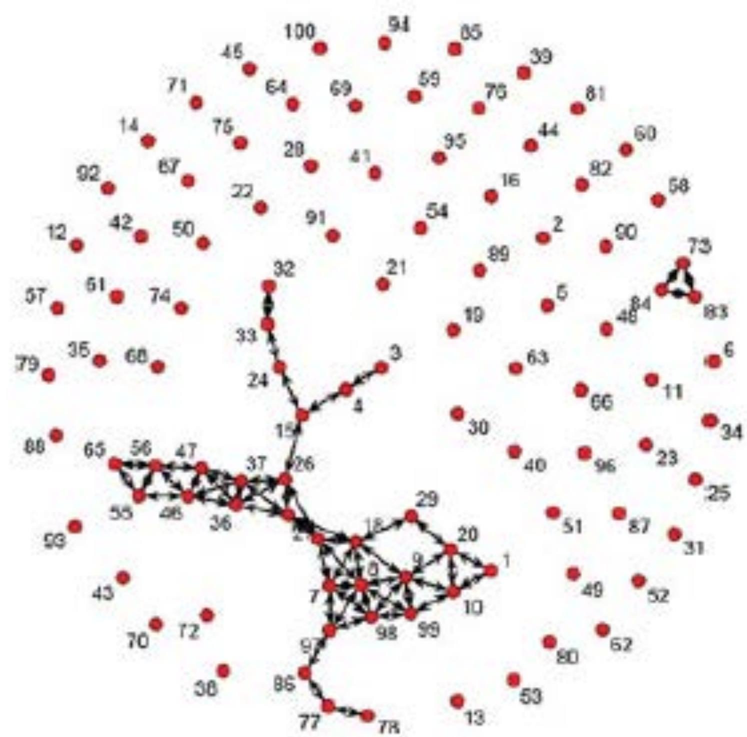
http://www.pnas.org/content/suppl/2009/04/27/0812644106.DCSupplemental/SM1.mov

Movie: Chaotic pattern formation in spatial ecological public goods. A sequence of snapshots demonstrates the spatial density distribution of **cooperators (green)** and **defectors (red)** over time.

- In social dilemmas individual selection favors defectors, but for the community, it is best if everybody cooperates.
- Benefits of the common resource enable cooperators to maintain higher population densities.
- There is a natural feedback between population dynamics and interaction group sizes as captured by ''**ecological public goods**.''

Reference: *Spatial dynamics of ecological public goods, J. Y. Wakanoa, M. A. Nowakb, and C. Hauert, PNAS, 2009, vol. 106, no.19, 7910–7914.*

Spatial Prisoner's Dilemma



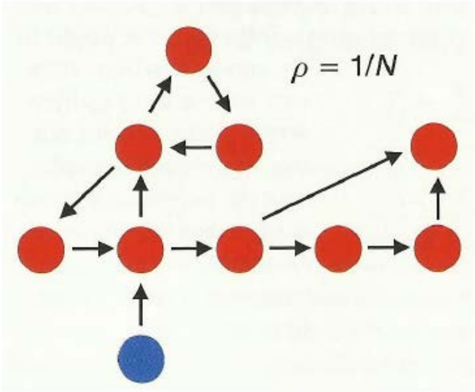
Graph plot for b= 1.5 – network composed by 35 Cooperate Agents (Strategy A) and 65 Defect Agents (Strategy B)

Innovation

Vincent van Gogh, *Irises, 1889, Getty Center, Los Angeles*

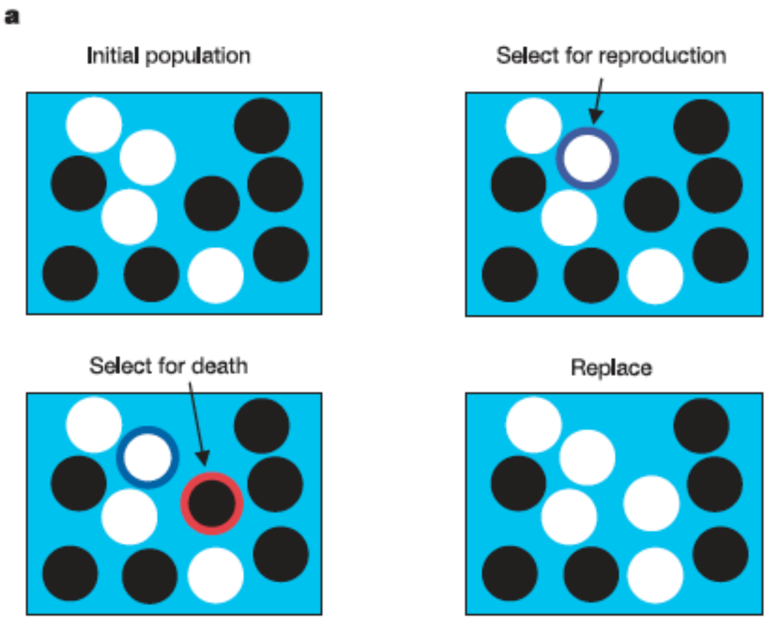
What is the probability that a single mutant generates a lineage that takes over the entire population?

Lieberman, E., Hauert, Ch. & Nowak, M. (2005) Evolutionary Dynamics on Graphs. Nature 433, 312-316.

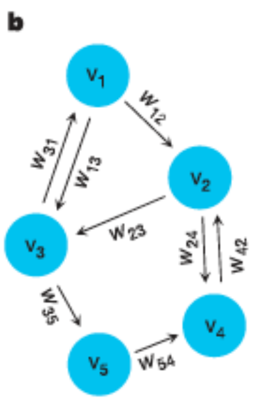


- Population structure can be generalized by arranging individuals on a graph. Each vertex represents an individual.
- The fitness of an individual denotes its reproductive rate which determines how often offspring is placed into adjacent vertices.
- A homogenous population corresponds to a fully connected graph and spatial structures are represented by lattices where each node is connected to its nearest neighbors.

Models of evolution



a. The Moran process describes stochastic evolution of a finite population of constant size.
b. The process is described by a stochastic matrix **W**, where $\mathbf{w_{ij}}$ denotes the probability that an offspring of individual **i** will replace individual **j**.
c. At each time step, an edge **ij** is selected with a probability proportional to its weight and the fitness of the individual at its tail.



$$W=\begin{bmatrix} 0 & w_{12} & w_{13} & 0 & 0 \\ 0 & 0 & w_{23} & w_{24} & 0 \\ w_{31} & 0 & 0 & 0 & w_{35} \\ 0 & w_{42} & 0 & 0 & 0 \\ 0 & 0 & 0 & w_{54} & 0 \end{bmatrix}$$

The fixation probability of the new mutant with relative fitness **r**, as compared to the residents, whose fitness is 1, is:

$$R_1 = (1-1/r)/(1-1/r^N).$$

Lieberman, E., Hauert, Ch. & Nowak, M. (2005) Evolutionary Dynamics on Graphs. Nature 433, 312-316.

Circulations and isothermal graphs

A circulation theorem

A graph is a circulation if for each vertex the sum of incoming weights equals the sum of outgoing weights.

$$\sum_{j=1}^{N} \omega_{kj} = \sum_{j=1}^{N} \omega_{jk}, \quad k = 1, 2, \ldots N.$$

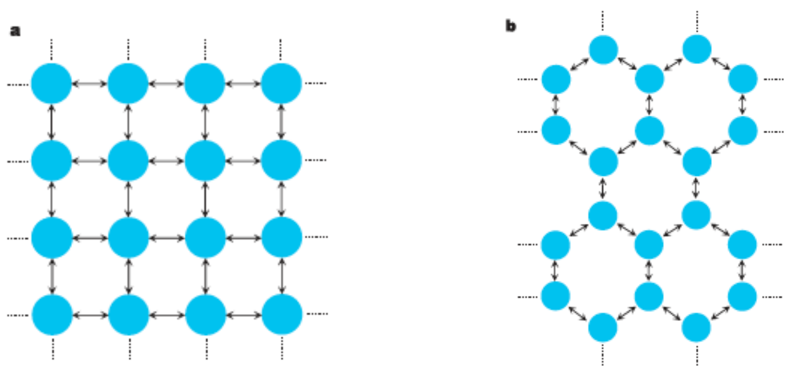
A graph has the same fixation behavior as the Moran process if and only if it is a circulation.

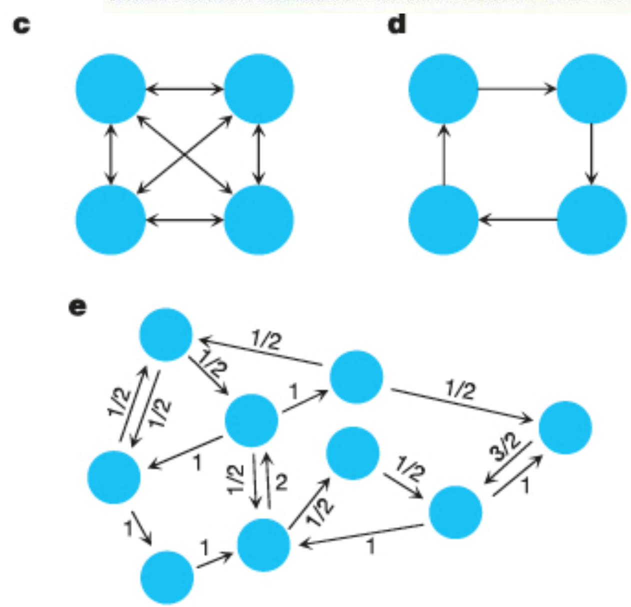
The isothermal theorem

The temperature of a vertex is the sum of all weights leading into that vertex

$$T_j = \sum_i w_{ij}$$

If all vertices have the same temperature, then the fixation probability is equivalent to the Moran process

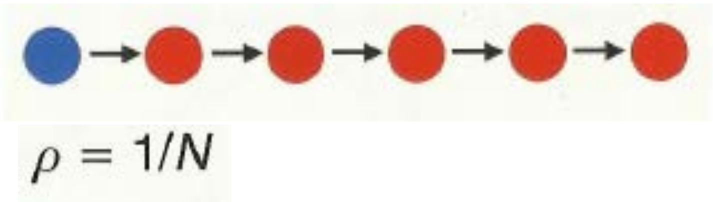




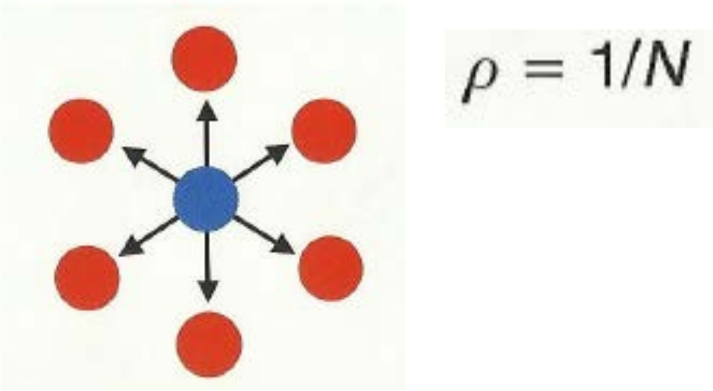
a the square lattice; **b** hexagonal lattice;
c complete graph; **d** directed cycle;
e irregular circulation.

Lieberman, E., Hauert, Ch. & Nowak, M. (2005) Evolutionary Dynamics on Graphs. Nature 433, 312-316.

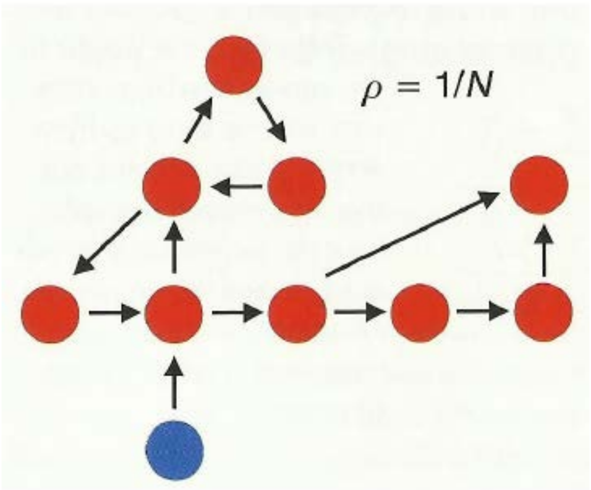
Graphs suppressors of selection



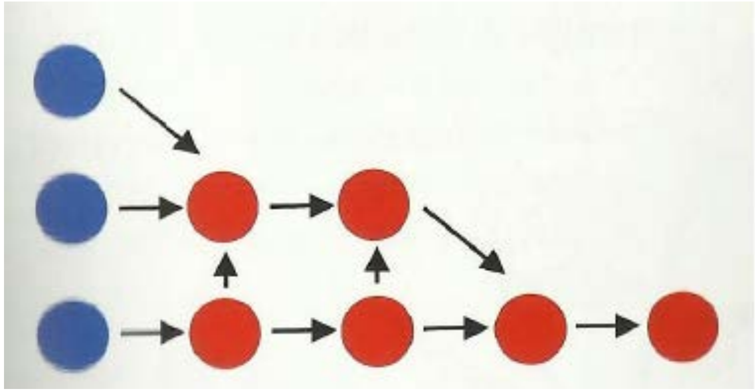
a. The line graph



b. The burst graph suppresses selection



c. The one-rooted graph suppresses selection, a root has zero temperature.

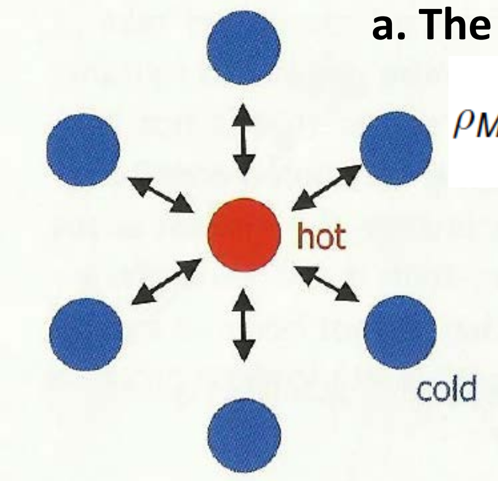


d. The multiple-rooted graph suppresses selection

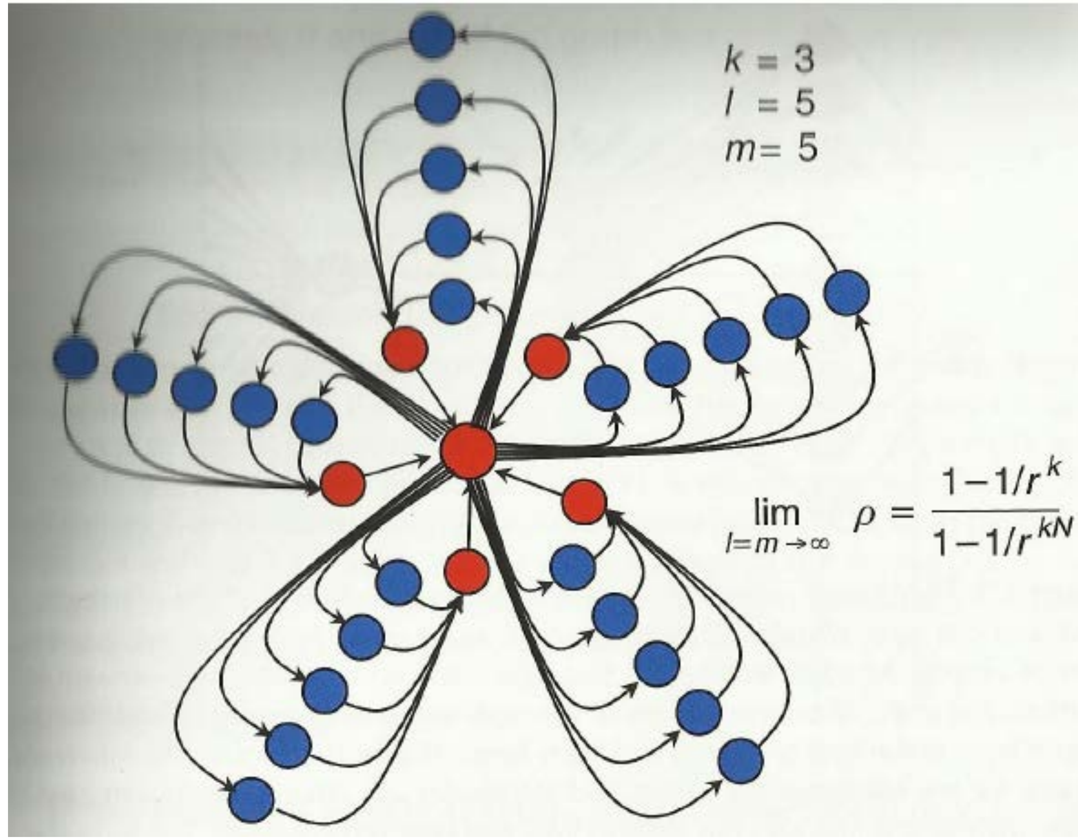
Lieberman, E., Hauert, Ch. & Nowak, M. (2005) Evolutionary Dynamics on Graphs. Nature 433, 312-316.

Graphs amplifiers of selection

a. The star graph

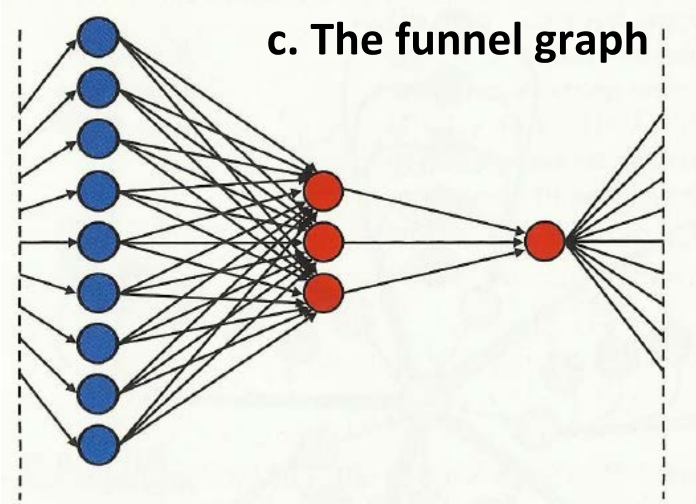


$$\rho_M = \frac{1 - 1/r^2}{1 - 1/r^{2N}}$$



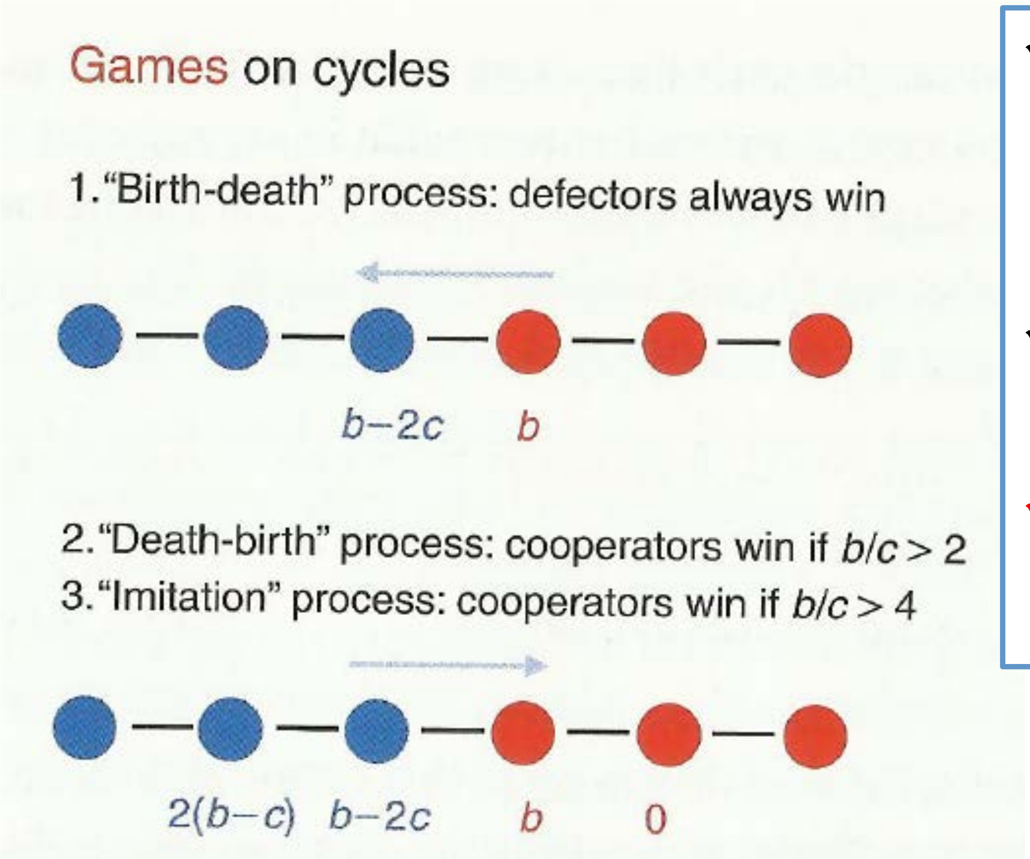
b. The superstar graph amplifies a selective difference r to **r**^{k} where **k** is the length of each loop in the graph.

c. The funnel graph



Lieberman, E., Hauert, Ch. & Nowak, M. (2005) Evolutionary Dynamics on Graphs. Nature 433, 312-316.

The evolution of cooperation on a one-dimensional graph cycle



- ✓ In the game between **cooperators (strategy A)** and **defectors (strategy B)** the replacement graph and the interaction graph are the same **H=G**.
- ✓ The **cooperator** pays a cost c, and each neighbor receives a benefit b.
- ✓ **Defectors** have no costs, but they can benefit by receiving help from adjacent cooperators.

In a regular graph of degree k each individual has exactly k neighbors (for a cycle k=2).

A simple rule for evolution of cooperation on graphs

For many graphs including cycles, spatial lattices, random regular graphs, random graphs and scale-free networks **natural selection favors cooperation if the benefit of the altruistic act b, divided by the cost c, exceeds the average number of neighbors k.** $b/c > k$

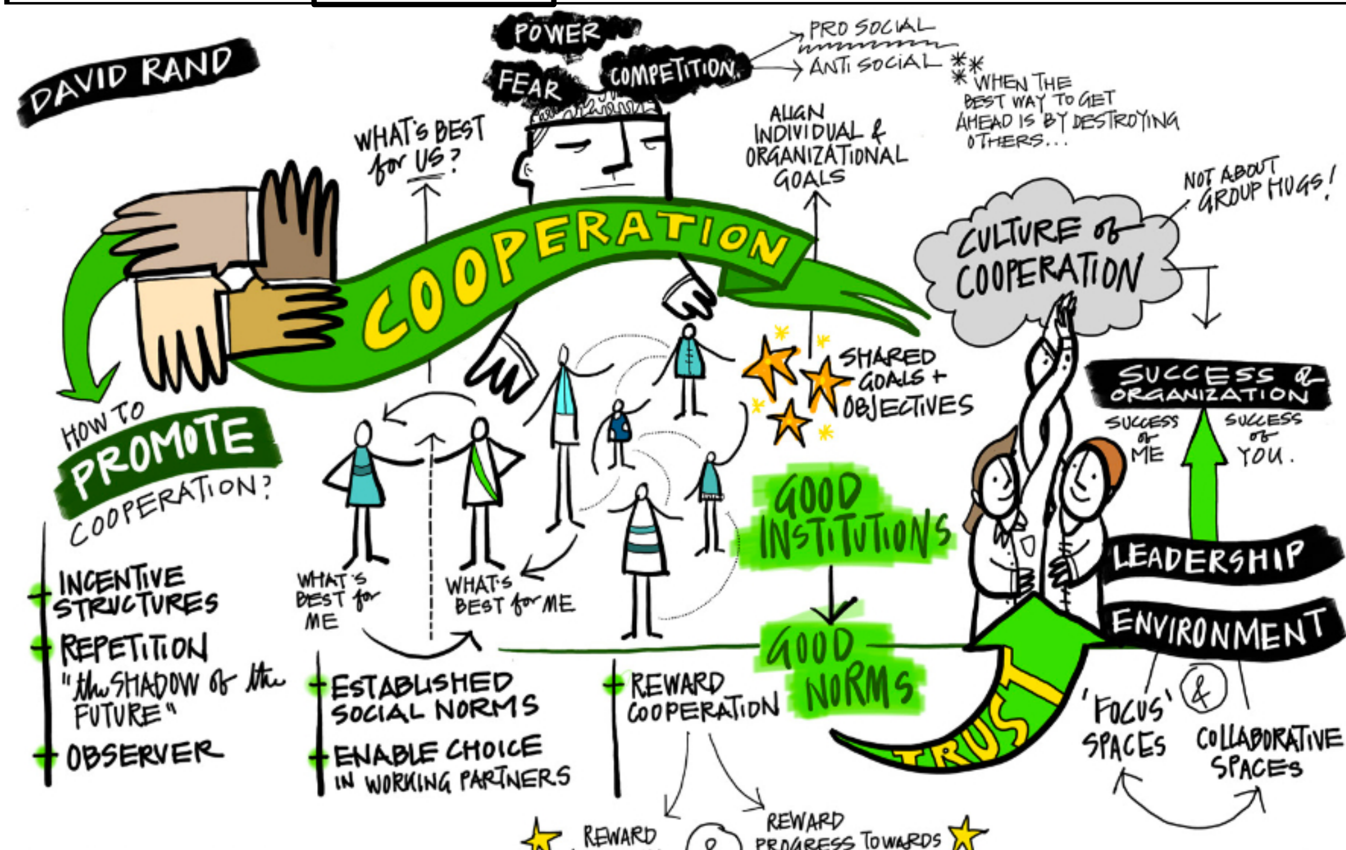


Image courtesy of the Learning Innovations Laboratory at the Harvard Graduate School of Education and Sita Magnuson, Dpict

1) A simple rule for the evolution of cooperation on graphs, H. Ohtsuki, C. Hauert, E. Lieberman, M. A. Nowak, Nature. 2006 May 25; 441(7092): 502–505.

2) Who cooperates in repeated games: The role of altruism, inequity aversion, and demographics, A. Dreber, D. Fudenberg, D. G. Rand, Journal of Economic Behavior & Organization. 2014 98, 41– 55.

The games on graphs

- **The general task** is to calculate the fixation probability of a certain **strategy A** , competing with another **strategy B**.

- **The interaction graph H**, determines who plays with whom.

- **The replacement graph G**, specifies the reproductive events: who learns from whom or who is replaced by whose offspring.

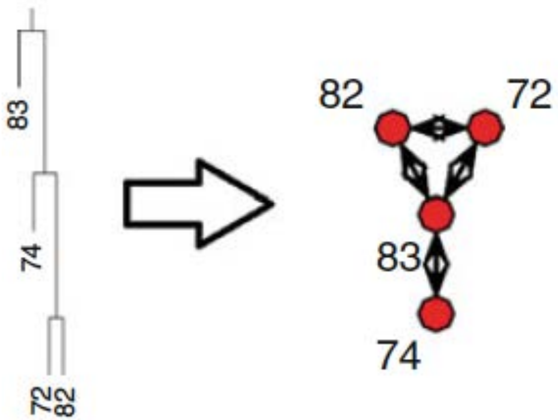
✓ For games on a **regular graph of degree k > 2** calculations for the game between **cooperators (C)** and **defectors(D)** can be done using **techniques of "pair-approximation"**: the average frequency of cooperators and defectors as well as the average frequency of all pairs, **CC**, **CD**, **DC** and **DD**.

✓ **A simple rule:** selection favors cooperation if the benefit-to-cost ratio exceeds the number of neighbors hold for random graphs and scale-free networks.

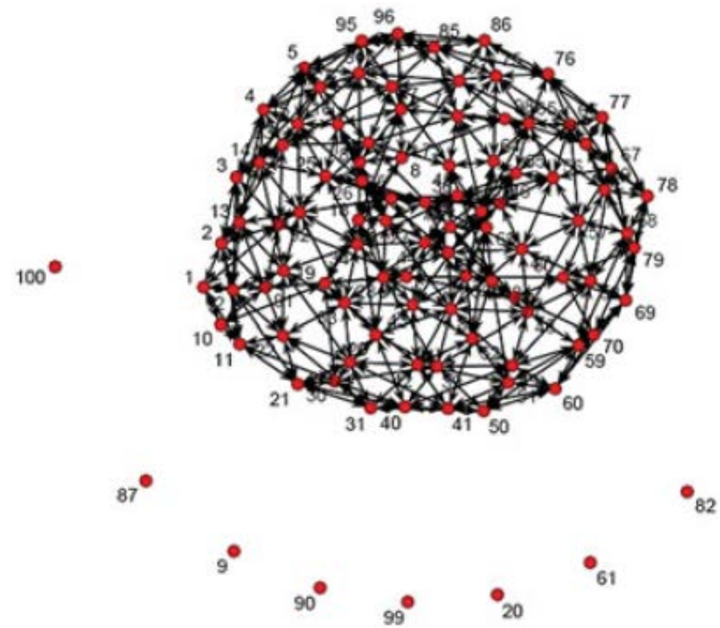
$$b/c > k \text{ and } b/c > k+2$$

A simple rule for the evolution of cooperation on graphs, H. Ohtsuki, C. Hauert, E. Lieberman, M. A. Nowak, Nature. 2006 May 25; 441(7092): 502–505

Demographic Prisoner’s Dilemma



1. Data example:
Cluster Dendrogram and Graph for corresponding cooperator cluster, following Innovation Strategy A.



2. Graph plot for b D 1:1 – network composed by 92 Cooperate Agents (Strategy A) and 8 Defect Agents (Strategy B).

Epstein, J.M.: Zones of cooperation in demographic prisoner’s dilemma. Complexity 4, 36–48 (1996).
Epstein, J.M.: Zones of cooperation in demographic prisoner’s dilemma. Working Papers, 97-12-094, Santa Fe Institute,(1997).

Social Dilemmas and Climate

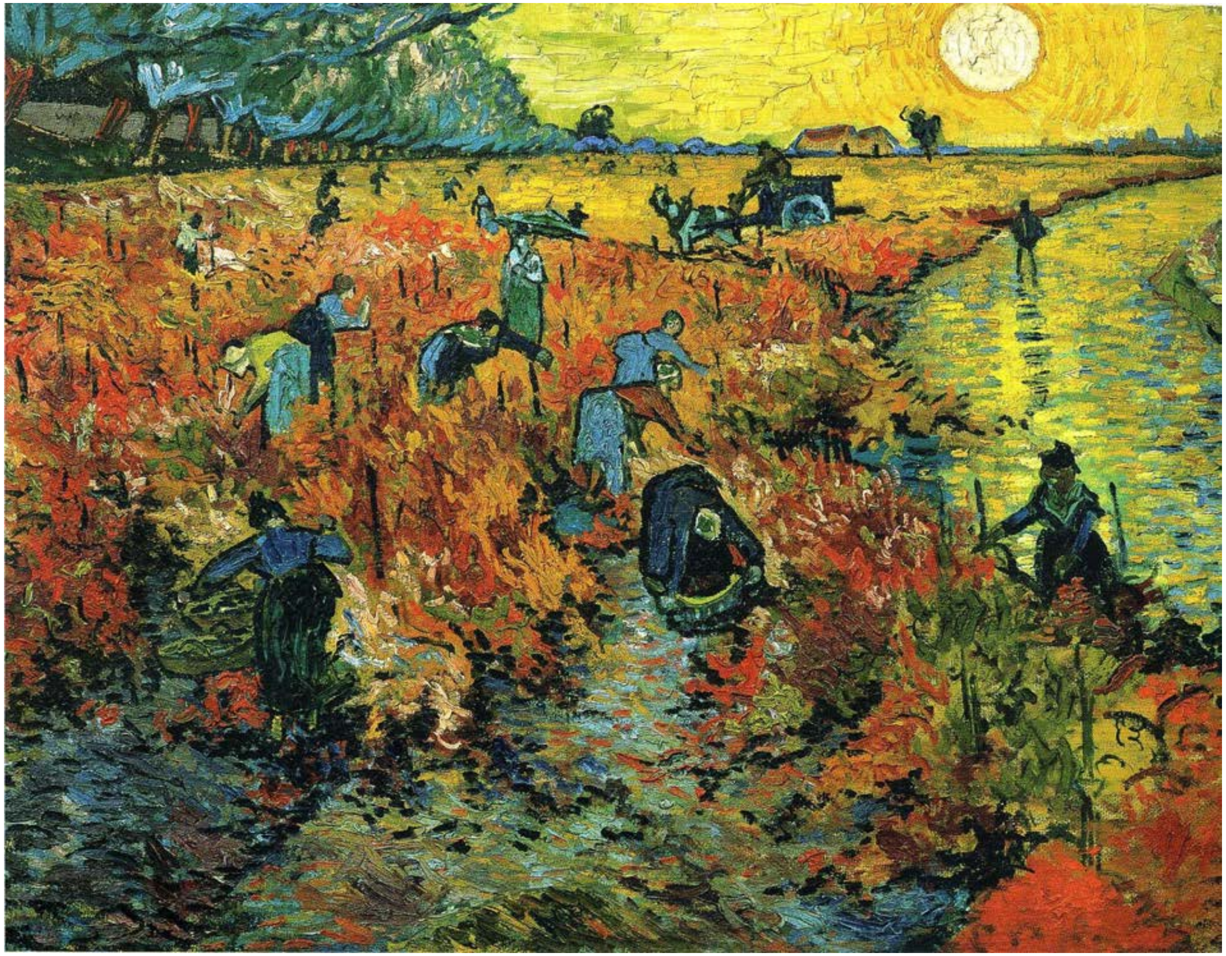
Vincent van Gogh,
The Starry Night,
June 1889,
The Museum of Modern Art,
New York

Intra- and intergenerational discounting in the climate game, J. Jacquet, K. Hagel, C. Hauert, J. Marotzke, T. Röhl, and M. Milinski,
Nature Climate Change, Vol.3, 2013.

- The production, consumption, and exploitation of common resources ranging from extracellular products in microorganisms to global issues of climate change refer to **public goods interactions**.

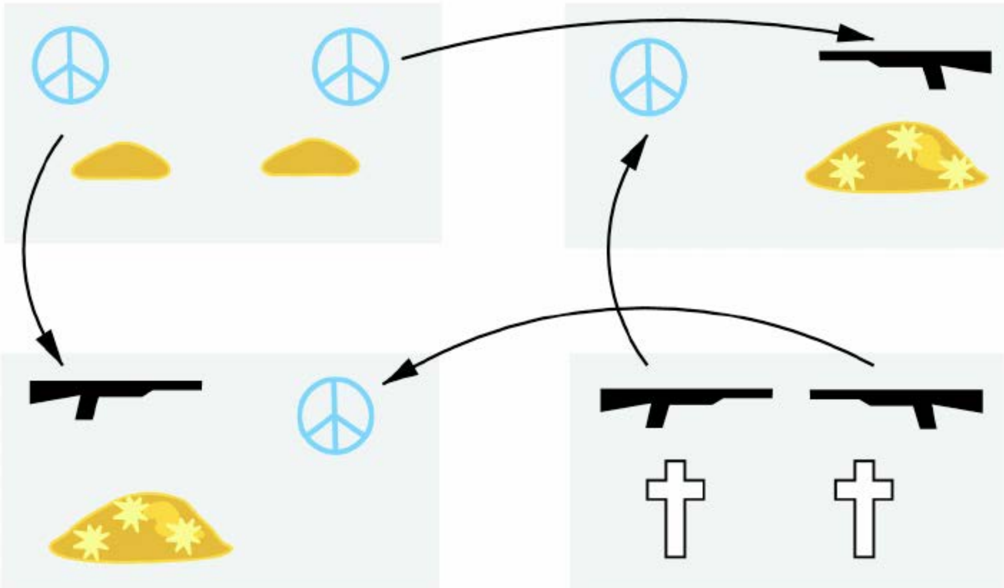
- This generates a **conflict of interest**, which characterizes **social dilemmas**: Individual selection favors defectors, but for the community, it is best if everybody cooperates.

Ideas for future development



Vincent van Gogh *The Red Vineyard, November 1888, Pushkin Museum, Moscow). Sold to Anna Boch, 1890*

Idea #1: Hawk and Dove Game



win	lose
win much	lose much

	2 : dove	2 : hawk
1 : dove	v/2	0
1: hawk	v	(v-c)/2

Gain of the resource v,
Cost of injury c,
Assumption is that c>v

Players compete for a common resource (for instance food) and can choose between two strategies termed “hawk” and “dove”.

Best choice: In Hawk-Dove game a player should always respond with the opposite strategy.

Invasion and expansion of cooperators in lattice populations: Prisoner’s dilemma vs snowdrift games, F. Fu, M. A. Nowak, C. Hauert, Journal of Theoretical Biology, 266 (2010), 358-366.

Idea #2: Empirical Games of Network Effects

Dynamic Games of Network Effects,

Filomena Garcia(Lisbon, Portugal) and Joana Resende (Porto, Portugal).

http://link.springer.com/chapter/10.1007/978-3-642-14788-3_25

Chapter in the book by M.M. Peixoto et al. (eds.), Dynamics, Games and Science II, Springer Proceedings in Mathematics 2, DOI 10.1007/978-3-642-14788-3 25.

The strategic complementarity between consumers' actions has several implications on the behavior of firms. For instance, firms need to gain advantage from early marketing stages. **Main results on pricing and evolution of market shares are exposed**.

- **Result #1** gives general formulations for the innovation of network effects in a dynamic setting.
- **Result #2** gives recent developments in the literature on firms' strategies in the context of dynamic network effects.

Idea #3: Stochastic Games

- ✓ Fudenberg D, Yamamoto Y. The Folk Theorem for Irreducible Stochastic Games with Imperfect Public Monitoring. Journal of Economic Theory. 2011;146:1664-1683.

- ✓ Fudenberg D, Yamamoto Y. Learning from Private Information in Noisy Repeated Games. Journal of Economic Theory. 2011;146:1733-1769.

- ✓ Fudenberg D, Yamamoto Y. Repeated Games Where the Payoffs and Monitoring Structure Are Unknown. Econometrica. 2010;(78): 1673-1710.

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Summary

Vincent van Gogh
Still Life: Vase with Twelve Sunflowers, August 1888, Neue Pinakothek, Munich

- ✓ Game theory to reason about situations with multiple decision-makers.
- ✓ Empirical game theory to conduct experiments.
- ✓ Spatial Prisoner's Dilemma to analyze evolution of cooperation using games on graphs.
- ✓ Selection favors cooperation if the benefit-to-cost ratio exceeds the number of neighbors.
- ✓ Online resources:

www.comlabgames.com

http://www.powertac.org
http://tradingagents.org/
http://tac.sics.se/page.php?id=1

http://library.duke.edu/rubenstein/collections/economists